Hydraulics of Pipeline Systems

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Each of the three authors has had a long and continuing involvement with some, indeed most, of the material in this book in one way or another since the time they first became acquainted with each other as graduate students at Stanford University in the 1960s. At that time the teaching and enthusiasm of John K. Vennard for the subject of flow in pipes made a positive impression on us, his then-young students. In the years since then each of us has taught portions of this material to students in university courses, or in short courses, or in both. This book is a long-delayed product of that early enthusiasm, now tempered by years of experience and aided by advances in knowledge about hydraulics and numerical methods and advances in computer hardware. We acknowledge here his influence on our lives and on this book.

We thank Nicole Newman for her skill in preparing, and sometimes repairing, almost three hundred figures for this book, and we particularly appreciate her patience and tolerance as the authors continued to revise some of them even after they were drawn. Her careful attention to detail has resulted in a better book.

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Bruce E. Larock
Roland W. Jeppson
Gary Z. Watters

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CHAPTER 1

INTRODUCTION

Pipeline systems range from the very simple ones to very large and quite complex ones. They may be as uncomplicated as a single pipe conveying water from one reservoir to another or they may be as elaborate as an interconnected set of water distribution networks for a major metropolitan area. Individual pipelines may contain any of several kinds of pumps at one end or at an interior point; they may deliver water to or from storage tanks. A system may consist of a number of sub-networks separated by differing energy lines or pressure levels that serve neighborhoods at different elevations, and some of these may have pressurized tanks so that pumps need not operate continuously. So these conveyance systems will adequately fulfill their intended functions, they may require the inclusion of pressure reducing or pressure sustaining valves. To protect the physical integrity of a pipeline system, there may be a need to install surge control devices, such as surge relief valves, surge tanks, or air-vacuum valves, at various points in the system.

How do these systems work? What principles are involved, and how are the systems successfully analyzed and understood? How can the behavior of a preliminary design be evaluated, and how can the design be modified to correct deficiencies? These are some, of many, questions that immediately confront any engineer who is involved in creating the physical infrastructure to satisfy a basic need of mankind: the delivery of water when and where it is wanted at a price that is affordable. It is the primary objective of these engineers to develop and apply their knowledge to make the system work. Success at this task first requires an adequate knowledge of some fundamental principles of fluid mechanics. Some experience with the solution of hydraulic flow problems is certainly desirable, and it will come with time and effort. These days an understanding of some particular numerical methods and the ability to implement them on a computer, sometimes for the solution of very large problems, is also a vitally needed skill. Computations associated with engineering practice have changed dramatically in the past quarter century from the estimation of a few key values by using a slide rule to the generation of pages of computer output that are the result of detailed simulations of system performance in response to various alternative designs, so that the consequences of various ideas can be ascertained quantitatively. The volume of computer output can overwhelm one's ability to glean the most pertinent information from the numbers. The purpose of this book is to empower the reader with the knowledge, experience, and tools to accomplish this objective.

This book will present to the reader a comprehensive and yet relatively practical study of pipeline hydraulics, with a substantial component being the use of computers for detailed computations that are not practical to perform by hand. The intent of the authors was to create a book, and an accompanying CD, that will serve well any of the following roles: (1) as a text for senior-level courses for BS students electing to specialize in fluid mechanics, hydraulics, water supply and distribution, and/or water resources; (2) as a text for graduate engineering courses in the same subject areas; (3) to provide instructional material for professional practicing engineers who wish to update their knowledge of specialties associated with the distribution, conveyance, and control of fluids in pipelines; (4) to provide resource material for engineers in governmental agencies at all levels who have responsibilities to design and/or approve plans for pipeline systems; and (5) to provide reference material for consultants who are asked to solve problems, review plans, or suggest project alternatives in the subject areas of this book.

The study of the hydraulics of pipeline systems builds on a small number of fundamental principles that are found in a first course on fluid mechanics; such a course is normally taken in the third full year of college study by all students in civil and environmental engineering, mechanical engineering, agricultural and irrigation engineering, and in some related engineering fields. Ideally this course is a judicious mix of the development of basic theory and its application, but it is not uncommon for such a course to emphasize theory over practice or *vice versa*. The authors will assume that readers have already acquired some knowledge of fundamental fluid mechanics principles; it is hoped that they also have in their individual libraries an elementary text on fluid mechanics that can be a resource for (1) refreshing their understanding of the basic concepts and (2) finding an occasional supplementary equation when it is needed to enhance the understanding and application of the developments in this book. Such a reference will also be useful as a source of data on fluid properties.

To establish a base on which to build in subsequent chapters, the authors begin in Chapter 2 with a review of elements of basic fluid mechanics that are pertinent to pipe system hydraulics. Because pumps are such a common part of (especially the larger) pipeline systems, Chapter 2 includes a short primer on pump behavior and the summary of such behavior by pump characteristic curves. Chapter 2 concludes with several basic flow examples that are much like those that are usually found in a first course. The remainder of the book then addresses three general categories of pipeline system analysis. The first category, examined in Chapter 3, considers pipe manifolds, relatively the least complex type of pipe system. Although any pipe manifold is basically a relatively large pipe which delivers fluid to many outflow points or ports, it is an example of a spatially varied flow; such flows are often not studied in undergraduate books on fluid mechanics, so some care is needed to avoid conceptual errors. A single manifold pipe is examined at several levels of completeness, and the chapter ends with a design example and some comments about developing a manifold design with the aid of a computer program.

The second category is steady-state pipe network analysis. The largest single segment of the book is devoted to this topic. Relative to the coverage of this topic in other books, the exploration of the topic here is both broad and thorough (or, as some say, 'in-depth'). (Even so, much that is known about optimal design technniques could not be included here, owing to limitations on the size and cost of this book!) The study of networks progresses from the simple to the complex. The simple networks are used to emphasize the principles, and the larger networks allow one to experience a taste of the real world and to learn to cope with additional complexity. Enough details of the numerical and programming techniques are presented so the reader can see how the entire analysis works. Chapter 4 concentrates on analysis techniques and completely describes the three primary alternative approaches to the formulation of a mathematical model for a pipeline system; then a method for solving each of them is presented. The primary elements of each solution method, in this and subsequent chapters, are implemented in Fortran and C programs that are contained in the CD that accompanies this book. The logic that is required to integrate the relatively complicated pressure reducing and back pressure valves into a system is carefully described. Chapter 5 goes on to describe effective approaches to the design of pipe networks; the first objective of most pipeline system designs is to determine the smallest acceptable, and commercially available, pipe diameters to fulfill specified delivery requirements, and in this chapter one finds out how to formulate a problem with some of the pipe diameters as unknown variables. This approach is in distinct contrast to the usual design approach of initially estimating (guessing?) all of the pipe sizes, conducting an analysis of the resulting network, and then iteratively adjusting the sizes until a satisfactory design is found. Methods will be described that allow one to decide rationally which component(s) of a large network should be altered to eliminate most effectively a deficiency in the network's performance; this decision process is based on the quantification of the sensitivities of dependent variables to independent variables. For example, the pumping station (with power as the independent variable) that produces the largest sensitivity to pressure at the node with the lowest pressure (the dependent variable) should be enlarged to eliminate a problem involving excessively low pressures. Finally, in Chapter 6 the reader is introduced to extended time simulations and additional economic considerations in network design.

The last of the major topics in this book is the analysis of several types of transient flow in pipelines and in networks. These chapters begin with a relatively brief section in Chapter 7 on slowly-varying flows that can be called quasi-steady. Chapter 7 then goes on to introduce two types of true transients, those in which only inertial effects are important and those for which the additional consideration of the elasticity of both pipe and fluid is essential to capture the true behavior of these flows. In Chapters 8 through 13 various transient flows in systems that range from single pipelines to entire pipeline networks are examined, as well as procedures and devices for controlling these transients.

Even if it is not already clear to the reader at the outset, it will become clear during the reading and study of this book that the solution of pipeline hydraulics problems, especially as the systems become larger, can require substantial computational effort. The routine computation of solutions to larger problems in either networks or transients can involve the heavy use of a modern desktop computer or a workstation. This type of computation, which normally requires the solution of either a moderate to large set of initially nonlinear algebraic equations or one to many differential equations, depends heavily on the use of reliable and reasonably efficient methods from numerical analysis, a branch of applied mathematics that also has some input from computer science.

In the steady-state analysis and design of networks, large systems of nonlinear algebraic equations must be solved; this book will emphasize the relatively reliable Newton method for the solution of these equation sets. The inclusion of inertia in unsteady flows will require us to solve a system, which can become very large for networks, of differential and algebraic equations, also called DAE's. Although research papers on the solution of DAE's began to appear in the 1980s, relatively little of this subject appears to have been previously applied to pipeline hydraulics problems, so far as the authors can tell, even though there are many applications in engineering practice in which such combined systems of equations govern. The presentation of a technique for the solution of these systems of equations is one of the contributions of this book. As the future requires more sophisticated simulations of engineering problems, similar solution techniques will become commonplace.

An exposition on the hydraulics of pipeline systems can approach this topic in any of several ways, ranging from one extreme where only hydraulic theory and the accompanying descriptive mathematical equation sets are presented, to the other extreme where an array of problem descriptions, computer files and fill-in-the-data sets of instructions for the use of computer programs is presented. In the authors' opinions neither extreme is deserving of commendation. But it is also understood by the authors that individual readers will have goals that do not agree entirely with those of either the authors or other readers of this book. After some deliberation the authors have chosen an intermediate approach to the subject. The first step in each major topic is to present the governing principles and their expression in mathematical equations. The examples of the application of the principles will usually progress from the smaller and simpler to the larger, more realistic and more difficult, both in the text and in the problem sections at the end of most chapters. Most of the numerical and procedural detail of problem solving will be examined when the smaller problems are discussed. Some readers may desire to know more in the way of details in the numerical analysis and/or the computer coding than is presented in the body of the text. To some extent this outcome is an unavoidable consequence of the authors' choice to take the intermediate approach, but those who desire more details on the numerical techniques and the actual computer programming can learn more! Appendix A presents a primer on some numerical analysis techniques. We also encourage readers to extract the source code of computer programs from the CD to list them, to study them, and to use them to solve a variety of problems. The CD contains approximately 250 separate files (not including the executable elements); seven files are document files to explain the use the major executable programs such as USU-NETWK which are on the CD. With few exceptions the source programs are provided in both Fortran and C. The CD contains slightly under one hundred Fortran source programs, ranging in size from less than a page (when listed) up to several pages. Among these are subroutines (also written as C functions) to perform numerical solutions of single, or systems of, ordinary differential equations or tasks such as cubic spline interpolations. In solving many of the problems at the ends of the chapters the reader will find it advantageous to use the vast additional resources on the CD. An INSTALL program on the CD permits the user to extract and decompress the files on the CD by type, or to make individual or group selections.

While this book has been written primarily to describe the hydraulics of pipeline systems, an important secondary objective is to describe with care, and to present examples of the application of, some reliable numerical methods for the solution of the larger, more complex problems that the practicing engineer encounters. Although the examples herein are all pipeline problems, the numerical methods themselves have potentially a far wider range of applications to any topic that can be modeled with similar sets of equations. Engineering colleges everywhere have for many years been debating the relative merits of teaching to their students a procedural programming language such as Fortran, C, or Pascal, vs. the teaching of the use of spreadsheets and interpretative languages as implemented in MathCAD, TK-Solver, or Mathematica. The authors' opinion, formed by observing many students during their university years and after graduation, is that computer programming is a very important, if not a vital, skill today when computers have become an integral part of our professional and personal lives. Individuals who can effectively use a procedural programming language seem to assimilate the use of application software packages more readily than those whose university experience was only with application packages. Consequently the authors conclude that there is much merit is learning how to program effectively not only to complete a task but also because programming requires a concise and correct application of fundamental principles, and the experience enhances an understanding of these principles even more than the solution of small problems that can be done by hand. But if a programming language is to be employed in this book, which language is *the* language? With the years, more and more languages appear, in some respects like the seasons. For example, depending on one's year of birth, the readers and the authors have seen one to several generations of Basic and Fortran, and then Pascal, and more recently C and C++, Java and still other languages appear, each with its own special attributes. How do the authors create a text that addresses the issues without forcing literacy in a particular programming language on the reader? (The answer probably is, with some difficulty, but the authors have tried.) The 'solution' follows in the next paragraph.

The authors have started from the premise that nearly all readers of this book will have some knowledge of computing methods. The authors have also assumed that many readers will be familiar with either Fortran or C as a programming language; however, it is also assumed that not all readers will have this background. Hence, included on the CD are executable program elements which can be used directly, without compilation, for the solution of some but not all of the problems in this book. In addition, the CD contains a few TK-Solver models; they are included because they present equations and the selection of dependent variables in a clear way. It was tempting to include not only more TK-Solver models but also MathCAD models in the text, until it was realized that page limitations would not permit more. It would be a valuable experience for readers to develop their own TK-Solver, MathCAD, Mathematica, spread-sheet, or other software models with interpretative capabilities to solve some of the example problems and problems at the ends of the chapters. The source programs have already been mentioned; of course, each of them may serve as a base from which the reader may create new, specialized programs for their own individual purposes. Any modification of a program will, of course, require its recompilation which, in turn, requires access to the appropriate

Pcompiler. As a reminder to the reader that these programs, which the authors believe are correct, are nevertheless provided as a service to the readers without a guarantee, some of the text programs explicitly contain the following caution:

```
*
*    THIS PROGRAM HAS BEEN INCLUDED FOR THE CONVENIENCE OF THE READER.
*    THE AUTHOR ACCEPTS NO RESPONSIBILITY FOR ITS CORRECTNESS.
*    USERS OF THIS PROGRAM DO SO AT THEIR OWN RISK.
*
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The authors intend that the reader understand that this caution applies to all of the codes in this book and on the CD, although the caution is not repeated on every file.

The authors are confident that the reader will find the many applications of the basic principles of hydraulics to a wide range of practical problems to be challenging, yet manageable, and useful in either advanced education or professional practice. The authors further hope that the considerable number and range of applicability of the computer programs will provide the user with the tools to analyze a wide range of pipeline systems.

CHAPTER 2

REVIEW OF FUNDAMENTALS

This chapter will review the fundamental concepts and principles upon which the hydraulics of pipeline systems is based. The review is intended to be sufficiently complete that readers who have taken a good first course in elementary fluid mechanics, but not necessarily recently, will be reminded of, and updated in, the essential conceptual building blocks that are the foundation of the material in this book. We will begin with an introduction to the fundamental equations that are the foundation of most of the subsequent developments in the book. Because the concept of the energy grade line (EGL or simply EL) and the hydraulic grade line (HGL) is so useful, we shall look at this idea separately. Next we look at some length at various head loss formulas. How turbomachines with rotating impellers, particularly pumps, function is vitally important to the understanding of many parts of this book, so their theory of operation and basic characteristics will be examined. The chapter will conclude with several steady-flow examples and a range of problems that will allow readers to test their readiness for the coming chapters. If a thorough review is desired, one might consult Miller (1984).

2.1 THE FUNDAMENTAL PRINCIPLES

2.1.1. THE BASIC EQUATIONS

Conservation of mass is the most basic principle. In general, the fluid density ρ may vary in response to changes in the fluid temperature and/or pressure. For a fixed control volume $\forall$ enclosed by a surface S, a general statement of mass conservation is

$$\frac{\partial}{\partial t}\int_{\forall} \rho\, d\forall + \int_{S} \rho \vec{v} \cdot \vec{n}\, dS = 0 \tag{2.1}$$

in which $\vec{v}$ is a velocity at a point and $\vec{n}$ is an outer normal unit vector to the surface S, and t is time. The first term represents the accumulation of mass over time in the control volume; for steady flows it is zero. At a surface point the dot product $\vec{v} \cdot \vec{n}$ gives the component of the velocity which crosses the surface, so the second term computes the net outflow of fluid across the entire control surface. For steady incompressible flow of a liquid in a pipe, the conservation of mass is generally referred to as the continuity principle, or simply **continuity**, and it is written

$$Q = \int_{A} v\, dA = V_1 A_1 = V_2 A_2 \tag{2.2}$$

in which Q is the volumetric discharge through a pipe cross section, which can also be written as the product of the mean velocity V and cross-sectional area A of the pipe.

The second, equally important, principle is the **work-energy** principle, sometimes called simply the energy principle. Some also call it the Bernoulli equation, but in general it is distinctly more than that. For the steady one-dimensional flow of a liquid in a pipe, per unit weight of fluid, the principle can be written between two sections or stations as

$$\frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 = \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2 + \sum h_{L_{1-2}} - h_m \tag{2.3}$$

In this equation $V^2/2g$ is the velocity head or kinetic energy, p/γ is the pressure head or flow work, and z elevation head or potential energy, all per unit weight. If the last two terms on the right were absent, the equation would be the classical Bernoulli equation. The last two terms, however, are extremely important in the study of the hydraulics of pipe lines. The head loss term, or the accumulated energy loss per unit weight, Σh_L, is the sum, between sections 1 and 2, of the individual head losses in the reach caused by frictional effects. The last term, h_m, is the mechanical energy per unit weight added to the flow by hydraulic machinery. A pump adds energy to the flow so h_m is then positive and called h_p; a turbine extracts energy from the flow so h_m would then be negative and called h_t.

Fluid power, sometimes denoted by P, is the product of the energy gain or loss per unit weight h_m and the weight rate of flow $Q\gamma$, or $P = Q\gamma h_m$. A unit conversion factor can be applied to this result to express the power in, say, horsepower or kilowatts. Depending on the purpose of the computation, an efficiency factor η may be used as a multiplier or divisor of the power.

The last of the major principles considers **linear momentum**, which is governed by the impulse-momentum equation

$$\frac{\partial}{\partial t}\int_{\forall} \rho\vec{v}\, d\forall + \int_S \vec{v}(\rho\vec{v}\cdot\vec{n})\, dS = \vec{F}_{net} = \vec{F}_s + \vec{F}_b \tag{2.4}$$

in which the net force on the contents of the control volume, fluid and solid, which can be divided into surface forces and body forces, is equal to the rate of accumulation of momentum within the control volume plus the net flux of momentum through the surface of the control volume. In a steady flow the first term is again zero. For steady, incompressible, one-dimensional flow through a pipe, the component momentum equation along the direction of flow is

$$\vec{F}_{net} = \rho Q\left(\vec{V}_2 - \vec{V}_1\right) \tag{2.5}$$

in which we assume flow into the pipe at the left section, section 1, and flow from the pipe at the right section, section 2. If the pipe cross-sectional area is constant between the end sections and the pipe is straight, then the velocities are equal, and the equation simplifies further to $\vec{F}_{net} = 0$. Since Eq. 2.5 is a vector equation, it can always be written in component form; for two-dimensional flow in the x-y plane, the components of this equation are

$$\begin{aligned} \sum F_x &= \left(\rho Q V_x\right)_2 - \left(\rho Q V_x\right)_1 = \left(\rho A V_x^2\right)_2 - \left(\rho A V_x^2\right)_1 \\ \sum F_y &= \left(\rho Q V_y\right)_2 - \left(\rho Q V_y\right)_1 = \left(\rho A V_y^2\right)_2 - \left(\rho A V_y^2\right)_1 \end{aligned} \tag{2.5a,b}$$

2.1.2. ENERGY AND HYDRAULIC GRADE LINES

The Energy Grade Line, also called the Energy Line or simply EL, is a plot of the sum of the three terms in the work-energy equation, which is also the Bernoulli sum:

$$EL = \frac{V^2}{2g} + \frac{p}{\gamma} + z \tag{2.6}$$

Since each term has units of length, we can conveniently superimpose a diagram of the behavior of each energy term, and the sum, on a drawing of the physical flow problem. For example, a Pitot tube, inserted into a flow to cause locally at its tip a point of zero velocity so the velocity head is converted into additional pressure head there, will cause the liquid to rise to the elevation of the EL for that point in the flow.

The Hydraulic Grade Line, or HGL, is the sum of only the pressure and elevation heads. The sum of these two terms is also called the piezometric head, which can be conveniently measured by a piezometer tube inserted flush into the side of a pipe. It is also important to recognize that any HGL can quickly be located on a diagram if the EL has already been located; we simply measure downward by the amount of the local velocity head from the EL to locate the HGL.

Figure 2.1 portrays the relation of the individual head terms to the EL and HGL and the head that is lost between sections 1 and 2.

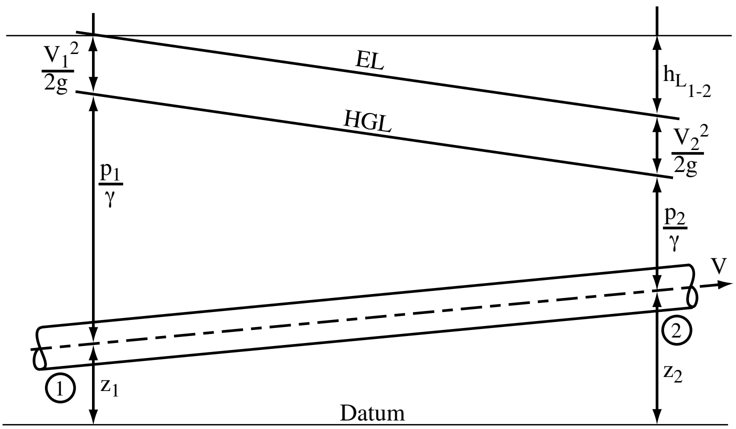


Figure 2.1 The EL and HGL in relation to individual heads and the head loss.

2.2 HEAD LOSS FORMULAS

The head loss term in Eq. 2.3 is responsible for representing accurately two kinds of real-fluid phenomena, head loss due to fluid shear at the pipe wall, called pipe friction, and additional head loss caused by local disruptions of the fluid stream. The head loss due to pipe friction is always present throughout the length of the pipe. The local disruptions, called local losses, are caused by valves, pipe bends, and other such fittings. Local losses may also be called minor losses if their effect, individually and/or collectively, will not contribute significantly in the determination of the flow; indeed, sometimes minor losses are expected to be inconsequential and are neglected. Or a preliminary survey of design alternatives may ignore the local or minor losses, considering them only in a later design stage. Each type of head loss will now be considered further.

2.2.1. PIPE FRICTION

If we were to select a small cylindrical control volume within a section of circular pipe, with coordinates s in the flow direction and r radially, in steady flow and subject this volume to analysis by the momentum equation, Eq. 2.4, we would find that the mean fluid shear stress τ, as a function of the radius r from the pipe centerline, is

$$\tau = -\frac{r}{2}\gamma\frac{\partial}{\partial s}\left(\frac{p}{\gamma} + z\right) \tag{2.7}$$

from which we learn two important facts:

1. The fluid shear stress τ varies linearly in a pipe cross-section, from zero at the centerline to a maximum, called τ_w, at the pipe wall where $r = D/2$.

2. In the absence of a streamwise gradient of the piezometric head $(p/\gamma + z)$, the fluid shear stress will be zero, and consequently no flow will exist at that section.

If we now expand the control volume to fill the pipe cross-section and integrate Eq. 2.7 over a length L of pipe of constant diameter, we learn with a bit of further work that the frictional head loss h_L over that length is directly related to the wall shear stress τ_w via

$$\tau_w = \gamma\, h_L \frac{D}{4L} \tag{2.8}$$

But this equation does not relate head loss to the mean velocity V or the discharge Q.

2.2.2. DARCY-WEISBACH EQUATION

The completely general functional relation $\tau_w = F(V, D, \rho, \mu, e)$ between the wall shear stress τ_w and the mean velocity V, pipe diameter D, fluid density ρ, and viscosity μ, and the equivalent sand-grain roughness e can be reduced by dimensional analysis to

$$\frac{\tau_w}{\rho V^2} = F\left(\frac{VD\rho}{\mu}, \frac{e}{D}\right) = \frac{f}{8} \tag{2.9}$$

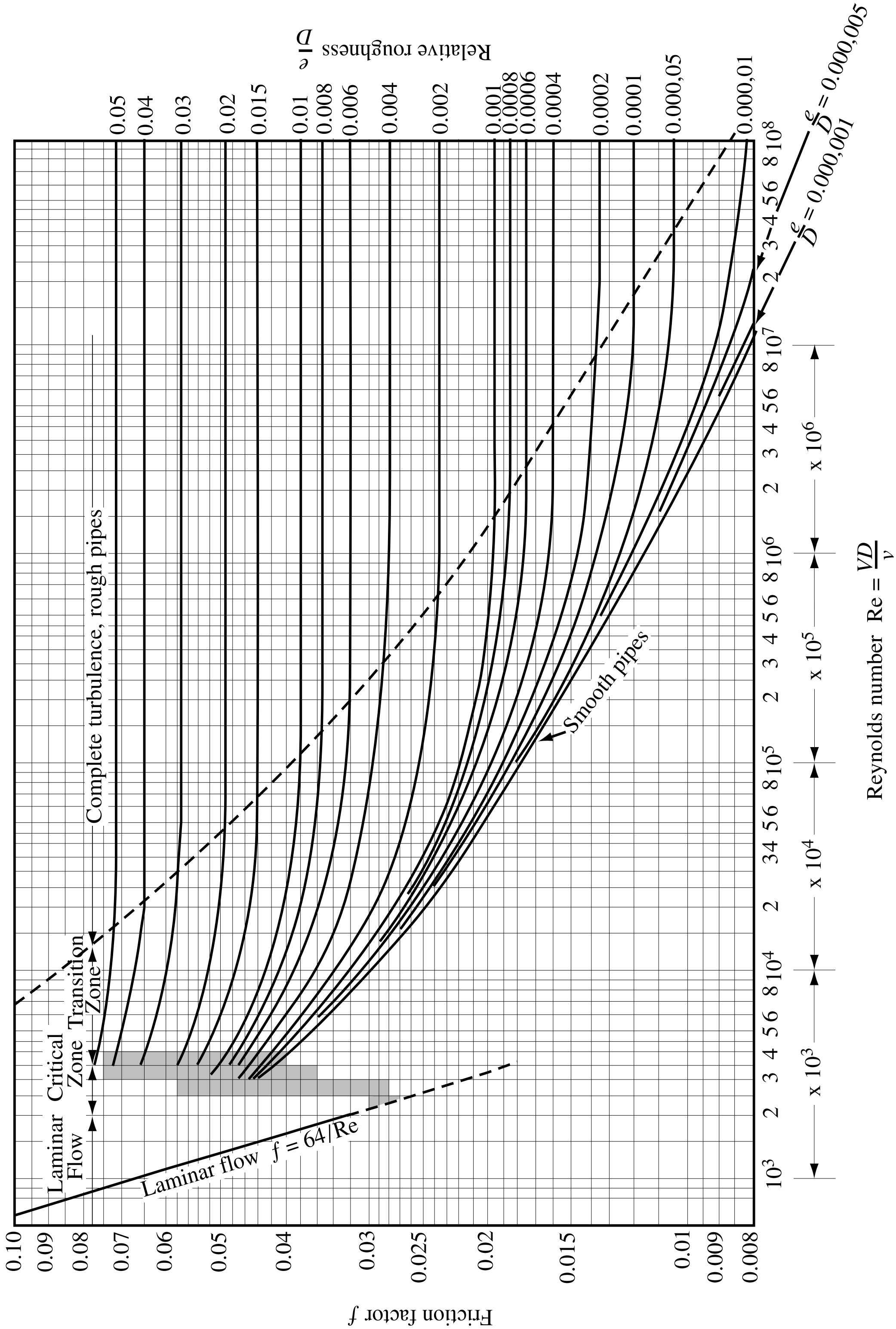
The combination of Eqs. 2.8 and 2.9 to eliminate the wall shear stress produces the fundamentally most sound and versatile equation for frictional head loss in a pipe, the Darcy-Weisbach equation:

$$h_f = f\frac{L}{D}\frac{V^2}{2g} = f\frac{L}{D}\frac{Q^2}{2gA^2} \tag{2.10}$$

In Eq. 2.9 the friction factor f (and the factor 8 to coincide with the historical development of the subject) is introduced as a shorthand notation for the function F. It is a function of the pipe Reynolds number $Re = VD\rho/\mu = VD/\nu$ and the equivalent sand-grain roughness factor e/D. For each pipe material either a single value or range of e/D values has been established; Table 2.1 presents common values for several materials.

Table 2.1 PIPE ROUGHNESSES

Material	e, mm	e, in
Riveted Steel	0.9 - 9.0	0.035 - 0.35
Concrete	0.30 - 3.0	0.012 - 0.12
Cast Iron	0.26	0.010
Galvanized Iron	0.15	0.006
Asphalted Cast Iron	0.12	0.0048
Commercial or Welded Steel	0.045	0.0018
PVC, Drawn Tubing, Glass	0.0015	0.000 06



From L. F. Moody, "Friction factors for Pipe Flow," *Trans.* A.S.M.E., Vol. 66, 1944, with permission.

Figure 2.2 The Moody diagram for the Darcy-Weisbach friction factor f.

Because commercially available pipes of any material display some heterogeneity or unevenness in roughness, any friction factor or its empirical equivalents can not be known with multiple-digit precision. The functional behavior of f is displayed fully in the Moody diagram in Fig. 2.2.

In the Moody diagram, which is Fig. 2.2, we see several zones that characterize different kinds of pipe flow. First we note that the plot is logarithmic along both axes. Below the Reynolds number $Re = 2100$ (some authors prefer 2300) there is only one line, which can be derived solely from the laminar, viscous flow equations without experimental input; the resulting friction factor for laminar flow is $f = 64/Re$. Because there is only one line in this region, we say all pipes are hydraulically smooth in laminar flow. Then for Reynolds numbers up to, say, 4000 is a so-called "critical" zone in which the flow changes from laminar flow to weakly turbulent flow. For still larger Reynolds numbers we find three flow zones that deserve comment:

1. A dashed line borders the upper right portion of the plot. In that zone, called wholly rough flow or the region of complete turbulence for rough pipes, the friction factor f is a function only of the roughness e/D and not of Re. For relatively rough pipes and/or large discharges this is a common flow type. Thus, if the pipe material is known so e/D is known, then the value of f follows immediately.

2. The lowest line is called the smooth-pipe line and is described by the empirical equation

$$1/\sqrt{f} = 2\,log_{10}\left(Re\sqrt{f}\right) - 0.8 \tag{2.11}$$

This line continually slopes and never becomes horizontal, as in the wholly rough flow zone, so f always depends on Re. Since the flow in PVC pipe is described by this line, it has become increasingly important in some fields in recent years.

3. Between zones 1 and 2 is an important transitional band, called the turbulent transition zone, in which f depends on both Reynolds number and e/D. The Colebrook-White equation

$$\frac{1}{\sqrt{f}} = 1.14 - 2\,log_{10}\left(\frac{e}{D} + \frac{9.35}{Re\sqrt{f}}\right) \tag{2.12}$$

is used, especially in computer codes, to replicate numerically the data in this zone of the Moody diagram. In spite of our prior caution about limited precision in friction factors, we sometimes need to allow more significant figures in computations to assure that the computer algorithms do indeed converge. And additional significant figures in computed values are also an aid in checking the success of computational examples, so we will sometimes present results in this book with more digits for these reasons, even though practical considerations may not seem to warrant it.

Table 2.2 summarizes the relations that describe the Darcy-Weisbach friction factor f.

Early in Chapter 5 procedures will be described for the computer solution of the Colebrook-White and Darcy-Weisbach equations as an alternative to the use of the Moody diagram itself. Readers who own a pocket calculator with the ability to solve implicit equations should seriously consider writing the Colebrook-White equation, Eq. 2.12, into the calculator memory for use in routinely computing friction factor values.

2.2.3. EMPIRICAL EQUATIONS

Empirical head loss equations have a long and honorable history of use in pipeline problems. Their initial use preceded by decades the development of the Moody diagram, and they are still commonly used today in professional practice. Some prefer to continue to use such an equation owing simply to force of habit, while others prefer it to avoid some of the difficulties of determining the friction factor in the Darcy-Weisbach equation.

As is common with empirical equations, each contains a constant that depends on the chosen unit system. Possibly the most widely used of these equations is the Hazen-Williams equation.

Table 2.2 DARCY-WEISBACH FRICTION EQUATIONS

Type of Flow	Equation for f	Range
Laminar	$f = 64/Re$	$Re < 2100$
Smooth pipe	$1/\sqrt{f} = 2\,log_{10}\left(Re\sqrt{f}\right) - 0.8$	$Re > 4000$ and $e/D \rightarrow 0$
Transitional Colebrook-White Eq.	$\frac{1}{\sqrt{f}} = 1.14 - 2\,log_{10}\left(\frac{e}{D} + \frac{9.35}{Re\sqrt{f}}\right)$	$Re > 4000$
Wholly Rough	$\frac{1}{\sqrt{f}} = 1.14 - 2\,log_{10}\left(\frac{e}{D}\right)$	$Re > 4000$

To compute the discharge, the equation takes the forms

$$Q = 1.318\, C_{HW} A\, R^{0.63} S^{0.54} \qquad \text{ES units} \tag{2.13}$$

or

$$Q = 0.849\, C_{HW} A\, R^{0.63} S^{0.54} \qquad \text{SI units} \tag{2.14}$$

in which C_{HW} is the Hazen-Williams roughness coefficient, $S = h_f/L$ is the slope of the energy line, $R = A/P$ is the hydraulic radius, A is the cross-sectional area, and P is the wetted perimeter, so that pipes flowing full will always have $R = D/4$. Table 2.3 gives values for C_{HW} for some common pipe materials.

Another empirical equation, which was originally and primarily developed for flow in open channels, is the Manning equation

$$Q = \frac{1.49}{n} A\, R^{2/3} S^{1/2} \qquad \text{ES units} \tag{2.15}$$

or

$$Q = \frac{1}{n} A\, R^{2/3} S^{1/2} \qquad \text{SI units} \tag{2.16}$$

The pipe boundary roughness is described by the Manning n, for which some values are listed in Table 2.3.

Table 2.3 HAZEN-WILLIAMS AND MANNING ROUGHNESSES

Pipe Material	C_{HW}	n
PVC	150	0.009
Very Smooth	140	0.010
Cement-lined Ductile Iron	140	0.012
New Cast Iron, Welded Steel	130	0.014
Wood, Concrete	120	0.016
Clay, New Riveted Steel	110	0.017
Old Cast Iron, Brick	100	0.020
Badly corroded Cast Iron	80	0.035

A comparison of the Hazen-Williams and Manning equations with the Darcy-Weisbach equation would show conclusively that the empirical equations are much more limited in their ranges of applicability. Each is applicable only to the turbulent flow of water. The Manning equation is only valid for flows which correspond to the wholly rough flow regime in pipes. If the Hazen-Williams equation were plotted on the doubly-logarithmic Moody chart, it would appear as a family of sloping (the slope is - 0.15) straight lines across the turbulent transitional flow portion of the Moody diagram; hence each choice of a Hazen-Williams coefficient can at most replicate only a part of an individual *e/D* line on the Moody diagram.

2.2.4. EXPONENTIAL FORMULA

It will later be advantageous to express the head loss in each pipe in a network by an exponential formula so one presentation of the theory covers all cases, regardless of whether the Darcy-Weisbach equation, the Hazen-Williams equation or the Manning equation is used to express the head loss as a function of discharge:

$$h_f = KQ^n \tag{2.17}$$

The values for K and n change, depending on whether the Darcy-Weisbach, Hazen-Williams, or Manning equation is used.

The Hazen-Williams and Manning equations can be solved for h_f and put in the form of the exponential formula. For the Hazen-Williams equation the exponent is $n = 1.852$ and the coefficient K is

$$K = \frac{C_K L}{C_{HW}^{1.852} D^{4.87}} \tag{2.18}$$

For the Manning equation the exponent is $n = 2$ and K is

$$K = \frac{C_K n^2 L}{D^{5.33}} \tag{2.19}$$

in which the dimensional constant C_K is given for various choices of units in Table 2.4.

Table 2.4 The Coefficient C_K

Units of *D*	Units of *L*	**Hazen-Williams** C_K in Eq. 2.18	**Manning** C_K in Eq. 2.19
ft	ft	4.73	4.66
in	ft	8.53×10^5	2.65×10^6
m	m	10.67	10.29

To find K and n for the Darcy-Weisbach equation, we note that f can be approximated over a limited range on the Moody diagram by an equation of the form

$$f = a/Q^b \tag{2.20}$$

This equation plots as a straight line on the Moody diagram (a log-log plot) if a and b are constant. Substituting Eq. 2.20 into the Darcy-Weisbach equation and grouping terms gives

$$n = 2 - b \tag{2.21}$$

and

$$K = \frac{aL}{2gDA^2} \tag{2.22}$$

Hence a determination of K and n for use in Eq. 2.17 is equivalent to a selection of values for a and b in Eq. 2.20 which cause that equation to approximate f over the expected discharge range. If the chosen range is too large, then K and n will cause Eq. 2.17 to produce frictional head losses that differ slightly from predictions that are obtained directly from the Darcy-Weisbach and Colebrook-White equations. If the chosen range is too small, then the actual discharge may fall outside this range, and K and n should be redetermined. To obtain a and b, select an appropriate Reynolds number (discharge, or velocity) range that brackets the expected discharge Q. Solve the Colebrook-White equation with these two Reynolds numbers Re_1 and Re_2, obtaining both f_1 and f_2 and the corresponding discharges Q_1 and Q_2. Taking the logarithm (either natural or base-10 logarithms can be used) of both sides of Eq. 2.20 now gives two equations for a and b:

$$\begin{aligned} \ln f_1 &= \ln a - b \ln Q_1 \\ \ln f_2 &= \ln a - b \ln Q_2 \end{aligned} \tag{2.23}$$

Subtracting the second equation from the first and solving for b produces

$$b = \frac{\ln(f_1/f_2)}{\ln(Q_2/Q_1)} \tag{2.24}$$

Then a can be obtained as

$$a = f_1 Q_1^b \tag{2.25}$$

Calculations to determine K and n can readily be done with a pocket calculator, but if many are needed, the computations should be implemented in a spreadsheet or a computer program. FORTRAN program 2.1, PIPK_N, is included on the CD for this purpose.

Example Problem 2.1

Determine the values of K and n in the exponential formula for the three pipes in the table which follows ($\nu = 1.217 \times 10^{-5}$ ft^2/s or $\nu = 1.13 \times 10^{-6}$ m^2/s):

Pipe	**Type**	**Length**	**Diameter**	$e \times 10^4$	C_{HW}	n	Approx. Q
1	PVC	1000 ft	8 in	0.08 in	150	0.009	2.5 ft^3/s
2	Riveted steel	1000 ft	8 in	2.5 ft	110	0.015	0.8 ft^3/s
3	Ductile iron	3000 m	300 mm	500 mm	140	0.011	0.4 m^3/s

Only the solution details for pipe 1 are given here, but for practice the other answers should be verified. For the Hazen-Williams and Manning equations K and n are computed from Eqs. 2.18 and 2.19, respectively. For the Hazen-Williams equation n_1 =

1.852 and $K_1 = 4.73(1000)/(150^{1.852}0.667^{4.87}) = 3.17$; for the Manning equation $n_1 = 2.000$, $K_1 = 4.66(0.009)^2 1000/0.667^{5.33} = 3.27$. For the Darcy-Weisbach equation first select two discharges that span the expected range, say $Q_1 = 2$ ft^3/s and $Q_2 = 3$ ft^3/s. Next, from the Colebrook-White equation find the friction factors f corresponding to these two discharges, or $f_1 = 0.01435815$, $f_2 = 0.01332301$. For the accuracy that we require, these values must be obtained from a pocket calculator or other computational equipment and not just read from a Moody Diagram. Next compute $b = \{\log f_1/f_2\}/\{\log Q_2/Q_1\} = 0.18454$, leading to $n = 2 - b = 1.81546$, and $a = f_1 Q_1^b = 0.016317$, from which $K = aL/(2gDA^2) = 3.2649$. The remainder of the computations for each determination of K and n for these three pipes is summarized in the following pair of tables:

Pipe	Q_1	Q_2	Re_1	Re_2	f_1	f_2	b	a
1	2.0	3.0	314000	471000	0.0145	0.0133	0.1845	0.0163
2	0.4	1.2	62800	188000	0.0200	0.0160	0.1993	0.0166
3	0.2	0.6	751000	2250000	0.0146	0.0134	0.0543	0.0133

	Darcy-Weisbach		Hazen-Williams		Manning	
Pipe	K	n	K	n	K	n
1	3.2649	1.8155	3.1773	1.852	3.2649	2.000
2	9.0692	1.8007	5.6431	1.852	9.0691	2.000
3	2296.1	1.9957	1194.7	1.852	2296.1	2.000

* * *

In summary, the best equation for computing the frictional head loss in a given pipe for a given discharge, or the best equation for the discharge if the head loss is known, regardless of the fluid, is the Darcy-Weisbach equation. The range of applicability for the empirical equations is much more restricted. Consequently, all engineers should consider using the Darcy-Weisbach equation in professional practice even if it is sometimes more difficult to use than the empirical equations.

2.2.5. LOCAL AND MINOR LOSSES

A local loss is any energy loss, in addition to that of pipe friction alone, caused by some localized disruption of the flow by some flow appurtenances, such as valves, bends, and other fittings. The actual dissipation of this energy occurs over a finite but not necessarily short longitudinal section of the pipe line, but it is an accepted convention in hydraulics to lump or concentrate the entire amount of this loss at the location of the device that causes the flow disruption and loss. If a loss is sufficiently small in comparison with other energy losses and with pipe friction, it may be regarded as a minor loss. Often minor losses are neglected in preliminary studies or when they are known to be quite small, as will often happen when the pipes are very long. However, some local losses can be so large or significant that they will never be termed a minor loss, and they must be retained; one example is a valve that is only partly open.

Normally, theory alone is unable to quantify the magnitudes of the energy losses caused by these devices, so the representation of these losses depends heavily upon experimental data. Local losses are usually computed from the equation

$$h_L = K_L \frac{V^2}{2g} \tag{2.26}$$

in which $V = Q/A$ is normally the downstream mean velocity. For enlargements the following alternative formula applies:

$$h_L = K_L \frac{(V_1 - V_2)^2}{2g} \tag{2.27}$$

in which V_1 and V_2 are, respectively, the upstream and downstream velocities. In Eq. 2.27 the loss coefficient K_L is unity for a sudden enlargement and takes on values between 0.2 and 1.2 for assorted gradual conical enlargements. The head loss for flow from a pipe into a reservoir is a special but important case of Eq. 2.27, called the exit loss; in this case $K_L = 1$ and $V_2 = 0$, independent of the geometric details of the pipe exit shape.

Local loss coefficients K_L for some common valve and pipe fittings are listed in Table 2.5. The energy losses for these fittings are mostly a consequence of fluid turbulence caused by the device rather than by secondary motions which persist downstream. Normally a locally accelerating flow will cause much less energy loss than does a decelerating flow. If deceleration is too rapid, it causes separation, which results in additional turbulence and a high velocity in the non-separated region. Some additional loss coefficients from specific valve manufacturers and coefficient values as a function of the amount of the valve opening can be found in Appendix C.

Table 2.5 Loss Coefficients for Fittings

Fitting	K_L
Globe valve, fully open	10.0
Angle valve, fully open	5.0
Butterfly valve, fully open	0.4
Gate valve, fully open	0.2
3/4 open	1.0
1/2 open	5.6
1/4 open	17.0
Check valve, swing type, fully open	2.3
Check valve, lift type, fully open	12.0
Check valve, ball type, fully open	70.0
Foot valve, fully open	15.0
Elbow, 45°	0.4
Long radius elbow, 90°	0.6
Medium radius elbow, 90°	0.8
Short radius (standard) elbow , 90°	0.9
Close return bend, 180°	2.2
Pipe entrance, rounded, r/D < 0.16	0.1
Pipe entrance, square-edged	0.5
Pipe entrance, re-entrant	0.8

An abrupt contraction has first a region of accelerating flow, followed by a region of decelerating flow caused by flow separation. Though the region of accelerating flow may be larger, the head loss is attributable principally to the deceleration and separation which occurs immediately downstream from the contraction. The local loss coefficient for a pipe contraction is given in Fig 2.3.

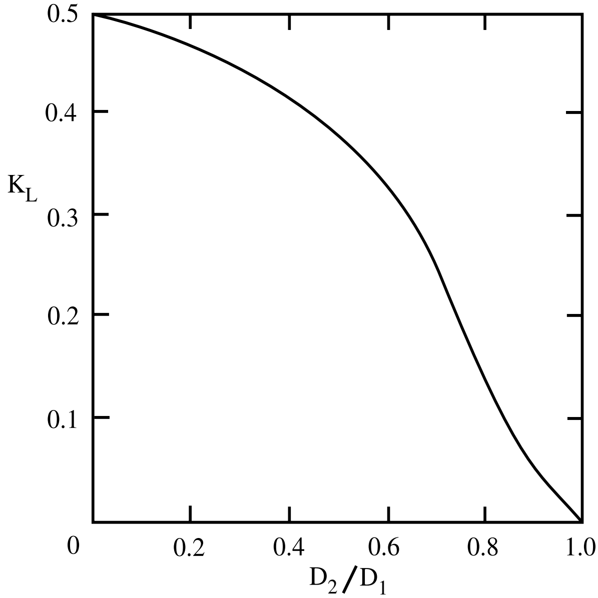


Figure 2.3 Local loss coefficient for a sudden contraction as a function of diameter ratio.

2.3 PUMP THEORY AND CHARACTERISTICS

The addition of mechanical energy $h_m = h_p$ per unit weight to a fluid stream is accomplished by pumps, as was mentioned with Eq. 2.3. Although positive displacement pumps sometimes play a role, by far the more important class of pumps contains a rotating impeller to inject energy, in the form of an increased pressure head, into the flowing fluid in the pipe. The characteristic shape of the impeller varies with the operating regime of the pump. The energy addition is called the net head h_p of the pump. The water power P_w that is delivered to the fluid stream is the product of the net head, the discharge, and the unit weight of the fluid, or $P_w = Q\gamma h_p$. The mechanical power to operate the pump must be larger; it is called the brake horsepower or $bhp = T\omega$, in which T and ω are the torque and angular velocity of the pump drive shaft. The ratio $\eta = P_w/bhp$ is the pump efficiency, which may be larger than 0.8 for large and/or efficient pumps that are operating near their best efficiency point (bep), also called the design point, but which may be much lower for small, old or worn pumps.

Pumps are sufficiently complex that they cannot be designed on the basis of theory alone. To refine a new or revised design, model experiments are first conducted, and after success is achieved with the model, then the full-scale or prototype pump is built. The results of dimensional analysis are used to relate the model and prototype to each other. First we assume that the model and prototype are similar in shape, called geometric similarity, and second that the velocity fields also have a similar shape, called kinematic similarity. Devices satisfying these requirements are called homologous. The nondimensional parameters that are used to complete the scaling process are called affinity or scaling laws. They are three in number and are called the head, discharge, and power coefficients C_H, C_Q, and C_P, respectively:

$$C_H = \frac{gh_p}{N^2 D^2}; \quad C_Q = \frac{Q}{ND^3}; \quad C_P = \frac{P}{\rho N^3 D^5} \tag{2.28}$$

The diameter of the rotating impeller is D. These coefficients may be computed in any consistent set of units. If plots of one nondimensional coefficient vs. another are constructed, homologous units having different sizes and/or rotative speeds can be related to each other. Or one can say for homologous units that

$$\left(\frac{h_p}{N^2D^2}\right)_1 = \left(\frac{h_p}{N^2D^2}\right)_2; \quad \left(\frac{Q}{ND^3}\right)_1 = \left(\frac{Q}{ND^3}\right)_2; \quad \left(\frac{P}{\rho N^3D^5}\right)_1 = \left(\frac{P}{\rho N^3D^5}\right)_2 \tag{2.29}$$

In a way these relations are more versatile than Eqs. (2.28) because the units no longer must lead to a truly nondimensional group so long as each variable is measured in the same units. Thus rotative speed can be in rad/s, rev/s or rev/min. If pumps 1 and 2 have the same diameter, Eqs. 2.29 show how h_p, Q, and P respond to changes in N, or for fixed N we see how the variables scale with the diameter D.

The specific speed N_S is a parameter for homologous pumps that contains the important pump variables, the discharge Q and head h_p, without containing the unit size D; different ranges of this parameter therefore capture the essential differences in shape, not mere size, that separates the performance of one kind of pump from another type of pump. The nondimensional form of pump specific speed, with N in rad/s, is

$$N_S = \frac{NQ^{1/2}}{\left(gh_p\right)^{3/4}} \tag{2.30}$$

In the United States, however, for many years it has been customary instead to use

$$N_S^{'} = \frac{(rev/min)(gal/min)^{1/2}}{\left[h_p(ft)\right]^{3/4}} \tag{2.31}$$

which is clearly far from dimensionless. Based on specific speed, pumps can be classified into three categories, based on impeller shape, as given in Table 2.6.

Table 2.6
Pump Type vs. Specific Speed

	Radial Flow	**Mixed Flow**	**Axial Flow**
N_S	$N_S < 1.46$	$1.46 < N_S < 3.7$	$3.7 < N_S$
N'_S	$500 < N'_S < 4000$	$4000 < N'_S < 10000$	$10000 < N'_S$

For relatively low specific speed the most efficient pump uses a radial-flow impeller, that is, the primary flow direction through the impeller is radially outward from the axis of rotation of the impeller; this pump type has several names but is usually called a centrifugal pump. For the highest specific speed range, the flow through the impeller is nearly parallel to the axis of rotation and is called axial flow in pumps that are termed propeller pumps. The transition from radial to axial flow occurs over the intermediate range called mixed flow; the pumps are called turbine pumps. Certainly there is some overlap between regions, and different authorities cite somewhat differing values for the ends of the ranges.

The performance of an individual pump, or a family of pumps having the same pump casing and several impellers that differ only in size, is usually described by a set of pump characteristic curves, or simply pump curves, that are developed by manufacturers. Appendix B contains eight sets of pump characteristic curves. Across the upper portion of each figure is a plot of head (per stage) vs. discharge; although these curves are usually approximated as straight lines or parabolic curves for subsequent analysis, the reader will quickly notice that the actual head curves are more complex. A change in the shape of a curve normally means that the flow pattern within the pump has also changed. Crossing the set of head curves are contour lines of constant efficiency. By each contour is the numerical percentage value of the efficiency; usually the values are between 70 and 85%. Across the bottom of each plot is a set of curves that relate brake horsepower to the discharge; we see that straight lines would fit most of these lines rather well but not perfectly. Finally, in the upper right corner of each plot is a plot of NPSH vs. discharge.

The Net Positive Suction Head (NPSH) for a pump is used to determine the head z_i that is needed at the pump inlet so that cavitation is avoided in the pump. Cavitation is the conversion of liquid into vapor by locally low absolute pressure. The onset of cavitation can also be inferred from tests to note impaired operational efficiency, excessive noise and possibly damage to the pump. A useful form of the NPSH relation is

$$NPSH = \frac{p_{atm}}{\gamma} - \frac{p_v}{\gamma} - h_L - z_i \tag{2.32}$$

in which p_{atm} and p_v are the atmospheric and vapor pressure of the liquid, h_L is the head loss in the inlet piping (often included in NPSH itself), and z_i is the highest allowable or safe elevation for the pump impeller inlet. For the operating discharge, read NPSH from the pump curve, and z_i can then be computed.

2.4 STEADY FLOW ANALYSES

This section will touch on several kinds of steady flow problems. Although the exponential formula or the empirical head loss equations could be used for this purpose, we choose to employ the versatile Darcy-Weisbach formula here, sometimes simplifying by assuming the value of the friction factor. We will look at series pipe flow first, with and without consideration of local losses and a pump in line. Flow through parallel pipes will follow, and the section concludes with a look at multiple-reservoir problems.

2.4.1. SERIES PIPE FLOW

The basic tools for analysis here are Eqs. 2.2, 2.3 and 2.10, which are the continuity, work-energy and Darcy-Weisbach equations. All series pipe flow problems fit one of three computational categories, depending on which factors are known or given and which is sought, as listed in Table 2.7:

Table 2.7 Problem Types

Category	Known Quantities:	To Find:
1	Q, pipeline properties	h_L
2	h_L, pipeline properties	Q
3	Q, h_L	Smallest size D

The problems in categories 1 and 2 are analysis problems; analysis of type 1 problems is direct, without iteration, but iteration may be required for the second group. Category 3 is a design problem, which normally requires more assumptions and more iterative computations to solve. Pipeline properties include the length, diameter and material type so that the relative roughness is known.

Example Problem 2.2

A cast iron pipe connects two reservoirs. The line is 1200 ft long and has a diameter of 12 in. If it were to convey 8 ft^3/s, what would be the frictional head loss for this pipe? [In this and following examples in this chapter, we assume the fluid is $60^{o}F$ water with a kinematic viscosity $\nu = 1.2x10^{-5}$ ft^2/s.]

This problem is a type 1 problem. The mean velocity in the pipe is

$$V = \frac{Q}{A} = \frac{8}{\pi/4} = 10.18 \text{ ft/s.}$$

Thus the pipe Reynolds number is

$$Re = \frac{VD}{\nu} = \frac{(10.18)(1)}{1.2x10^{-5}} = 8x10^5$$

Upon consulting Table 2.1 for cast iron pipe, we determine $e/D = 0.010/12 = 0.00083$. From the Moody diagram, Fig. 2.2, we find $f = f(Re, e/D) = 0.0185$. The Darcy-Weisbach equation, Eq. 2.10, then produces

$$h_f = f\frac{L}{D}\frac{V^2}{2g} = (0.0185)\frac{1200}{12/12}\frac{(10.18)^2}{2(32.2)} = 35.7 \; ft$$

* * *

Example Problem 2.3

The pipe in Example Problem 2.2 actually connects two reservoirs having a difference in water surface level of only 20 ft, so that pipe is clearly incapable of conveying 8 ft^3/s.

Now a new pipe has been installed between the reservoirs. It is made of welded steel and has a diameter of 18 in.

(a) If only pipe friction is considered, what is the new discharge?

(b) If local losses for a sharp-edged entrance, a fully open gate valve near the pipe exit, and the pipe exit itself are also considered, how much does the computed discharge change?

(c) If the gate valve in part (b) were only 1/4 open, what would then be the discharge?

All parts of this problem belong to category 2, since now Q and not h_L is sought.

(a) We are told to assume in this case that

$$z_1 - z_2 = 20\,ft = h_f = f\frac{L}{D}\frac{V^2}{2g}$$

From Table 2.1 for welded steel, we find $e/D = 0.0018/18 = 0.0001$. If the flow is assumed to be in the wholly rough flow zone of the Moody diagram, Fig. 2.2, $f = 0.012$. Hence

$$h_f = 20 = (0.012)\frac{1200}{18/12}\frac{V^2}{2(32.2)}$$

and $V = 11.6$ ft/s. Now we must check $Re = VD/\nu = 11.6(18/12)//1.2x10^{-5} = 1.4x10^6$, which is not in the wholly rough zone; this Re and the value of e/D imply $f = 0.013$. Using 0.013 in place of 0.012 leads to $V = 11.1$ ft/s. The small change in Re will cause no further change in f, so the discharge can now be computed as

$$Q = VA = (11.1)\frac{\pi}{4}\left(\frac{18}{12}\right)^2 = (11.1)(1.77) = 19.6 \; ft^3/s$$

(b) In this case

$$20 = \sum h_L = \left(K_{ent.} + f\frac{L}{D} + K_{valve} + K_{exit}\right)\frac{V^2}{2g}$$

The velocity head factors out only because each loss term is associated with the same pipe size, area and velocity. Table 2.5 supplies 0.5 and 0.2 for the entrance and valve loss coefficients; always $K_{exit} = 1.0$. From part (a) we take our first estimate of the friction factor as 0.013, leading to

$$20 = \left(0.5 + 0.013\frac{1200}{18/12} + 0.2 + 1.0\right)\frac{V^2}{2g}$$

and yielding V = 10.3 ft/s. Again check $Re = VD/\nu = 10.3(18/12)//1.2\text{x}10^{-5} = 1.3\text{x}10^6$, so the initial estimate of f is adequate. Now Q = (10.3)(1.77) = 18.2 ft^3/s so the discharge has decreased by 1.4 ft^3/s, a bit under 8%, as a consequence of considering the local losses.

(c) When the gate valve is only 1/4 open, we find from Table 2.5 that the valve loss coefficient has increased from 0.2 to 17.0. The valve loss remains a local loss, but it is no longer in any way a minor loss, since it will cause more head loss than the pipe friction term. Replacing 0.2 in part (b) by 17.0, we recompute and find V = 6.68 ft/s. The new, lower Reynolds number is $Re = 8.4\text{x}10^5$, so the new friction factor is $f = 0.0135$. A recomputation of the velocity gives V = 6.63 ft/s, and so Q = 11.7 ft^3/s, a decrease of about one third from the discharge in part (b).

* * *

2.4.2. SERIES PIPE FLOW WITH PUMP(S)

The solution of pipeflow problems involving pumps normally requires us to read data from pump characteristic curves. However, if we prefer to use a computer to solve these problems, such readings can no longer be done in this way. But the resolution of this problem is not difficult. As part of the computer solution of this kind of problem, we supply sufficient data to the program so that the head h_p can be expressed as a polynomial in discharge that fits the pump-curve data.

Let the pump characteristic curve for the head h_p be expressed by a second-order polynomial $h_p = AQ^2 + BQ + C$, in which the coefficients A, B, and C are determined by the use of three (h_p, Q) data pairs that bracket the expected range of operation of the pump. To obtain the coefficients, we write three equations by substituting each data pair into the polynomial to obtain

$$\begin{aligned} AQ_1^2 + BQ_1 + C &= h_{p1} \\ AQ_2^2 + BQ_2 + C &= h_{p2} \\ AQ_3^2 + BQ_3 + C &= h_{p3} \end{aligned} \tag{2.33}$$

In matrix notation Eq. 2.33 becomes

$$\begin{bmatrix} Q_1^2 & Q_1 & 1 \\ Q_2^2 & Q_2 & 1 \\ Q_3^2 & Q_3 & 1 \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = \begin{Bmatrix} h_{p1} \\ h_{p2} \\ h_{p3} \end{Bmatrix} \tag{2.34}$$

which can be solved in various ways to determine the coefficients.

An alternative approach is to use the Lagrangian interpolation. Lagrange's formula is

$$h_p = \frac{(Q-Q_2)(Q-Q_3)}{(Q_1-Q_2)(Q_1-Q_3)} h_{p1} + \frac{(Q-Q_1)(Q-Q_3)}{(Q_2-Q_1)(Q_2-Q_3)} h_{p2} + \frac{(Q-Q_1)(Q-Q_2)}{(Q_3-Q_1)(Q_3-Q_2)} h_{p3} \quad (2.35)$$

The head h_p is again expressed as a quadratic equation in Q, but the terms are rearranged from the earlier approach. The coefficients A, B, and C can be found by expanding the numerators. Letting

$$\begin{aligned} c_1 &= h_{p1}/(Q_1-Q_2)(Q_1-Q_3) \\ c_2 &= h_{p2}/(Q_2-Q_1)(Q_2-Q_3) \\ c_3 &= h_{p3}/(Q_3-Q_1)(Q_3-Q_2) \end{aligned} \quad (2.36)$$

we find

$$\begin{aligned} A &= c_1 + c_2 + c_3 \\ B &= -2\left[(Q_2+Q_3)c_1 + (Q_3+Q_1)c_2 + (Q_1+Q_2)c_3\right] \\ C &= Q_2Q_3c_1 + Q_3Q_1c_2 + Q_1Q_2c_3 \end{aligned} \quad (2.37)$$

Irrespective of which approach is used, the results can be inserted in a computer program so that the need to read data from a pump characteristic curve during the solution process is avoided. Additional uses of such polynomial representations and interpolations will be found in later chapters, including Chapters 4, 5, and 10.

Example Problem 2.4

A single-stage Ingersoll-Dresser 15H277 pump, outfitted with the largest impeller (Refer to Appendix B for the pump characteristic curves), is used to pump water from a reservoir at elevation 1350 ft to another reservoir at elevation 1425 ft. The line is 6000 ft long and 18 in. in diameter with an equivalent sand grain roughness $e = 0.015$ in. ($\nu = 1.14\text{x}10^{-5}$ ft^2/s) Neglecting local losses, compute the discharge in the pipeline.

We begin by applying the work-energy equation, Eq. 2.3, between the two reservoir water surfaces, points 1 and 2:

$$1350 = 1425 + h_f - h_p$$

or

$$h_p = 75 + f\frac{L}{D}\frac{Q^2/A^2}{2g} = 75 + f\frac{6000}{1.5}\frac{Q^2}{2g(1.767)^2} = 75 + 19.9fQ^2$$

There are three unknowns in this equation: h_p, Q, and f. They must be determined by using this equation, the pump curve and the Colebrook-White equation. We shall obtain the solution in two ways, first by hand and then with the aid of a computer.

The hand solution begins by (a) selecting a trial discharge, (b) solving the Colebrook-White equation, Eq. 2.12, for f, (c) calculating h_p from the above work-energy equation, (d) comparing this h_p with the value that is read from the pump characteristic curve, and (e) repeating the process until the h_p's match, as summarized in the table:

(a) Q gal/min	Q ft^3/s	(b) f	(c) h_p ft	(d) hp, curve ft
3000	6.68	0.01961	92.4	103
3500	7.80	0.01950	98.6	88
3300	7.35	0.01951	96.0	95
3280	7.31	0.01954	95.8	95.8

The discharge is 3280 gal/min by this method.

The pump curve must be defined by an algebraic equation if the computer is to be used in solving for h_p, Q, and f. A second-order polynomial can be fit to the Ingersoll-Dresser 15H277 pump curve by applying Eqs. 2.33 and 2.34 and using the three data pairs (103.0, 6.68), (95.0, 7.35), and (88.0, 7.80). Equation 2.34 gives the matrix form of this problem as

$$\begin{bmatrix} Q_1^2 & Q_1 & 1 \\ Q_2^2 & Q_2 & 1 \\ Q_3^2 & Q_3 & 1 \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = \begin{bmatrix} 44.62 & 6.68 & 1 \\ 54.02 & 7.35 & 1 \\ 60.84 & 7.80 & 1 \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = \begin{Bmatrix} 103 \\ 95 \\ 88 \end{Bmatrix}$$

yielding a solution $A = -3.224$, $B = 33.293$, and $C = 24.472$ so that the pump curve is approximately

$$h_p = -3.224Q^2 + 33.293Q + 24.472$$

Using MathCAD, TK-Solver or some other mathematics application software to solve our three simultaneous equations leads to $h_p = 95.7$ ft, $Q = 7.30$ ft^3/s = 3280 gal/min, and $f = 0.019546$.

* * *

Example Problem 2.5

Repeat the problem in Example Problem 2.4 with two three-stage Ingersoll-Dresser 15H277 pumps in parallel; assume the smallest of the three impellers is used in each pump stage.

The pipeline analysis itself in unchanged; hence

$$h_p = 75 + 19.9 f Q^2$$

In this case h_p is the total head developed in the three stages of either of the two pumps. In addition, only half of the pipeline discharge passes through each pump. The table of trial solutions can be developed as

Pump Q gal/min	**Pipe Q** gal/min	f	**Right Side h_p** ft	**h_p/stage** ft	**Total h_p** ft
3000	6000	0.01921	143.3	67	201
3500	7000	0.01915	167.7	45	135
3300	6600	0.01917	157.5	54	162
3320	6640	0.01917	158.5	53	159

The total discharge is 6640 gal/min.

To set up the computer solution for this problem, we first obtain the polynomial approximation to the pump curve by setting up the matrix

$$\begin{bmatrix} Q_1^2 & Q_1 & 1 \\ Q_2^2 & Q_2 & 1 \\ Q_3^2 & Q_3 & 1 \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = \begin{bmatrix} 44.69 & 6.685 & 1 \\ 54.02 & 7.35 & 1 \\ 60.84 & 7.80 & 1 \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = \begin{Bmatrix} 67 \\ 55 \\ 45 \end{Bmatrix}$$

resulting in $h_{p1} = -3.74643Q_1^2 + 34.536Q_1 + 3.552$ for one stage. To account for the number of stages, we multiply the coefficients by 3 so that $h_p = 3h_{p1}$. Since only half of the pipe flow passes through each of the parallel pumps $Q_1 = Q/2$. The final composite pump curve is therefore

$$\begin{aligned} h_p &= 3(-3.7464)(Q/2)^2 + 3(34.536)Q/2 + 3(3.552) \\ &= -2.8098Q^2 + 51.804Q + 10.656 \end{aligned}$$

Solving this equation, the Colebrook-White equation and the work-energy equation simultaneously gives $h_p = 159.4$ ft, $Q = 14.878$ ft^3/s = 6680 gal/min, and $f = 0.01917$.

* * *

2.4.3. PARALLEL PIPE FLOW, EQUIVALENT PIPES

In the flow of fluid in parallel pipes the roles of energy loss and discharge are reversed from their roles in series pipe flow: for a series of pipes, as we have seen earlier, the discharge is identical in each pipe of the series while the head losses are additive; for a set of parallel pipes between two common junctions the head loss between the two junctions is identical for each pipe while the total discharge is the sum of the individual discharges.

Since the analysis of flow in a series of pipes is more straightforward than the analysis of flow through a combination of pipes that includes parallel pipes as a part of the combination, it is advantageous to replace the set of parallel pipes by a single "equivalent pipe." This equivalent pipe, which is devised so it has the same head loss as the original set of parallel pipes and conveys the same total discharge, will in some cases allow the analyst to avoid the use of iteration in seeking a solution. In other cases the amount of iteration will be reduced.

The equivalent pipe formula can be constructed so it can be used with any pipe combination having head loss characteristics that can be described by the exponential formula, Eq. 2.17. Assume that pipes 1 and 2 are two parallel pipes with frictional losses described by KQ^n; then the equivalent pipe (unsubscripted) must satisfy

$$h_f = KQ^n = K_1Q_1^n = K_2Q_2^n \tag{2.38}$$

and

$$Q = Q_1 + Q_2 \tag{2.39}$$

By solving Eqs. 2.38 for Q_1 and Q_2 and inserting the results into Eq. 2.39, we find

$$\left(\frac{1}{K}\right)^{1/n} = \left(\frac{1}{K_1}\right)^{1/n} + \left(\frac{1}{K_2}\right)^{1/n} \tag{2.40}$$

For the remainder of the problem the equivalent variables K and Q are then used in place of the original parallel pipes. Once Q has been found, then a back-substitution into Eq. 2.38 determines Q_1 and Q_2. To treat several parallel pipes rather than two, simply add one additional term to the right side of Eq. 2.40 for each pipe that is in parallel.

Example Problem 2.6

Two reservoirs have a difference in water surface elevation of 40 ft. Water flows from the higher reservoir through 4000 ft of 12-in-diameter pipe, which then joins a pair of parallel 2000-ft-long pipes which end at the lower reservoir. One parallel pipe has a 10-in diameter; the diameter of the other pipe is 8 in. For simplicity, assume $f = 0.02$ for all pipes. Find the discharge in each pipe between the two reservoirs.

In this problem we use the exponential formula for head loss. For each pipe $n = 2$ and

$$K = f\frac{L}{D}\frac{1}{2g}\frac{1}{A^2}$$

With the given data $K_{12} = 2.01$, $K_{10} = 2.51$ and $K_8 = 7.65$. The equivalent pipe coefficient K_e is found from

$$\left(\frac{1}{K_e}\right)^{1/2} = \left(\frac{1}{K_{10}}\right)^{1/2} + \left(\frac{1}{K_8}\right)^{1/2} = \left(\frac{1}{2.51}\right)^{1/2} + \left(\frac{1}{7.65}\right)^{1/2}$$

or $K_e = 1.014$. Omitting local losses, the work-energy equation for the change in water surface elevation ΔWS between the reservoirs is

$$\Delta WS = 40 = \left(K_{12} + K_e\right)Q^2$$

and $Q = 3.64$ ft^3/s. From Eq. 2.38 we then find $Q_{10} = 2.31$ ft^3/s and $Q_8 = 1.33$ ft^3/s.

* * *

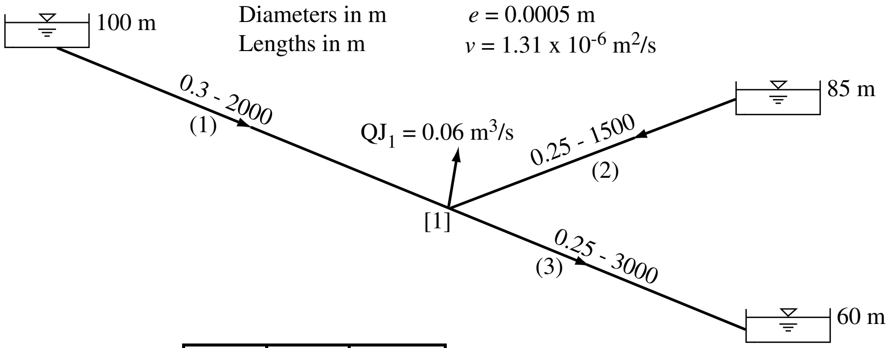
If the friction factor is known, no iteration is needed in such a problem. This will be the case for problems involving large discharges and rough pipes, for the friction factor will then come from the wholly rough flow region of the Moody diagram. For problems in which the friction factors are found in the transitional turbulent region of the Moody diagram, some iteration to determine the friction factors will be required, but it is iteration only to determine the correct friction factors.

2.4.4. THREE RESERVOIR PROBLEM

Problems involving pipe flow between more than two reservoirs will always require some form of iterative solution. Here we examine briefly an economical solution strategy for these problems. In Chapter 4 a computer-oriented solution to such problems will be detailed.

Example Problem 2.7

The figure below is a diagram of the three reservoir problem; the reservoirs are connected by three pipes with an external demand at the common junction of the pipes. The highest reservoir has a water surface elevation of 100 m; the middle reservoir water surface elevation is 85 m; and the lowest reservoir has a water surface elevation of 60 m. Determine the discharge in each pipe.



Pipe	K	n
1	1469	1.974
2	2432	1.927
3	5646	1.971

It is clear that flow is out of the upper reservoir and into the lowest reservoir. What is unclear is the direction of flow in the pipe that connects the middle reservoir to the system. The key step is to determine that direction in only one trial.

Let H_J be the head at the junction. The discharges in pipes 1 and 2 can then be found from these two head loss equations:

$$100 - K_1 Q_1^{n_1} = H_J \qquad H_J - K_3 Q_3^{n_3} = 60$$

Now select $H_J = 85$ m, the water surface elevation of the middle reservoir, so that there is no flow in pipe 2 for the first trial solution. Inserting values of K and n from the table, we find $Q_1 = 0.0980$ m^3/s and $Q_3 = 0.0639$ m^3/s. These values, combined with the external demand QJ_1, do not satisfy continuity at the junction J. To satisfy junction continuity we need more inflow to the junction, so H_J must be less that 85 m; thus we find that the flow in pipe 2 will be toward the junction and will be governed by

$$85 - K_2 Q_2^{n_2} = H_J$$

The junction continuity error for each trial will be $Q_e = Q_1 + Q_2 - Q_3 - QJ_1$. Now we select trial values for H_J, use the three head loss equations to compute the discharges and finally compute the error Q_e. Each trial outcome can be compactly recorded in a table:

H_J m	Q_1 m^3/s	Q_2 m^3/s	Q_3 m^3/s	Q_e m^3/s
85.0	0.0980	0.0	0.0639	- 0.0259
80.0	0.1134	0.0403	0.0571	0.0366
83.0	0.1045	0.0251	0.0613	0.0083
83.5	0.1029	0.0216	0.0620	0.0025
83.7	0.1023	0.0200	0.0622	0.0001

The systematic assignment of values to the head at the junction, which is itself usually not of great interest, is the step which allows us to search methodically for the solution. This approach can also be applied productively to similar problems which may even contain more than three reservoirs. The repeated manual intervention in selecting the trial values of H_J may make other procedures more attractive for solutions by computer, however.

2.5 PROBLEMS

2.1 For the following pipe flows determine whether the flow is laminar, turbulent smooth, turbulent rough, or turbulent transition, using the Moody diagram, Fig. 2.2.

(a) A velocity of 3.05 m/s (10 ft/s) occurs in a cast iron pipe having $e = 2.6\times10^{-4}$ m (8.5×10^{-4} ft) which is 2.54 cm (1 in) in diameter. The fluid kinematic viscosity is $\nu = 9.29\times10^{-5}$ m^2/s (10^{-3} ft^2/s).

(b) A velocity of 2.44 m/s (8 ft/s) occurs in a cast iron pipe having $e = 2.6\times10^{-4}$ m (8.5×10^{-4} ft) which is 0.15 m (6 in) in diameter. Use $\nu = 9.29\times10^{-8}$ m^2/s (10^{-6} ft^2/s).

(c) The velocity is 2.44 m/s (8 ft/s) in a 0.91 m (3 ft) diameter welded steel pipe having $e = 4.6\times10^{-5}$ m (1.5×10^{-4} ft). Use $\nu = 9.29\times10^{-5}$ m^2/s (10^{-3} ft^2/s).

(d) The velocity is 2.44 m/s (8 ft/s) in a 0.91 m (3 ft) diameter welded steel pipe having $e = 4.6\times10^{-5}$ m (1.5×10^{-4} ft). Use $\nu = 9.29\times10^{-7}$ m^2/s (10^{-5} ft^2/s).

2.2 A 250 mm diameter pipe is 1500 m long. When the discharge is 0.095 m^3/s in this pipe, the pressure drop between the ends of the pipe is measured as 98.06 kPa. The elevation at the end of the pipe is 10 m below its beginning. What type of flow is this? What is the equivalent sand-grain roughness of the pipe wall? What is the Hazen-Williams roughness coefficient? How much energy is dissipated by fluid friction during each hour that this flow continues? Use $\nu = 1.31\times10^{-6}$ m^2/s.

2.3 Find the pressure drop in 1000 m (3280 ft) of 0.10 m (0.33 ft) diameter pipe carrying 0.015 m^3/s (0.53 ft^3/s) of olive oil at 10 ^{o}C (50 ^{o}F). The downstream end of the pipe is 10 m (32.8 ft) below the upstream end.

2.4 Determine the discharge that will occur in a 450 mm diameter pipe that is 1000 m long connecting two reservoirs with a difference in water surface elevations of 25 m. The wall roughness of the pipe is $e = 0.12$ mm, and $\nu = 1.31\times10^{-6}$ m^2/s. How much head must a pump supply to reverse the flow, i.e. cause the same discharge to flow from the lower to the upper reservoir? What power must be supplied by electricity to this pump if the combined efficiency of the pump and motor is 75%?

2.5 A 0.305 m (1 ft) diameter concrete pipe that is 366 m (1200 ft) long carries water from a reservoir with surface elevation 1086 m (3560 ft) to a ditch at elevation 1041 m (3415 ft). If the Hazen-Williams roughness coefficient is 120, find the discharge through the pipe.

2.6 Determine the minimum pipe size to convey 0.028 m^3/s (1 ft^3/s) of water at 15^{o}C (60^{o}F) for new cast iron pipe that is 914 m (3000 ft) long with a change in HGL between the ends of 15.2 m (50 ft).

2.7 Determine the values of K and n in the exponential formula $h_f = KQ^n$, based on the Darcy-Weisbach and Hazen-Williams formulas for these pipes:

(a) $L = 1000$ ft, $D = 6$ inches, $e = 0.002$ inches, $C_{HW} = 110$, $V \approx 8$ ft/s.

(b) $L = 1000$ m, $D = 0.2$ m, $e = 0.005$ m, $C_{HW} = 140$, $V \approx 2$ m/s.

(c) $h_f = 50$ ft, $L = 3000$ ft, $D = 8$ inches, $e = 0.0102$ inches, $C_{HW} = 120$.

2.8 Plot the K and n values that were found in Example Problem 2.1 from the Darcy-Weisbach equation on a Moody diagram. How close are the slopes of these lines on the Moody diagram to the slopes of the Hazen-Williams and Manning equations? From this comparison develop some guidelines for when the Hazen-Williams equation is most appropriate, and when the Manning equation may be more appropriate.

2.9 Use the K and n values that were found in Example Problem 2.1 from the Darcy-Weisbach, Hazen-Williams and Manning equations to compute the head losses associated with discharges that are 50 and 200% of the given approximate Q, and compare the results.

2.10 If the friction factor is held constant, show that the Darcy-Weisbach equation indicates that the head loss is proportional to the velocity squared, or the discharge squared, just as the Manning equation does. For what flow type(s) is such a relation appropriate?

2.11 Determine the coefficient K and the exponent n in $h_f = KQ^n$ for the pipes in the table which follows by using both the Darcy-Weisbach and Hazen-Williams equations. The water flows in the pipe at about 6 m/s and has a temperature of 10°C.

Pipe No.	**Type**	**Dia.** m	**Length** m	**Darcy-Weisbach** K	n	**Hazen-Williams** K	n
1	Smooth concrete	2.50	1000				
2	PVC	0.25	1500				
3	Old cast iron	0.08	800				
4	e = 0.005 mm	0.35	2000				

2.12 For pipes 1 and 3 in Problem 2.11, determine the equivalent length of pipe that could be used to replace the minor loss caused by a globe value (K = 10). If needed, assume a velocity of 6 m/s in the pipe.

2.13 Determine the discharge of water at 20°C (68°F) through a 10 cm (4 in) diameter concrete pipe that is 457 m (1500 ft) long. Assume the wall roughness is e = 0.61 mm (0.002 ft). The pipe connects two reservoirs with a 6.1 m (20 ft) difference in water surface elevation.

2.14 One-tenth m^3/s (3.53 ft^3/s) of water at 20°C (68°F) flows through a 0.25 m (0.82 ft) diameter cast iron pipe. Find the head loss in 200 m (656 ft) of this pipe.

2.15 Compare the head loss for a discharge of 0.1 m^3/s (3.53 ft^3/s) of water at 20°C (68°F) through (a) a 0.20 m (8 in) diameter concrete pipe with (b) a 0.20 m (8 in) diame-ter PVC pipe.

2.16 Water at 10°C (50°F) flows between reservoirs through a 0.30 m (1 ft) diameter cast iron pipe that is 1 km (3280 ft) long. Find the difference in elevation between the reservoirs if the discharge is 0.2 m^3/s (7.1 ft^3/s).

2.17 Water is to be pumped from a lake to a canal which is 200 m (656 ft) distant and 20 m (65.6 ft) higher in elevation. If 0.5 m^3/s (17.66 ft^3/s) of water at 20°C (68°F) is to be delivered through a 0.5 m (1.64 ft) concrete pipe, what power must the pump deliver to the water?

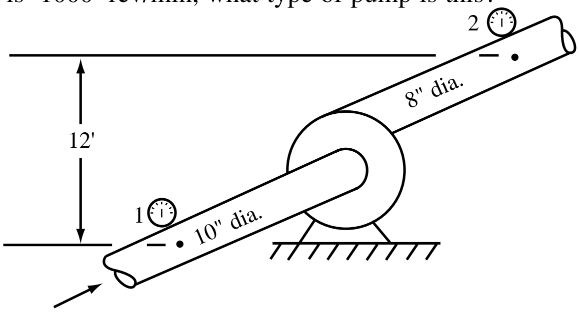
2.18 Find the power which pumps must supply to 3 m^3/s (106 ft^3/s) of water at 20°C (68°F) which is to be delivered from the Snake River to the plateau 180 m (591 ft) above the river through 1100 m (3610 ft) of 1 m (3.28 ft) asphalt-dipped cast iron pipe.

2.19 Use the Hazen-Williams formula to find h_f when $0.013\ m^3/s$ ($0.46\ ft^3/s$) of water at 20°C (68°F) flows through 300 m (984 ft) of 75 mm (0.25 ft) diameter smooth pipe.

2.20 A power plant is 16 km (52,500 ft) from a reservoir. A discharge of $25\ m^3/s$ ($883\ ft^3/s$) is to be delivered to the plant at an elevation that is 1120 m (3,670 ft) below the reservoir surface. What size of riveted steel pipe is required? Assume a temperature of 4°C (40°F).

2.21 What diameter of commercial steel pipe will convey $0.003\ m^3/s$ ($0.106\ ft^3/s$) of crude oil at 40°C (104°F) with a pressure drop of 15 kPa ($2.18\ lb/in^2$) per 30 m (98 ft)?

2.22 The pump shown below delivers $8\ ft^3/s$ of water. The recorded pressures at sections 1 and 2 on the gauges are $-5.0\ lb/in^2$ and $+35.0\ lb/in^2$. (a) Draw a diagram of the system and locate the EL and HGL at sections 1 and 2 in the diagram. (b) Determine the required h_p and power that must be supplied by the pump to the water to deliver this discharge. Neglect pipe friction and local losses. (c) If the rotative speed of the pump impeller is 1000 rev/min, what type of pump is this?



2.23 You are asked to design a pipe line for a farmer which will carry $0.2\ m^3/s$ of water from a lake on a mountainside at elevation 1905 m to a farm sprinkler system 6 km away at elevation 1795 m. The sprinklers require a pressure of 400 kPa to operate properly. PVC pipe is to be used. Assume a temperature of 10°C.

2.24 A farmer wants you to design his irrigation pipe line so it can be used in the winter to generate electricity for his home. He wants to run a 20 kW turbine-generator (70% efficient) from the $0.05\ m^3/s$ stream. The PVC pipe line is 1050 m long, and the up-stream end is 75 m above the turbine. What pipe diameter should be selected? Assume a temperature of 10°C.

2.25 Use a computer program to generate several tables of f versus Re for different values of relative roughness e/D, and use these to plot several curves on a Moody diagram with a spreadsheet or other graphing software.

2.26 How much energy per unit weight would be saved by using a long radius elbow instead of a short radius elbow in a 0.30 m (1 ft) diameter pipe with a discharge of 0.23 m^3/s ($8\ ft^3/s$) of water at 20°C (68°F)?

2.27 What loss is caused by a close return bend in a 0.15 m (0.49 ft) diameter pipe carrying a discharge of $0.1\ m^3/s$ ($3.53\ ft^3/s$) of gasoline at 20°C (68°F)? How does this loss compare with the use of two short radius bends? Two long radius elbows?

2.28 A discharge of $0.283\ m^3/s$ ($10\ ft^3/s$) flows in a 0.30 m (1 ft) diameter pipe. Compare the head losses for a completely open (a) angle valve, (b) gate valve, and (c) globe valve. Under what conditions would you select the gate valve? One of the other valves?

2.29 An irrigation siphon tube is 76 mm (3 in) in diameter and 3 m (9.84 ft) long. Estimate the discharge for a head difference of 0.5 m (1.64 ft), assuming a re-entrant entrance, an equivalent sand-grain roughness $e = 0.06$ mm (2.36×10^{-3} in), and two bends with loss coefficients of 0.2. Draw the system, including the EL and HGL.

2.30 To obtain more electrical energy during the day when there is a shortage and use it during the late night when there is a surplus, a power company plans to pump water from a lake to a reservoir through a 0.5 m diameter pipe that is 2500 m long ($e = 0.001$ m); when the power is needed, the company will run that water through a turbine. The elevation difference between the reservoir and lake water surfaces is 90 m. Surplus electrical energy costs \$0.02/kWh, prime time energy is worth \$0.10/kWh, and the efficiencies of the pump and turbine are 80 percent. Analyze the hydraulics and economics of the proposed plan. Suggest the discharges that should be used.

2.31 Write a program for a computer or calculator for determining the unknown discharge Q in a pipe line (Category 2), including local losses.

2.32 Write a program for a computer or calculator for determining the unknown diameter of a pipe (Category 3), including local losses.

CHAPTER 3

MANIFOLD FLOW

3.1 INTRODUCTION

Every hydraulic manifold consists of one relatively large pipe, or several in some kind of series configuration, which may be called the barrel or main. Along each main pipe there are numerous junctions with small pipes or there are numerous ports, all allowing flow from the main or (less common) all allowing flow into the main. One characteristic of manifolds is the presence of many junctions or ports, usually relatively closely spaced but not so close that the flow at adjacent ports interacts. Every flow in a manifold is a spatially varied flow, and flows in manifolds are almost always analyzed as steady flows, as we will do in this chapter.

Although manifold flow is a less-frequently studied topic than the flow in networks or the behavior of hydraulic transients, this flow type does have numerous practical applications. Manifold flow has several kinds of applications to farm irrigation systems (Jensen, M. E., 1983; U. S. Soil Conservation Service, 1984; James, L. G., 1988; Cuenca, R. H., 1989; Keller, J. and Bliesner, R. D., 1990), including recent research on trickle and sprinkler systems (e.g., Scaloppi, E. J. and Allen, R. G., 1993; Hathoot, H. M. et al., 1994). Protective fire sprinkler systems in buildings are another application. Marine outfall systems (Vigander, S. et al., 1970; Grace, R. A., 1978) rely on manifolds for the initial distribution of the wastewater into the receiving water body through multi-port diffuser manifolds. The filling and emptying systems for large locks on navigable waterways are basically manifolds (Richardson, G. C., 1964, 1969; Stockstill, R. L. et al., 1991). And the ventilation of vehicle tunnels also relies in part on an understanding of manifold flow (Pursall, B. R. and King, A. L., 1976).

This chapter will first describe several levels of analysis that are applicable to manifold flow; they differ in whether friction is considered and whether junction losses are considered. We will then look at one example of an analysis of the internal hydraulics of a marine outfall diffuser and show how this approach can easily be aided with a short computer program. Articles by McNown (1954) and Rawn et al. (1961) and the book by Miller (1984) are good places to begin further study of this topic.

3.2 ANALYSIS OF MANIFOLD FLOW

In this section we will look at the analysis of flow in a manifold on three levels. The first level will ignore all energy losses; although this assumption is unrealistic, it will serve as an introduction to manifold flow and allow us to unlearn some flow behavior from the flow in pipes which is not spatially varied. In the second and third levels we progressively add friction in the barrel or main, and a consideration of energy losses at junctions or ports. At the end of these analyses we can draw some conclusions about the importance of barrel friction and junction losses in various applications.

3.2.1. NO FRICTION

Primarily as an introduction to the subject, let us look briefly at the schematic diagram of a small, simple manifold having only a few equally-spaced circular exit ports, all of the same diameter, as shown in Fig. 3.1. The downstream end of the main is a dead end. In the complete absence of real-fluid effects, the reservoir level on the left sets the elevation of the horizontal energy line along the entire manifold, which is shown here as having five single exit ports that are relatively closely spaced. A sectional view is shown on the right,

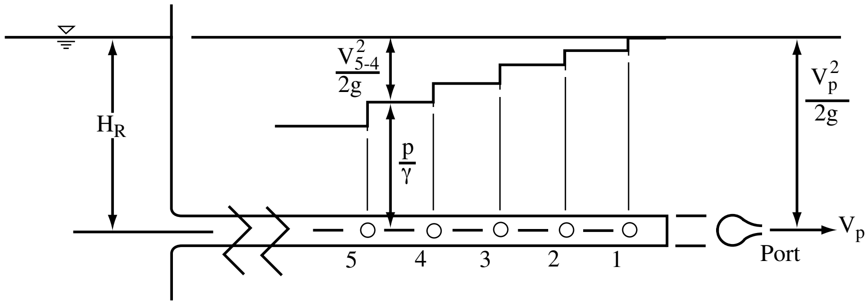


Figure 3.1 A small manifold, no pipe friction or junction losses.

with the transition region from the main to the exit point being rounded to suggest that energy losses in the port region can indeed be neglected as a first approximation. For convenience in the analyses, the ports are numbered from the downstream end toward the reservoir, beginning with 1. The key feature here is the behavior of the hydraulic grade line for this flow. As always, we can locate the hydraulic grade line by measuring down a distance of $V^2/2g$ from the energy line to it, in which V is the mean velocity in the barrel segment. Since this mean velocity becomes progressively larger as we move from the lower- to higher-numbered ports, the hydraulic grade line, and therefore the pressure head p/γ, is farthest below the energy line at the upstream end of the barrel. Since it is usual to think of the pressure in a horizontal pipe as decreasing in the direction of flow, we have an immediate indication that some care will be required if we are to avoid reaching incorrect conclusions as we study manifold flow.

In the absence of energy losses, the velocity from each port is $V_p = [2gH_R]^{1/2}$. The discharge from each port is then identical if the ports are all the same. With identical ports, only two factors can cause the discharge to change from port to port: differing energy levels from port to port, and junction energy losses. We shall look at both factors in the next two sections.

A reading of past literature will reveal two points of view on the physics of the flow out of a port: Some articles assume that only the pressure head in the main is responsible for driving the fluid out of an adjacent port. Others, including this text, write a work-energy equation between the main and the exit point of the port; this approach assumes that the full distance between the EL in the main and the exit point drives the flow. The existence of loss coefficients and discharge coefficients, which play somewhat differing roles, depending on the point of view, allows the two approaches to be made compatible with one another.

3.2.2. BARREL FRICTION ONLY

When barrel friction is considered, then the energy line slopes downward as a sequence of straight-line segments in the direction of flow in the barrel, as shown in Fig. 3.2. As we look from port 1 to port 5, we find the velocity head in the barrel grows as it did in the absence of friction, and each segment of the hydraulic grade line along the barrel also has a slope that is parallel to the energy line above it. We find the port velocity is $V_{pi} = [2gH_i]^{1/2}$, in which H_i is the vertical distance from the centerline of port i to the local energy line above that port. And the discharge in the barrel changes in each segment, in accordance with continuity at each junction.

In the manifold section of length L with five equally-spaced ports, we may record the frictional head loss as

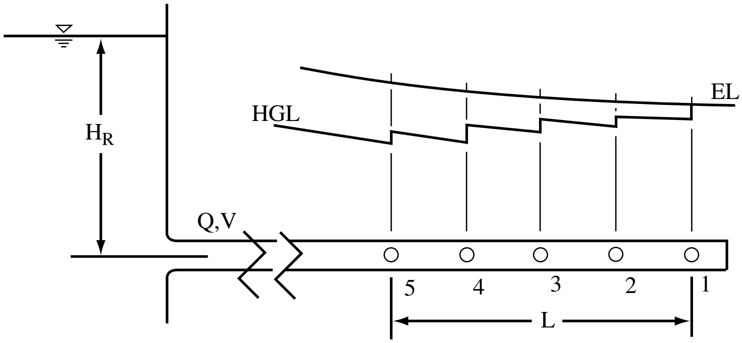


Figure 3.2 A small manifold, n = 5 ports, with barrel friction but no junction losses.

$$\Sigma h_L = f\frac{L/4}{D}\frac{V_{5-4}^2}{2g} + f\frac{L/4}{D}\frac{V_{4-3}^2}{2g} + f\frac{L/4}{D}\frac{V_{3-2}^2}{2g} + f\frac{L/4}{D}\frac{V_{2-1}^2}{2g} \tag{3.1}$$

if we assume that the pipe Reynolds number is sufficiently high that the Darcy-Weisbach friction factor f is a constant over the range of flow in the barrel.

Various results can be developed from Eq. 3.1 or an equation like it, depending on the diameters of the ports. For example, with a total discharge Q_T and the assumption that the diameter of each port is chosen so that equal discharges issue from each of the five ports, that is, $Q_p = Q_T/5$, then in each barrel segment port continuity shows that $V_{5\text{-}4} = 4V/5$, $V_{4\text{-}3} = 3V/5$, $V_{3\text{-}2} = 2V/5$, and $V_{2\text{-}1} = V/5$. Then Eq. 3.1 will simplify to

$$\Sigma h_L = \frac{1}{4}\left[\left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2 + \left(\frac{2}{5}\right)^2 + \left(\frac{1}{5}\right)^2\right] f\frac{L}{D}\frac{V^2}{2g} = 0.3f\frac{L}{D}\frac{V^2}{2g} \tag{3.2}$$

with $V = Q_T/A$ and A = cross-sectional area of the main. If instead there were n ports with equal discharges $Q_p = Q_T/n$, then by using mathematical induction we find that the total frictional head loss for the section of the main containing the ports is

$$\Sigma h_L = \frac{1}{n-1}\left[\frac{1}{n^2}\sum_{i=1}^{(n-1)} i^2\right] f\frac{L}{D}\frac{V^2}{2g} = \frac{1}{n-1}\left[\frac{1}{n^2}\frac{n}{6}(n-1)(2n-1)\right] f\frac{L}{D}\frac{V^2}{2g} \tag{3.3}$$

or

$$\Sigma h_L = \left(\frac{1}{3} - \frac{1}{6n}\right) f\frac{L}{D}\frac{V^2}{2g} \tag{3.4}$$

However, the unhappy fact is that the diameter of each port must differ, if only slightly, from the diameters of the other port openings for this expression to be completely applicable. But Eq. 3.4 may still be useful in obtaining an approximation for the head loss over a group of n uniformly spaced ports in a distance L in a barrel.

3.2.3. BARREL FRICTION WITH JUNCTION LOSSES

Now the state of affairs at the intersection of the barrel and a pipe lateral of smaller diameter, both assumed here to lie in one horizontal plane, is relatively complex. We begin with a diagram, Fig. 3.3, of one such barrel-lateral junction that displays the energy line EL and hydraulic grade line HGL for the main and the lateral and also introduces a set of locally-numbered variables: subscript 1 denotes a variable that is defined upstream of the lateral in the main; subscript 2 denotes a variable that is defined downstream of the lateral in the main; and subscript 3 denotes a variable that is associated with the lateral itself. It is assumed that the spacing of the laterals is such that the flow to successive laterals does not interact. The energy line now has a loss $h_{L_{1-2}}$ along the main at the junction, and there is also an energy loss $h_{L_{1-3}}$ at the junction that is associated with the flow that passes into the lateral. The hydraulic grade line experiences a rise Δh along the main as it passes the junction. We must keep in mind that it is the art/science of approximation in hydraulics that expresses these energy and pressure changes as discrete jumps at a precise location; actually all three factors represent phenomena that occur over a larger but finite flow region, although we concentrate or lump the effect at a point. All parts of the energy line slope downward in the direction of flow in the main and in the lateral, owing to the

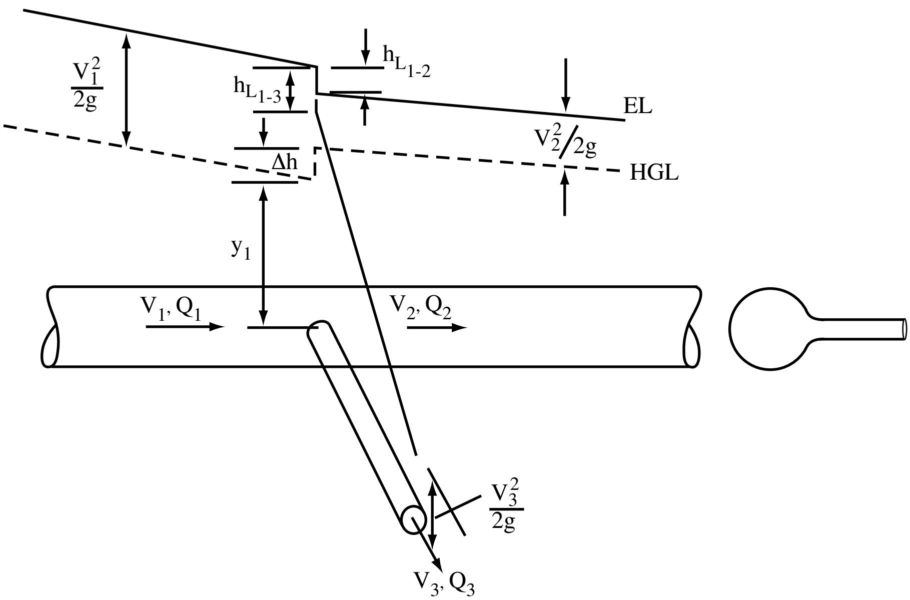


Figure 3.3 Diagram of a barrel-lateral junction with local notation.

effect of fluid friction. The flow from the lateral is presumed to exit as a jet into the atmosphere.

From Fig. 3.3 we see that the pressure head rise along the main is

$$\Delta h = \frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_{L_{1-2}} \tag{3.5}$$

Dividing all by the upstream velocity head produces a nondimensional pressure head rise coefficient

$$\frac{\Delta h}{V_1^2/2g} = 1 - \left(\frac{V_2}{V_1}\right)^2 - \frac{h_{L_{1-2}}}{V_1^2/2g} \tag{3.6}$$

or

$$\frac{\Delta h}{V_1^2/2g} = 1 - \left(\frac{Q_2}{Q_1}\right)^2 - \frac{h_{L_{1-2}}}{V_1^2/2g} \tag{3.7}$$

Applying continuity at the junction in the form $Q_1 = Q_2 + Q_3$ leads to

$$\frac{\Delta h}{V_1^2/2g} = 1 - \left(1 - \frac{Q_3}{Q_1}\right)^2 - \frac{h_{L_{1-2}}}{V_1^2/2g} = 2\left(\frac{Q_3}{Q_1}\right) - \left(\frac{Q_3}{Q_1}\right)^2 - \frac{h_{L_{1-2}}}{V_1^2/2g} \qquad (3.8)$$

If we employ the usual terminology, the last term in Eq. 3.8 is the head loss coefficient $K_{L_{1-2}}$. Hence we can conclude that the pressure head rise coefficient is a function of only two nondimensional factors, or

$$\frac{\Delta h}{V_1^2/2g} = \Phi_1\left(\frac{Q_3}{Q_1}, \frac{h_{L_{1-2}}}{V_1^2/2g}\right) \qquad (3.9)$$

in which Φ_1 is the function appearing in Eq. 3.8.

Statements about the functional behavior of the pressure head rise coefficient can be made if we hypothesize how $h_{L_{1-2}}$ behaves; Figure 3.4 is the outcome of such an inquiry.

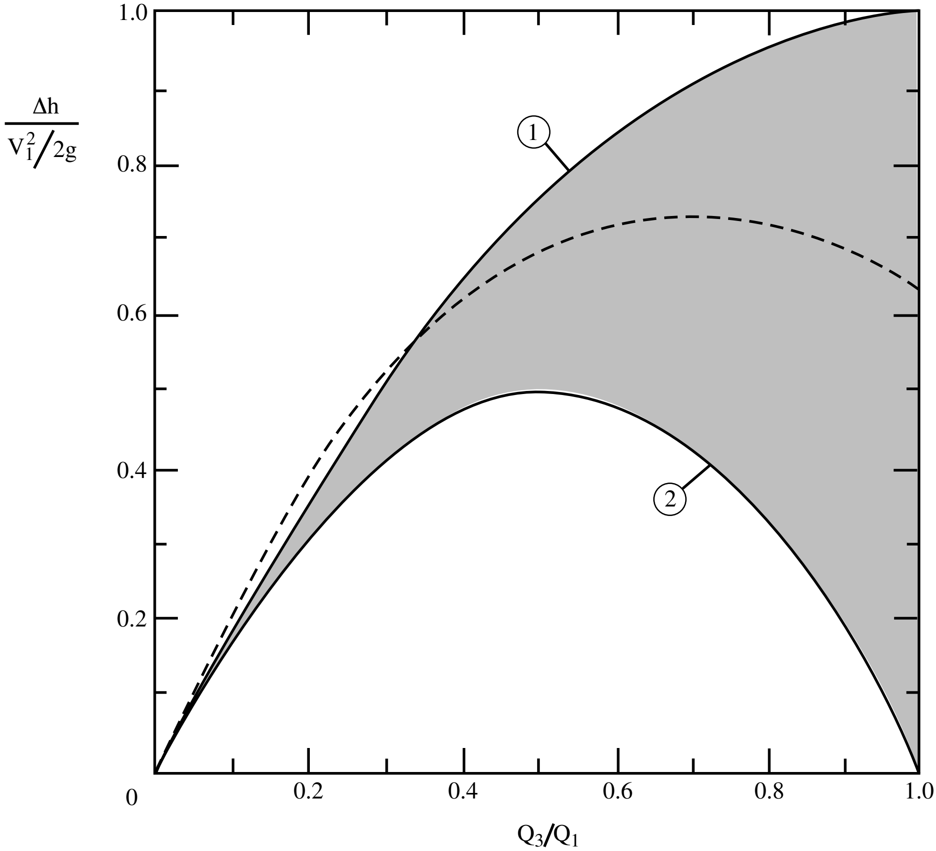


Figure 3.4 The expected range of pressure rise coefficients as a function of Q_3/Q_1.

To begin probing this point, it does not seem difficult to delimit the range of possible values for $h_{L_{1-2}}$. At the low end it seems reasonable simply to assume $(h_{L_{1-2}})_{min} = 0$, i.e., no loss. At the high end of the spectrum we note that the flow in the main at the junction displays some of the character of the flow at a sudden enlargement, as a rapid deceleration of the flow occurs in the barrel, accompanied by some increase in eddy motion and other turbulence phenomena. Hence we expect

$$\left(h_{L_{1-2}}\right)_{max} = \frac{\left(V_1 - V_2\right)^2}{2g} \tag{3.10}$$

This behavior for the pressure head rise coefficient is plotted in Fig. 3.4 as a function of the discharge ratio Q_3/Q_1; curve 1 is the curve for minimum head loss, and curve 2 is the result of using Eq. 3.10 to represent the head loss. Superimposed on Fig. 3.4 is a dashed curve that is taken from Fig. 3.5, which shows experimental data (unpublished) for the typical behavior of the pressure head rise coefficient as a function of the lateral-to-main diameter ratio D_3/D_1, assuming in this example that $f = 0.02$ and $L_3/D_3 = 5$ for the lateral.

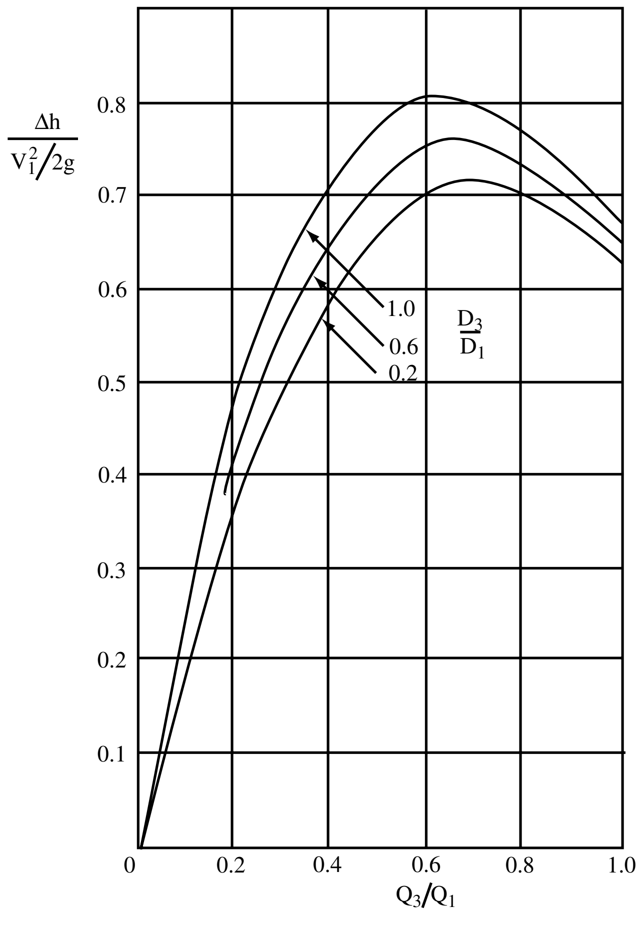


Figure 3.5 Typical experimental data for the pressure rise coefficient.

From Fig. 3.4 we find a remarkable result. The experimental pressure head rise coefficient data do not fall within the rather generous region of expected behavior for D_3/D_1 below about 1/3. The clear meaning is that the loss coefficient $K_{L_{1-2}}$ is negative for small values of D_3/D_1. How can this be? It is not simply caused by experimental error but is a real phenomenon, another of the peculiarities of manifold flow, the cause of which has been debated at some length. Experts conclude that the flow must be something like the diagram in Fig. 3.6, in which a small fraction of the upstream discharge is drawn into the lateral. This lateral fraction initially possessed less than the average kinetic energy per unit weight of fluid, since the fluid near the pipe wall moves more slowly than the central core fluid. Consequently the mean energy that remains to flow to section 2 appears to have been enhanced by a small amount. In a one-dimensional hydraulic representation of the flow, the effect shows up as a small "negative" loss coefficient, however unrealistic that may seem. Additional study would show that a loss in the overall flux of energy does still occur in this situation.

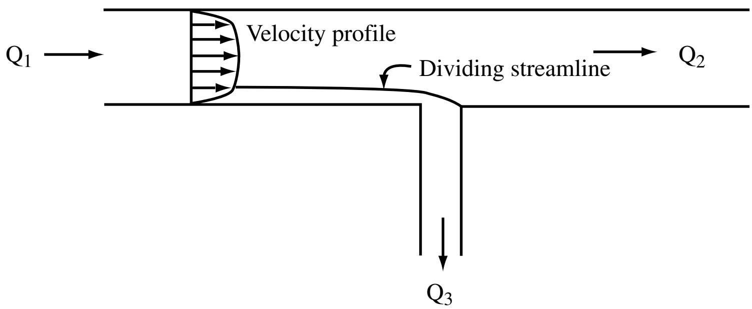


Figure 3.6 Flow at a lateral junction.

The behavior of the flow in the lateral must also be quantified. The habit began long ago of treating these laterals as if they were orifices and assuming for convenience that the flow through the lateral was driven by the pressure head y_1 that exists just upstream from the entrance to the lateral. Thus we write

$$Q_3 = CA_3\sqrt{2gy_1} \quad \text{or} \quad y_1 = \frac{1}{C^2}\frac{V_3^2}{2g} \tag{3.11}$$

in which $A_3 = \pi D_3^2/4$ is the cross-sectional area of a lateral of diameter D_3, the velocity $V_3 = Q_3/A_3$, and C is the orifice coefficient, which can depend on several variables, depending on the geometric details of the lateral.

We want to establish a relation between C and the head loss coefficient $K_{L_{1-3}}$ for flow from the main to the lateral

$$K_{L_{1-3}} = \frac{h_{L_{1-3}}}{V_1^2/2g} \tag{3.12}$$

since this coefficient can be found experimentally. Assuming that the lateral flows full and has length L_3 and a constant friction factor f_3, from Fig. 3.3 we can write

$$y_1 + \frac{V_1^2}{2g} = h_{L_{1-3}} + h_{f_3} + \frac{V_3^2}{2g} \tag{3.13}$$

with h_{f_3} being the frictional head loss in the lateral, or

$$\frac{1}{C^2}\frac{V_3^2}{2g} + \frac{V_1^2}{2g} = K_{L_{1-3}}\frac{V_1^2}{2g} + f_3\frac{L_3}{D_3}\frac{V_3^2}{2g} + \frac{V_3^2}{2g} \tag{3.14}$$

Dividing throughout by the velocity head in the lateral and rearranging,

$$\frac{1}{C^2} = \left(K_{L_{1-3}} - 1\right)\left(\frac{V_1}{V_3}\right)^2 + \left(1 + f_3 \frac{L_3}{D_3}\right) \tag{3.15}$$

or

$$\frac{1}{C^2} = \left(K_{L_{1-3}} - 1\right)\left(\frac{Q_1}{Q_3}\right)^2 \left(\frac{D_3}{D_1}\right)^4 + \left(1 + f_3 \frac{L_3}{D_3}\right) \tag{3.16}$$

In summary,

$$C = \Phi_2\left(\frac{Q_3}{Q_1},\ K_{L_{1-3}}\right) \tag{3.17}$$

in which Φ_2 is a shorthand notation for the function displayed in full in Eq. 3.16. When

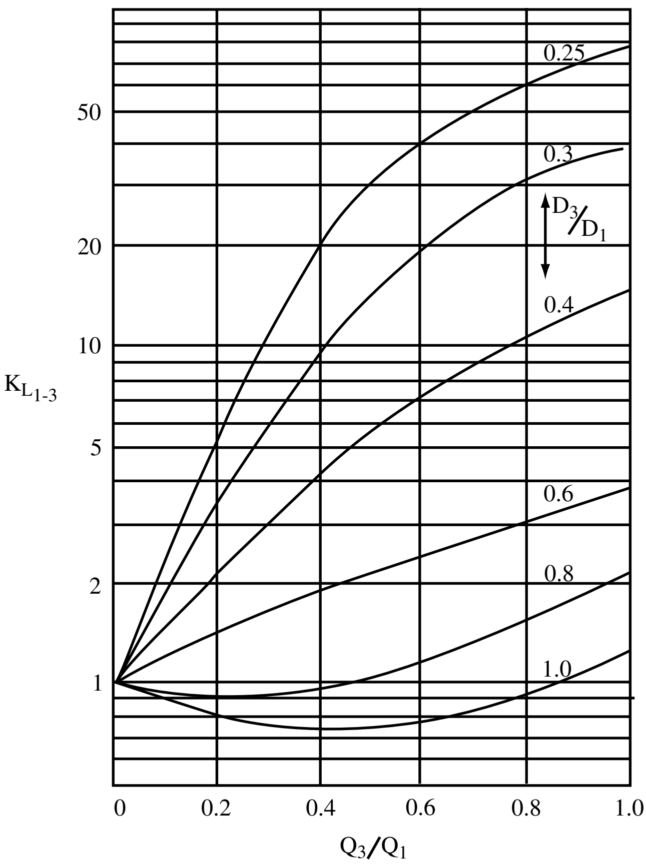


Figure 3.7 The loss coefficient $K_{L_{1-3}}$ as a function of D_3/D_1 and Q_3/Q_1.

appropriate experiments have been conducted to determine the behavior of $K_{L_{1-3}}$, one will usually find a relation that is similar to that shown in Fig. 3.7. And once f_3 and L_3/D_3 have been prescribed, then a plot of C vs. Q_3/Q_1 can be prepared; for example, Fig. 3.8 has been prepared from Fig. 3.7 by assuming $f_3 = 0.02$ and $L_3/D_3 = 5$. (Some will be surprised to see how large the lateral loss coefficient may become; keep in mind, however, that a lateral that is less than 1/3 the diameter of the main will normally convey

1/3 or less of the upstream discharge to that junction, so such high loss coefficients are rarely encountered in practice.)

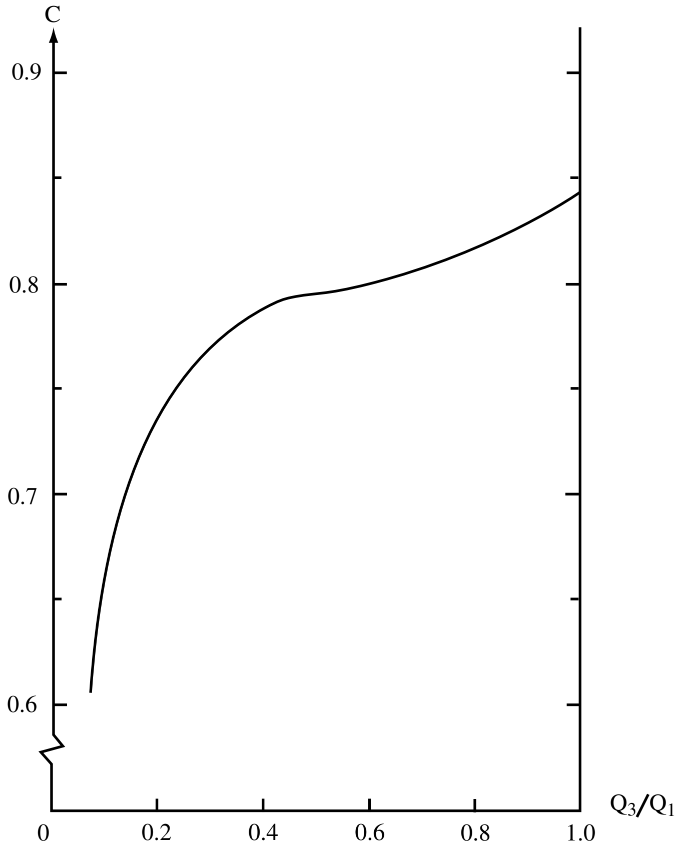


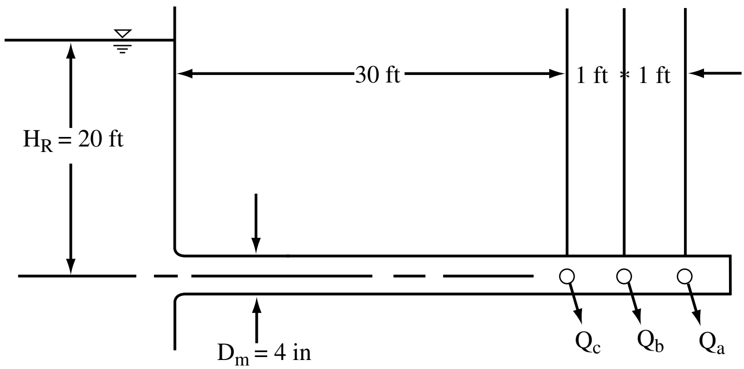
Figure 3.8 An example of the behavior of the orifice coefficient C.

Example Problem 3.1

The 3-port manifold shown in the next diagram has a port-to-main diameter ratio $D_3/D_1 = D_3/D_m = 0.4$, a friction factor $f = 0.02$ in the main and all laterals, and $L_3/D_3 = 5$ for each lateral. Considering fluid friction in the main and laterals and junction losses, as described by Figs. 3.5, 3.7, and 3.8, compute the port discharges Q_a, Q_b, and Q_c. The downstream end of the main is closed off by a blank plate.

This problem is more difficult than earlier problems where the junction losses were ignored, but the results are valuable in helping us decide whether to include or ignore junction losses in other similar problems.

Such a problem can be formulated in terms of a set of linear and nonlinear simultaneous equations, but in the past solutions to this problem were normally sought by following the



method that will now be used. The solution process typically begins by arbitrarily selecting an energy line elevation at the downstream end of the manifold, and computations are started there. Of course, the initial elevation will almost never be the correct final elevation, but it is easy to adjust for this later in the computations. So we begin by choosing EL = HGL = 10.0 ft downstream of port 1.

At port a the ratio $Q_{lateral}/Q_{main} = Q_3/Q_1 = Q_a/Q = 1.0$. Just before this port we see that $y_1 + \Delta h = 10.00$, with the discharge out the first port satisfying

$$Q_a = Q_3 = C\frac{\pi D_3^2}{4}\sqrt{2gy_1}$$

From Fig. 3.5 we read $\dfrac{\Delta h}{V_1^2/2g} = 0.63$, and from Fig. 3.8 we find $C = 0.84$. Hence

$$\left(\frac{Q_a}{0.84}\frac{4}{\pi D_3^2}\right)^2\frac{1}{2g} + 0.63\left(\frac{4Q_a}{\pi D_m^2}\right)^2\frac{1}{2g} = 10.00$$

with $D_3 = 0.4(4)/12 = 0.1333$ ft and $g = 32.2$ ft/s^2. From this equation we compute $Q_a = 0.296$ ft^3/s, from which $V_1^2/2g = 0.179$ ft and $\Delta h = 0.113$ ft, which establishes the values of the EL and HGL immediately before port a as 10.066 ft and 9.887 ft, respectively. With these values the frictional loss between port a and port b is 0.011 ft from the Darcy-Weisbach equation, giving EL and HGL elevations of 10.077 ft and 9.898 ft just downstream of port b.

With no prior experience upon which to anticipate the flow behavior at port b, the second port, the logical initial estimate for the discharge ratio is $Q_{lateral}/Q_{main} = Q_b/Q = 0.50$. Turning to the plots, from Fig. 3.5 we obtain $\dfrac{\Delta h}{V_1^2/2g} = 0.680$ and from Fig. 3.8 $C = 0.795$. With $Q = Q_b/0.5$, the equation $y_1 + \Delta h = 9.898$ ft at port b becomes

$$\left(\frac{Q_b}{0.795}\frac{4}{\pi D_3^2}\right)^2\frac{1}{2g} + 0.680\left(\frac{4(Q_b/0.5)}{\pi D_m^2}\right)^2\frac{1}{2g} = 9.898$$

or $Q_b = 0.274$ ft^3/s. The discharge in the main is then $0.296 + 0.274 = 0.570$ ft^3/s and $Q_b/Q = 0.274/0.570 = 0.481$, which is not 0.50 as assumed. Thus we repeat the calculation using $Q_b/Q = 0.481$, with 0.670 being read with some difficulty from Fig.

3.5 and $C = 0.795$ from Fig. 3.8. Also the discharge in the main shifts slightly to become $Q = Q_b/0.481 = 2.08Q_b$. The new result is $Q_b = 0.274$ ft^3/s again. For this discharge we can compute $V_1^2/2g =$ 0.662 ft and $\Delta h =$ 0.444 ft with $y_1 = 9.465$ ft, leading to EL and HGL elevations just before port b of 10.127 ft and 9.465 ft. The Darcy-Weisbach frictional loss between port b and port c is then 0.040 ft so that the EL and HGL elevations just downstream of port c are 10.167 ft and 9.505 ft.

At port c the uninformed initial estimate for the discharge ratio would be $Q_c/Q =$ 0.333. But from our experience at port b we may speculate that this ratio will be too high and instead choose our first estimate to be $Q_c/Q = 0.31$ so that $Q = Q_c/0.31 = 3.23Q_c$. Then Fig. 3.5 yields $\dfrac{\Delta h}{V_1^2/2g} = 0.545$, and $C = 0.770$ is obtained from Fig. 3.8. The equation $y_1 + \Delta h = 9.505$ ft at port c is then

$$\left(\frac{Q_c}{0.770}\frac{4}{\pi D_3^2}\right)^2\frac{1}{2g} + 0.545\left(\frac{4(3.23Q_c)}{\pi D_m^2}\right)^2\frac{1}{2g} = 9.505$$

giving $Q_c = 0.255$ ft^3/s. Then in the main $Q = 0.570 + 0.255 = 0.825$ ft^3/s, with a ratio $Q_c/Q = 0.255/0.825 = 0.31$. We have been fortunate in our choice of the estimate! Otherwise we must repeat the computational cycle of adjusting the discharge ratio and the coefficients that depend on it before again computing a new discharge at port c and checking the result for adequacy. By now it should be clear that a limiting factor in our ability to obtain an accurate solution that agrees with our starting estimates is the accuracy of the coefficients. Two factors affect this accuracy, the quality of the original experiments that led to the preparation of the coefficient plots and our limited ability to read those plots. As a result, Miller (1984) suggested that agreement within 2% is a reasonable goal. At least some of the computations in this example exceed this limit, but the results have been presented in this way so the computations can be followed more easily.

Some computations upstream of port c remain. With the discharge upstream of port c now computed, the velocity head in the main in this region is $V^2/2g = 1.388$ ft, and y_1 upstream of port c is

$$y_1 = \frac{1}{2g}\left(\frac{0.255}{(0.770)(0.01396)}\right)^2 = 8.739 \text{ ft}$$

The EL at this section is the sum of these two terms, or 10.127 ft. Just downstream of port c the EL was computed as 10.167 ft., so we observe the phenomenon of an apparent negative head loss occurring at port c. This effect is small, but it is not an error. Continuing, we compute the effect of the frictional head loss in 30 ft of pipe leading to the reservoir as

$$h_L = f\frac{L}{D}\frac{V^2}{2g} = 0.02\left(\frac{30}{4/12}\right)(1.388) = 2.498 \text{ ft}$$

so that the computed EL at the reservoir is $10.127 + 2.498 = 12.625$ ft. Alas, this value is actually specified as $H_R = 20$ ft. Our work is not wasted, however. Each velocity, and consequently each discharge, is proportional to the square root of the total head that is available in the problem, so long as f is assumed to be a constant. To adjust our estimated discharges to the true discharges, we need only multiply the estimates by the square root of the ratio of the true total head, 20 ft, to the computed head, 12.625 ft. The discharge from each port, in ft^3/s, is therefore

$$Q_{true} = Q_{est}\left[\frac{H_{true}}{H_{est}}\right]^{1/2}$$

$$Q_a = 0.296\left[\frac{20.0}{12.625}\right]^{1/2} = 0.373 \ \ \text{ft}^3/\text{s}$$

$$Q_b = 0.274\left[\frac{20.0}{12.625}\right]^{1/2} = 0.345 \ \ \text{ft}^3/\text{s}$$

$$Q_c = 0.255\left[\frac{20.0}{12.625}\right]^{1/2} = 0.321 \ \ \text{ft}^3/\text{s}$$

If it is desired, the actual elevation of each point on the EL and HGL could now be computed directly since the discharges are known.

The foregoing hand solution has acquainted us with the complexities that come with the inclusion of junction losses. The modern alternative to such a solution is to formulate the problem in terms of a set of equations that can be solved simultaneously for a chosen set of unknown variables. The CD contains both a MathCAD and a TK-Solver model of this problem, listed under the names PRB3_1.MCD and PRB3_1.TK, respectively, which are formulated in this way. For this example we denote the hydraulic grade line downstream ($y_1 + \Delta h$) from the three ports by HGL$_a$, HGL$_b$ and HGL$_c$. In a similar way the C's from Fig. 3.8 and the coefficients K from Fig. 3.5 will be given subscripts a, b, and c. The following three equations are the result of adding y_1 and Δh at the three ports:

$$\left(Q_a/C_a\right)^2/\left(2gA_a^2\right) + K_a Q_a^2/\left(2gA_m^2\right) = HGL_a$$

$$\left(Q_b/C_b\right)^2/\left(2gA_a^2\right) + K_b\left(Q_a + Q_b\right)^2/\left(2gA_m^2\right) = HGL_b$$

$$\left(Q_c/C_c\right)^2/\left(2gA_a^2\right) + K_c\left(Q_a + Q_b + Q_c\right)^2/\left(2gA_m^2\right) = HGL_c$$

in which $A_a = (\pi/4)D_3^2$ is the area of each equally-sized port, and $A_m = (\pi/4)D_m^2$. Along the main three energy equations

$$HGL_b = HGL_a + \left(fL_s/D_m - K_a\right)Q_a^2/\left(2gA_m^2\right)$$

$$HGL_c = HGL_b + \left(fL_s/D_m - K_b\right)\left(Q_a + Q_b\right)^2/\left(2gA_m^2\right)$$

$$H_R = HGL_c + \left(1 + fL/D_m - K_c\right)\left(Q_a + Q_b + Q_c\right)^2/\left(2gA_m^2\right)$$

can be written, in which $L_s = 1$ ft is the spacing between ports, $L = 30$ ft is the length of the upstream main, and $H_R = 20$ ft is the elevation of the reservoir water surface. These six equations can be solved for six variables, which could be chosen as Q_a, Q_b, Q_c, HGL$_a$, HGL$_b$, and HGL$_c$. Using any software that is capable of solving a nonlinear system of equations produces $Q_a = 0.373$ ft^3/s, $Q_b = 0.345$ ft^3/s, $Q_c = 0.321$ ft^3/s, HGL$_a$ = 15.844 ft, HGL$_b$ = 15.683 ft, and HGL$_c$ = 15.043 ft, if the coefficients that were determined in the hand solution are used. If the source of these coefficients must be Figs. 3.5 and 3.8, a solution can be obtained with trial coefficient values, the coefficients can then be adjusted and improved, and the problem can be solved again. However, an improved computational approach would use "list functions" that would obtain the coefficients from values that are found from tables that describe the curves in these figures. If a function subprogram that solves the Colebrook-White equation (so one Colebrook-White equation would be written to determine the friction factor f in each flow segment of the main; in this case three equations) is added to the equation system, then one could merely specify the pipe material (actually the equivalent sand grain roughness e for that material) rather than specifying a value for f itself. Talozi (1998) has analyzed manifold flow recently using some of these computational approaches.

* * *

A review of these computations allows us to come to several conclusions:

1. As the local ratio Q_3/Q_1 changes, the experimentally determined coefficients that describe the flow at each junction probably also change. The flow from a port cannot be determined accurately unless the lateral discharge coefficient C and the nondimensional pressure head rise coefficient are known reasonably well.
2. For practical manifold flows in which a large number (more than three or four is large) of ports are present, the negative head loss phenomenon will in theory be present at a large majority of the ports (all but the last few ports), but the actual amount of the energy change across such a port will almost always be very small. And if this energy change (gain) across a port along the main is neglected, the effect of this neglect is a conservative one in the design process.
3. The first end-of-chapter problems will demonstrate that ports of equal diameter, in the absence of the consideration of junction losses, display a trend of increasing port discharge with increasing EL in the main. But Example Problem 3.1 is one example where a consideration of junction losses leads to a decrease in port discharge with an increasing EL as one moves upstream. When this trend was observed many years ago along with a decrease in pressure head in the upstream direction, it was concluded that it was the pressure head, and not the energy line location, that determined the port discharge; old technical articles that emphasize the importance of pressure head alone in manifold behavior should be viewed with caution.

3.3 A HYDRAULIC DESIGN PROCEDURE

Whether the application is a submarine diffuser as part of a wastewater dispersal operation or a drip irrigation system, some elements of the design procedure change little. There are also some elements that vary from application to application, however. A submarine diffuser, for example, normally is laid on a slope in water of a different density than that of the wastewater, which leads to external pressure differences from port to port that must be incorporated into the design computations. And the physical shape of a submarine diffuser port differs substantially in size and other details from, say, a drip irrigation emitter (port). With some exceptions the trend in recent years is toward a larger number of smaller ports. And the ports within a major segment of the manifold, if not the entire manifold, will be uniformly spaced and of uniform diameter for ease of construction.

In the design of a manifold there are several goals:

1. To assure that the manifold functions in the intended manner, it must always flow full. For a simple manifold this is usually met by requiring the sum of the individual port cross-sectional areas to be less than, typically about 90% of, the cross-sectional area of the main. For larger manifolds with a stepped main, the ratio of the sum of port areas downstream from a particular section to the cross-sectional area of the main at that section is usually limited to some fraction between 1/2 and 2/3.

2. The ports and the main should both have a simple, clean design for several reasons. A simple design will often lead to low hydraulic losses, which will reduce operating expense by saving energy and will lead to a much simpler hydraulic analysis if junction losses can be neglected. It ought also to reduce maintenance costs.
3. The primary design goal, but not one that is strictly attainable, is an even or relatively even distribution of flow between ports.
4. The range of acceptable velocities in the main should be examined carefully for each application, especially if there is any possibility of some solids being conveyed in the manifold. The velocity of the carrier fluid must then be high enough to prevent the settlement of the solids, and it must also be low enough to avoid a scour or abrasion problem. When solids are borne in water, the acceptable range is between 2 and 15 ft/s but usually below 5 ft/s.

The computational sequence for manifold design that will be described in the following paragraphs was developed in the 1960s and 1970s by several investigators and authors, including Rawn et al. (1960), Vigander et al. (1970), and Grace (1978). A brief look into these publications, however, will show the continuing influence of N. H. Brooks on all these efforts. Notationally we follow the presentation of Grace (1978), which is diagramed in Fig. 3.9. The procedure is organized so the entire sequence can be converted relatively directly into computer code.

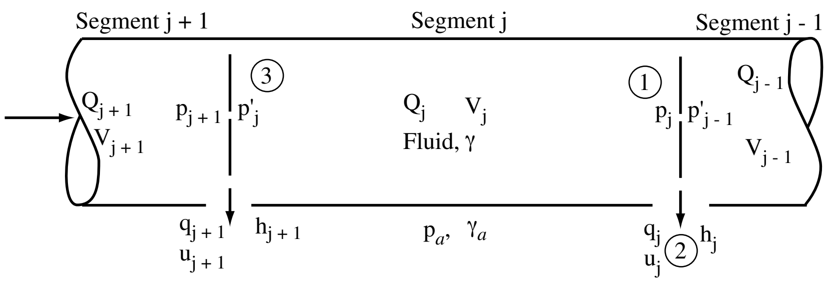


Figure 3.9 A two-port segment of a manifold, to display notation.

The manifold ports and barrel segments are numbered from the downstream end toward the upstream supply head or reservoir, with each port and segment number that is upstream of it denoted by j, which will also be used as a subscript on the other variables to indicate their location. Other variables are Q = discharge in the barrel segment, V = mean velocity in the barrel segment, A = cross-sectional area of the barrel segment, D = diameter of the barrel segment, q = discharge from a port, u = mean velocity through a port, a = cross-sectional area of a port, and d = diameter of a port.

To allow several different design environments, we assume that this design considers a manifold or diffuser conveying a liquid fluid of constant unit weight γ (the fluid is usually water, but the design procedure is not restricted to water only) that is submerged in an ambient body of fluid of constant unit weight γ_a. The horizontal surface of the exterior fluid body serves as a datum where $h = 0$; the submerged elevation of a port is then $-h$, which will change from port to port if the manifold slopes. The pressure outside a port is $p_a = \gamma_a h$. If the ambient fluid is air, then we choose $\gamma_a = 0$. The hydraulic model of flow in a manifold has a discrete jump in pressure across a port; just upstream of port j the internal pressure is p_j, and just downstream of the next port the pressure is p'_j. Flow is assumed to exit horizontally from ports having centerlines at the same elevation as the centerline of the barrel.

The manifold computations begin at the downstream end. Select an estimate of the average port discharge q_p as the total discharge through the manifold divided by the number of ports. If the fluid flow in the manifold is to carry with it any settleable material, then it is advisable to put a large port, with a discharge of roughly $4q_p$, at the downstream end to counteract the siltation that would otherwise occur in a dead end. The discharge through this port is governed by an orifice equation, but the total head at this port is not known; simply pick a value for the total head that is consistent with the port discharge that is chosen, and it will be corrected later.

The computations at a port, say port j, are basically the same for every port. We write an energy equation from a point inside the main, point 1, to a point in the port efflux stream, point 2:

$$-h_j + \frac{V_j^2}{2g} + \frac{p_j}{\gamma} = -h_j + \frac{u_j^2}{2g} + \frac{p_{aj}}{\gamma} + k_L \frac{u_j^2}{2g} \tag{3.18}$$

In this equation k_L is the port head loss coefficient. If we define an energy parameter E at port j as

$$E_j = \frac{p_j - p_{aj}}{\gamma} + \frac{V_j^2}{2g} \tag{3.19}$$

then the fluid exit velocity through the port is

$$u_j = \left(\frac{1}{1+k_L}\right)^{1/2} \left(2gE_j\right)^{1/2} \tag{3.20}$$

The discharge through this port is the product of the port velocity and the flow cross-sectional area of the jet from this port, or

$$q_j = u_j C_c a_j \tag{3.21}$$

or

$$q_j = C_D a_j \left(2gE_j\right)^{1/2} \tag{3.22}$$

with the discharge coefficient C_D combining the effects of the port head loss and the local contraction coefficient C_c into

$$C_D = C_c / \left(1 + k_L\right)^{1/2} \tag{3.23}$$

Since the local loss coefficient varies, for an unchanging individual port geometry, with the ratio of the local velocity head to total head or its surrogate E_j, these relations can be contracted to

$$C_{Dj} = C_D \left(\frac{V_j^2 / 2g}{E_j}\right) \tag{3.24}$$

This function must initially be determined experimentally, and the results can be summarized in any number of ways, in a graph or table, as an analytical curve fit or as data pairs that can be interpolated by a computer program subroutine. For example, Grace (1978) cites an empirical equation fitting data that describe the flow through a bell-mouth port that is part of the diffuser manifold in an ocean outfall for the city of Honolulu, valid only when $d_j/D_j < 0.1$, as

$$C_D = 0.975\left(1 - \frac{V_j^2/2g}{E_j}\right)^{3/8} \tag{3.25}$$

and Rawn et al. (1960, p. 94) graphically present analogous curves for bell-mouth and sharp-edged ports.

Along the main between points 3 and 1, we may write the energy equation as

$$-h_{j+1} + \frac{V_j^2}{2g} + \frac{p_j'}{\gamma} = -h_j + \frac{V_j^2}{2g} + \frac{p_j}{\gamma} + h_{Lj} \tag{3.26}$$

The velocity head terms cancel, the elevation terms are either known or zero, the pressure head term at port j has been computed, and the last term, the frictional loss term along the barrel, can be computed from the Darcy-Weisbach equation. Hence the pressure head term p_j'/γ downstream of port $j+1$ can be computed. Now we assume that it is acceptably accurate to assume no head loss across a port along the main, leading to

$$\frac{V_{j+1}^2}{2g} + \frac{p_{j+1}}{\gamma} = \frac{V_j^2}{2g} + \frac{p_j'}{\gamma} \tag{3.27}$$

This last assumption may be questionable for the first two or three ports at the downstream end, but thereafter it should be a very good and slightly conservative assumption. The right side of this equation is entirely known. Since p_{aj+1} can be determined, then E_{j+1} is known, but the two terms on the left side of Eq. 3.27 are not yet separately known. Consequently we can not immediately find C_D at port $j+1$, since Eq. 3.24 shows that we must know the upstream velocity head to do that. So we proceed as follows.

If we know q_{j+1}, then

$$Q_{j+1} = Q_j + q_{j+1} \tag{3.28}$$

and V_{j+1} can be found directly. But Eq. 3.22 clearly requires a value for C_{Dj+1}. We can iterate our way to a solution by first computing an estimate of q_{j+1} as

$$q_{j+1} = C_{Dj} a_{j+1} \left(2gE_{j+1}\right)^{1/2} \tag{3.29}$$

with C_{Dj} based on $\left(V_j^2/2g\right)/E_{j+1}$ instead of $\left(V_{j+1}^2/2g\right)/E_{j+1}$, and then in turn computing Q_{j+1} from Eq. 3.28, V_{j+1} from Q_{j+1}, and then an improved value of C_{Dj+1} based on V_{j+1}. This cycle will almost always converge in one or two iterations to give an accurate value of C_D at port $j+1$. This computational routine is used at each port.

This computational routine is repeated from one port to the next until the entire manifold has been traversed. At this point the total head has been computed at the upstream end of the manifold. For the manifold to function as the computations indicate, this head or a larger head must be supplied to this point. Commonly the goal is either to match some head here to a reservoir head or the head from a pumping plant, and the computed total will rarely be the same as the target head. Two approaches are available for the reconciliation of this difference: (1) Recognizing in the entire computational procedure that heads are proportional to the square of the velocities or discharges, all discharges can be proportionally scaled, as was demonstrated in Example Problem 3.1,

$$Q_{true} = Q_{est}\left[\frac{H_{true}}{H_{est}}\right]^{1/2} \tag{3.30}$$

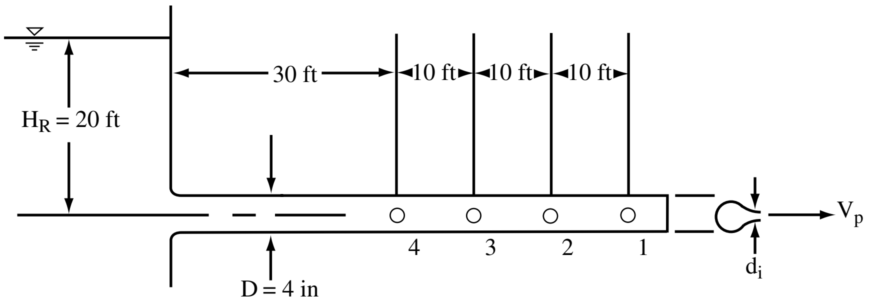
in which H_{true} is the desired target head, Q_{est} and H_{est} are the estimated discharges and heads that are the outcome of the computation, and Q_{true} are the discharges that will produce the desired head. (2) The other approach is simpler but still effective, and that is simply to raise or lower the original head at port 1 in proportion to the amount by which the target head is missed in the previous trial and to rerun the problem with the computer program; continue these adjustments until the target head is met with acceptable accuracy.

Computer programs that perform this sequence of computations have been developed by various individuals and organizations. Grace (1978, pp. 296-297) presents a typical set of plots that are the outcome of such studies; the plots display the relatively small variation of discharge from port to port that is attainable by good design. A relatively simple version of a typical manifold program has been written and will be found with the other programs on the CD in file MANIFOLD. A study of the program listing should help the reader understand the details of implementing the computational procedure. The current program follows the methodology in this section, including the neglect of head loss at a port along the main. But the code also indicates where modifications are needed to include this factor, if it is to be added to the program.

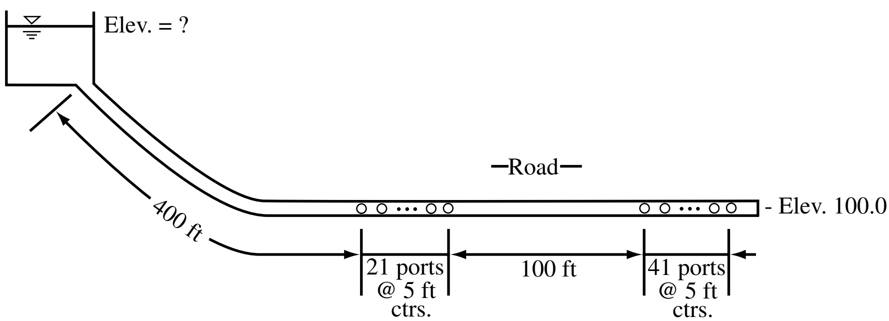
3.4 PROBLEMS

3.1 In the manifold shown below, neglect all losses except pipe friction in the barrel, and assume $f = 0.02$ is a reasonable estimate for the Darcy friction factor in the barrel.

(a) Assume the discharge from each port i is $Q_i = 0.35$ ft^3/s. Compute each port diameter d_i, $i = 1,4$, that is required so that the assumption of equal discharges is true.
(b) Now assume that all four port diameters are each the size d_1 that was computed in part (a) and compute the resulting discharge Q_i from each port.



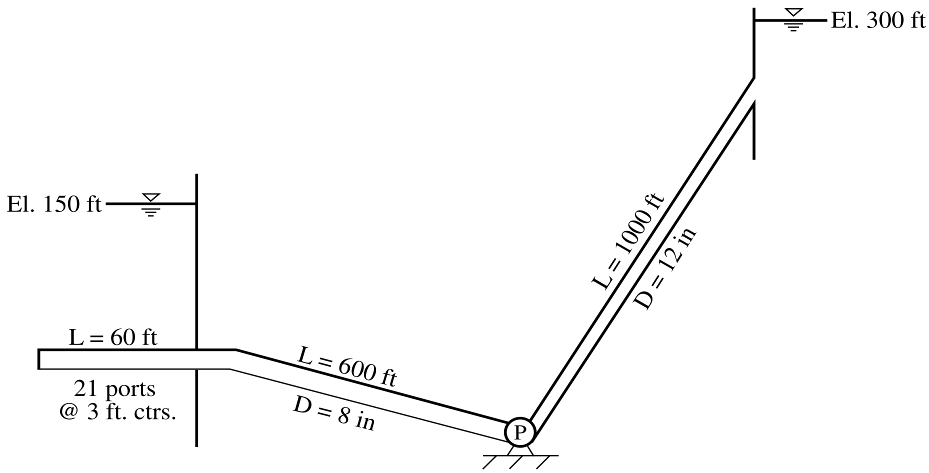
3.2 It is proposed to distribute water to irrigation furrows on two sides of a road, as shown in the next figure, by a system which is supplied by an elevated reservoir and consists of one 12-in-diameter used pipe (still in good condition) that serves both sides of the road via many circular holes or ports on 5-ft intervals. The largest port diameter is to be 2.0 in. Each port is to discharge 0.2 ft^3/s. Assuming for simplicity that $f = 0.02$ is a suitable friction factor and neglecting junction and other minor losses, estimate the required water surface elevation in the reservoir to fulfill these requirements.



3.3 Consider n equally-spaced ports in a length L of pipe having diameter D and friction factor f. Assume equal discharge q from each port.

(a) Including friction but not junction losses, is it possible for the hydraulic grade line to have the same elevation at both ends of the manifold section of the pipe? Conclude "yes" or "no" and then justify your answer by using equations.
(b) Does conclusion (a) depend on the overall discharge Q in the manifold, or is your conclusion independent of discharge?
(c) If condition (a) were realized and q is constant, does this mean that each port diameter must be the same? Respond "yes" or "no" only.
(d) Comment briefly on whether a consideration of junction losses would alter your reply to part (a).

3.4 Compute the discharge from reservoir A to reservoir B for the system shown below. Assume f = 0.02 and neglect local losses. The pump characteristic curve can be represented by $h_p = 300 - 20Q^2$ with h_p in ft and Q in ft^3/s. Although the diameters of the intake ports are not stated, assume as an approximation that they cause the inflow over this section to be uniformly distributed.



3.5 Consider again the manifold shown for Problem 3.1, but now do not neglect junction losses. The ratio of diameters between the laterals and the main is D_3/D_1 = 0.2, and the length of each lateral is 10 in. Assume f = 0.025.

(a) Using Fig. 3.5, develop and plot $K_{L1\text{-}2}$ vs. Q_3/Q_1. Does the coefficient become negative? Over what range of Q_3/Q_1?
(b) Develop a plot similar to Fig. 3.8 which displays C as a function of Q_3/Q_1.
(c) Starting with a trial EL of 10 ft, determine the discharge from each port and the total discharge from the manifold.
(d) What is the elevation of the actual EL downstream of the ports?

3.6 Certain assumptions are made in the analysis of a major submarine diffuser manifold for the disposal of wastewater. Indicate which of the following assumptions is both correct and justified, and why the others are in some way incorrect or not justified.

(a) All losses at a junction are ignored.
(b) At a junction only port losses are considered.
(c) Only losses along the main are considered at a junction.
(d) All losses at a junction usually should be considered.

3.7 A city treats at least some wastewater by overland flow. It is proposed to deliver 0.1 ft^3/s of dilute wastewater (same properties as water) through 50 ports, which are 5 feet apart, to the land surface. The main delivery line is old 8-inch-diameter metal pipe coming from a raised reservoir. You are asked to act as a consultant on the project.

(a) It is proposed that the diameter of each port opening be 1.25 inch because it is easy to build. Indicate whether this port size is an acceptable choice. Secondly, tell the project workers whether 0.1 ft^3/s can be delivered through each port this way.

(b) For a preliminary design assume $Q_p = 0.1$ ft^3/s from each port, $f = 0.02$ and neglect all local losses. Estimate the minimum reservoir surface elevation that can be used successfully here.
(c) Do you think a consideration of junction losses would significantly change your answer in part (b)? Do you think a more detailed analysis of the flow out each port is needed? In each case, why do you think so? Reply briefly to both questions, but do no additional calculations.

3.8 Devise a computational scheme to determine the head loss across a port in the main line of a manifold. Implement the scheme in the manifold program MANIFOLD on the CD, and test the scheme by running the program, using additional print statements to obtain enough information to verify that the program operates correctly.

3.9 Trickle irrigation of a field may involve a hierarchy of manifolds; that is, a delivery main can serve as the supply to several manifolds, and each manifold will in turn serve a number of laterals. Finally, each lateral will contain along its length a number of individual emitters. The manifold program on the CD is suitable for application to the pipes that are called manifolds in this application, so long as care is taken to treat the port exit pressures properly. However, each line called a lateral is itself a pipe containing numerous emitters or "ports" and so is itself a kind of manifold having two significant differences from the manifold which is modeled in the current manifold program: (1) At each port the trickle emitter usually (but not always) has a "barb" that projects into the main and causes a head loss at the port along the main; (2) Irrigation practitioners represent the discharge from an individual emitter by $q = KH^x$, in which K is a discharge coefficient that is characteristic of the emitter, H = pressure head = $(p - p_a)/\gamma$, and the exponent varies with the type of emitter over the range $0 \le x \le 0.8$. For example, for simple orifice or nozzle emitters $x = 0.5$. For more information see James (1988) or Keller and Bliesner (1990).

Modify the manifold program on the CD to simulate the flow in a trickle irrigation "lateral":
(a) For barb losses along the main, called the lateral, irrigation references (e.g. James 1988, p. 281) describe the head loss in terms of an equivalent additional pipe length. If the head loss along the main at a port is $h_L = KV^2/2g$, then the loss coefficient is of the form

$$K = Cf / D^m$$

with C being a pure number, f = Darcy friction factor, D = pipe inside diameter, and m = exponent, usually approximately 3.
(b) Replace the port discharge formula that is in the program with $q = KH^x$, and modify the program input statements to read the new data that are required.

CHAPTER 4

PIPE NETWORK ANALYSIS

4.1 INTRODUCTION

This chapter describes the analysis of steady flow in pipe systems. In any analysis problem all of the physical features of the network are known, and the solution process endeavors to determine the discharge in every pipe and the pressure, etc. at every node of the network. Therefore in this chapter the diameters of all pipes, their lengths and their roughnesses are known, as well as where reservoirs, pumps, pressure reduction valves, and other fittings are located. The ways in which these devices influence the hydraulics of the system will be specified. Design problems, on the other hand, try to select (wisely!) the diameters of pipes, the capacities of pumps, the water surfaces elevations of reservoirs, and so on. Thus, a design problem is distinguished from an analysis problem by the choice of the variables that are regarded as unknown. At some risk we dare to generalize by saying that design problems are usually more challenging to solve than are analysis problems, and design problems usually require the simultaneous solution of a larger system of equations than do analysis problems. A thorough understanding of the techniques of analysis for large networks that are composed of known physical features is a prerequisite to the understanding of the design of networks. The design of pipe networks is the focus of Chapter 5 and is not discussed directly in this chapter.

The analysis of a pipe network can be one of the more complex mathematical problems that engineers are called upon to solve, particularly if the network is large, as occurs in the water distribution systems of even quite small cities. A significant fraction of the entire set of equations consists of nonlinear equations, and a large number of these equations must be solved simultaneously. Before digital computers were widely used in engineering practice, it simply was not practical to solve such network problems, and consequently many existing water distribution systems have "grown" with time, based primarily on the best professional judgment of engineers, without any thorough or detailed analysis of the pressures and discharges that could exist in the pipes of the network in response to various combinations of demands on the system. The computer has made it possible to solve such large network problems with ease, and as a result many municipalities and water districts have benefited from the results of relatively detailed computer analyses of their systems in recent years. We believe it is important for an engineer to understand what is being accomplished in these computer solutions. To aid engineers in gaining this knowledge, we begin with the basic principles, and the equations that embody them, that interrelate the discharge in each pipe and the pressure at each node of the network.

The same few basic principles of fluid mechanics are the foundation of our work on pipe network analyses. These basic principles are (1) conservation of mass, or the continuity principle, (2) the work-energy principle, and (3) the relation between fluid friction and energy dissipation. Chapter 2 has already introduced these principles. The task here is to employ these ideas effectively in describing a large hydraulic system accurately by means of equations, and then to solve these simultaneous equations efficiently.

The oldest systematic method for solving the problem of steady flow in a pipe network is the Hardy Cross method, which is itself an early adaptation of the method of moment dis-tribution from structural engineering in 1936. Before the ready availability of digital computers in the late 1960s, this method was prized because it is so well suited for hand computations. Then it became the basis of most early computer software, but because of convergence problems for large systems containing pumps and other appurtenances, it will not be discussed herein. Over the past quarter century the Newton method has proven to be superior in solving the nonlinear equations, and now networks of 2500 pipes or more can be analyzed successfully with a desktop computer.

4.1.1. DEFINING AN APPROPRIATE PIPE SYSTEM

The first step in studying pipe systems is to decide what features are important and to retain them in defining the network of pipes. For large water distribution systems some "skeletonization" usually occurs in this process. In other words, not all pipes in the system are included in the analysis. This skeletonization can take on many forms, such as the following:

1. Not all connections to houses are considered as separate nodes or junctions, and all of the distributed demands along one block of a street, or even a small subdivision, may instead be aggregated or lumped at a single node;
2. Only those pipes that carry the water from the supply sources to the areas of demand are included, i.e., only the main transmission system is considered;
3. Only a few pipes and their associated appurtenances are considered; these components are regarded as vital to the proper operation of the system.

Any study of a pipe system may include one or even all of these levels of skeletonization; the first preliminary study may start with a model of type (3), and subsequent, more complete analyses may proceed back to type (1) as the adequacy of each is verified, or as components are adjusted. After these analyses have been obtained and studied, it may then be desirable to study intensively the network of pipes within a city block, or the pipes within the area of a major water user, such as a large structure or a manufacturing facility. Thus analyses can treat an entire delivery system, which is generally skeletonized, or a more detailed analysis of the piping system or plumbing within a large building, or a golf course, etc. When an analysis of a building's piping system is conducted, the exterior pressures that are supplied by the larger system can be specified with some degree of confidence since the analyses of the larger "delivery" networks provide this information. There are no hard rules that dictate which pipes should be omitted. Such decisions are often left to the professional judgment of the supervising engineer, and sometimes these decisions are called "art," but the insight gained from analyses at different levels of skeletonization often indicate which pipes should be included in the next level of analysis. Computers can now analyze a problem consisting of many more pipes (e.g., several thousand) than the human mind can visualize in detail when deciding which features should be changed to improve the performance and reduce the costs of the system.

Another vital part of defining the network problem is to determine which demands should be specified. The demands on an existing system can be obtained from water usage or billing records. Even for existing systems the data are seldom complete in describing how these demands vary during a day, or from day to day. Analyses are usually needed for a range of system demands, from peak hourly demands down to minimal demand periods (e.g., 2-3 a.m.). During above-average demand periods tanks will have their storage volumes partially depleted, but this loss of volume should be recovered when demands are small. Since a water system may be designed for a 50-year life, the specified demands must appropriately reflect future growth and increases (or possibly decreases) in per capita consumption. In the design of a new system, the demands may have to be based on comparisons with similar cities etc. However, if a system is to be designed to deliver known quantities at specified times, then the problem of determining appropriate demands does not exist. So we see that engineering experience, based on sound judgment, is often required in defining the most appropriate piping system problems to analyze.

After the analyst has obtained one or several apparently reasonable solutions, the next step is to verify by measurements in the actual system that reasonable agreement exists between the solution to the mathematical problem and the real system. This process is called network verification. If significant disagreements occur, their causes must be identified.

Are some valves in the real system unknowingly closed or partly closed; do some major leaks exist in the real system; has the skeletonization process inappropriately excluded some pipes that carry large flows? These and other possibilities should be explored until reasonable agreement does exist. Firms specialize in field flow measurements to verify that the actual pipe system is modeled properly.

After analysis has provided solutions to the network problem for various levels of demands, non-ideal or simply inadequate performance parameters can be identified. Some indicators of inadequate or poor performance consist of the following (many other possibilities that are peculiar to an individual system do exist):

1. Pressures at some nodes are too low;
2. Pressures are too high at some nodes (If water is pumped, excessively high pressures cost money, owing to larger power consumption than is needed, more frequent pipe ruptures and the premature replacement of facilities.);
3. Discharges are inadequate and/or pressures are too low to meet emergency demands, such as fire fighting;
4. Pumps are not operating near their peak efficiencies;
5. Some water storage facilities are always nearly empty, while others are nearly full or overtopping (Are the tanks under- or over-sized and located at the best elevations? Unless storage facilities perform near their capacities, they represent investments with cost/benefit ratios that are too large.);
6. Pressure reduction valves, or back pressure valves, are inactive or open (Perhaps they are not needed, or pipes should be removed or added.);
7. Too much of the supply is coming from expensive sources, etc.

Upon identifying deficiencies, the engineer's next task is to determine the best, most economical means of overcoming these deficiencies and improving the performance of the system. How best to accomplish this will again require sound professional judgment, but sound judgment seldom occurs in the absence of relevant information, i.e., the engineer must understand the system. Section 5.7 of the next chapter discusses sensitivity analyses, which could materially aid this evaluation process.

In the following work we will express the head loss in each pipe in a network by an exponential formula $h_f = KQ^n$, Eq. 2.17, so one presentation of the theory covers all cases, regardless of whether the Darcy-Weisbach equation, the Hazen-Williams equation or the Manning equation is used to express the head loss as a function of discharge. Only the values for K and n change, as we saw in Chapter 2.

4.1.2. BASIC RELATIONS BETWEEN NETWORK ELEMENTS

The two basic principles, upon which all network analysis is developed, are (1) the conservation of mass, or continuity, principle, and (2) the work-energy principle, including the Darcy-Weisbach or Hazen-Williams equation to define the relation between the head loss and the discharge in a pipe. The equations that are developed from the continuity principle will be called Junction Continuity Equations, and those that are based on the work-energy principle will be called Energy Loop Equations. The number of these equations that constitutes a non-redundant system of equations is related directly to fundamental relations between the number of pipes, number of nodes and number of independent loops that occur in branched and looped pipe networks. In defining these relations NP will denote the number of pipes in the network, NJ will denote the number of junctions in the network, and NL will denote the number of loops around which independent equations can be written. In defining junctions, a supply source will not be numbered as a junction. A supply source is a point where the elevation of the energy line, or hydraulic grade line, is established; a junction, or node, is a point where two or more pipes join. A node can exist at each end of a "dead end" pipe; this instance is an exception to the usual rule, where only one pipe is connected to a node. In a branched system there are by definition no loops, and thus $NL = 0$ for any branched system. In branched systems the number of nodes is always one larger than the number of pipes, or $NP = NJ - 1$, unless a reservoir is shown at the end of one pipe and this is not considered to be a junction. Then $NP = NJ$. (This situation actually occurs.) Figures 4.1a and 4.1b depict a small branched network and also a small looped network.

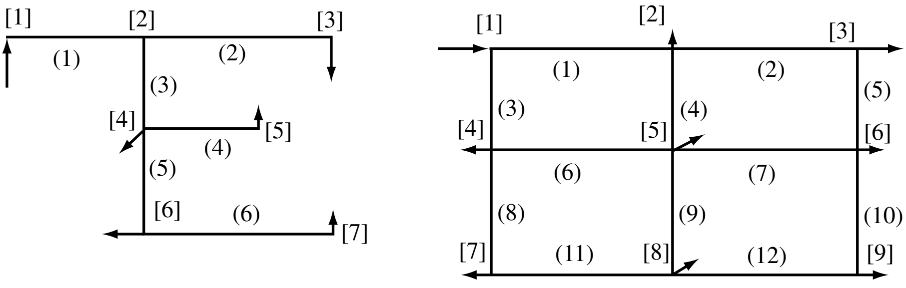


Figure 4.1 (a) A small branched system. 6 pipes, 7 nodes (b) A small looped system. 12 pipes, 9 nodes

In the branched system the number of nodes is 7 and the number of pipes is 6 (one less than the number of nodes), whereas in the looped system there are 12 pipes and 9 nodes, i.e., the number of nodes is less than the number of pipes.

For a looped network the number of loops (around which independent energy equations can be written) is given by

$$NL = NP - NJ \tag{4.1}$$

if the network contains two or more supply sources, or

$$NL = NP - (NJ - 1) = NP - NJ + 1 \tag{4.2}$$

If the network contains fewer than two supply sources and the flow from the single source is determined by adding all of the other demands, then this source is shown as a negative demand and the source is called a node. We note that this is the case in the small looped network in 4.1.b, so we have $NP = 12$, $NJ = 9$ and $NL = 12 - (9 - 1) = 4$.

Equation 4.2 also applies to a branched system with $NL = NP - NJ + 1 = 0$, since a branched system can have at most one supply source. Actually, every pipe system must have at least one supply source, but sometimes the source is not shown since the discharge from this supply source is known, and the source is replaced by a negative demand, which is a flow coming into this junction, equal to the sum of the other demands. When this is done, the elevation of the energy line (or HGL or pressure) must be specified at a node so the other HGL elevations can be determined. Energy loops that begin at one supply source and end at another are called pseudo loops, i.e., these loops do not close on themselves. The number of pseudo loops, which are numbered as part of NL, equal the number of supply sources minus one. In forming pseudo loops all supply sources must be located at the end of a pseudo loop. It is generally possible to form more loops than are needed to produce a set of independent equations. As each new loop is formed, see that at least one pipe in the new loop is not a part of any prior loop; in this way the formation of redundant loops can usually be avoided. For special devices, such as pressure reduction valves, this rule of experience must be modified slightly, as will be described later.

4.2 EQUATION SYSTEMS FOR STEADY FLOW IN NETWORKS

Three different systems of equations can be developed for the solution of network analysis problems. These systems of equations are named after the variables that are regarded as the principal unknowns in that solution method. These systems of equations are called the ***Q*-equations** (when the discharges in the pipes of the network are the principal unknowns), the ***H*-equations** (when the HGL-elevations, also simply called the heads H, at the nodes are the principal unknowns), and the ***ΔQ*-equations** (when corrective discharges, ΔQ, are the principal unknowns). Each of these three systems of equations will be studied separately.

4.2.1. SYSTEM OF *Q*-EQUATIONS

The analysis of flow in pipe networks is based on the continuity and work-energy principles. To satisfy continuity, the volumetric discharge into a junction must equal the volumetric discharge from the junction. Thus at each of the NJ (or NJ - 1) junctions an equation of the form of Eq. 4.3 is obtained:

$$QJ_j - \Sigma Q_i = 0 \tag{4.3}$$

In this equation QJ_j is the demand at the junction j, and each Q_i is the discharge in one of the pipes that join at junction j. These junction continuity equations are the first portion of the Q-equations. The work-energy principle provides additional equations which must be satisfied. These equations are obtained by summing head losses along both real and pseudo loops to produce independent equations. There are NL of these equations, and they are of the form of Eq. 4.4 or 4.5, depending upon whether the loop is a real loop or a pseudo loop, respectively, and they are the second portion of the Q-equations:

$$\Sigma h_{fi} = 0 \tag{4.4a}$$

$$\Sigma h_{fi} = \Delta WS \tag{4.5a}$$

When the head losses are expressed in terms of the exponential formula, then these equations take the forms

$$\sum K_i Q_i^n = 0 \tag{4.4b}$$

$$\sum K_i Q_i^n = \Delta WS \tag{4.5b}$$

in which the summation includes the pipes that form the loop. If the direction of the flow should oppose the direction that was assumed when the energy loop equations were written, such that Q_i becomes negative, then there are two alternatives: One is to reverse the sign in front of this term, i.e., correct the direction of the flow. The second, which is generally preferred when writing a program to solve these equations, is to rewrite the equations as follows:

$$\sum K_i \, Q_i \left| Q_i \right|^{n-1} = 0 \tag{4.4c}$$

$$\sum K_i \, Q_i \left| Q_i \right|^{n-1} = \Delta WS \tag{4.5c}$$

To illustrate the system of Q-equations, consider the small 5-pipe network shown in Fig. 4.2. Since no supply sources are shown for this network, only NJ - 1 junction continuity equations are available. Writing these continuity equations for nodes 1, 2, and 3 leads to

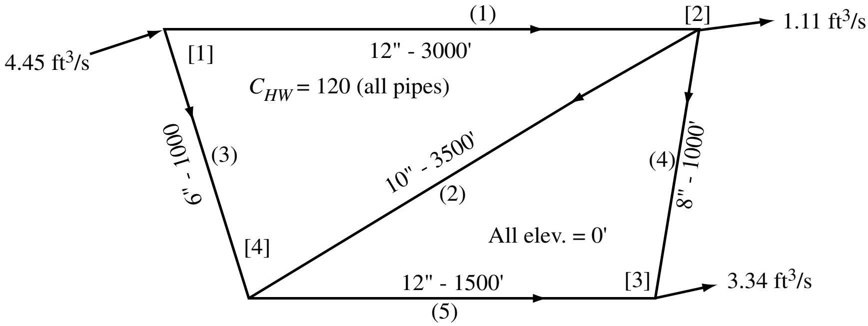


Figure 4.2 Small network.

$$
\begin{aligned}
F_1 &= Q_1 + Q_3 - 4.45 = 0 \\
F_2 &= -Q_1 + Q_2 + Q_4 + 1.11 = 0 \\
F_3 &= -Q_4 - Q_5 + 3.34 = 0
\end{aligned}
\tag{4.6}
$$

In these equations and throughout the text, F_i for any number i is any equation which has been arranged into the form $F_i = 0$; this format is useful for identification purposes and also for subsequent mathematical and numerical developments. The continuity equation at node 4 is $-Q_3 - Q_2 + Q_5 = 0$. However, this equation is not independent of the other three nodal equations since it is, except for sign, the sum of these three equations. Now let us use the Hazen-Williams equation to define the head loss in each pipe. In expressing these head losses, the exponential equation will be used. From Eq. 2.18 the coefficients are the following:

$$
K_1 = 2.018,\ K_2 = 5.722,\ K_3 = 19.674,\ K_4 = 4.847,\ K_5 = 1.009 \tag{4.7}
$$

The energy equations around the two loops may be written as

$$
\begin{aligned}
F_1 &= K_1 Q_1^{1.852} + K_2 Q_2^{1.852} - K_3 Q_3^{1.852} = 0 \\
F_2 &= K_4 Q_4^{1.852} - K_5 Q_5^{1.852} - K_2 Q_2^{1.852} = 0
\end{aligned}
\tag{4.8}
$$

or

$$
\begin{aligned}
F_1 &= 2.108 Q_1^{1.852} + 5.722 Q_2^{1.852} - 19.674 Q_3^{1.852} = 0 \\
F_2 &= 4.847 Q_4^{1.852} - 1.009 Q_5^{1.852} - 5.722 Q_2^{1.852} = 0
\end{aligned}
\tag{4.9}
$$

which might alternatively be written as follows if the directions of the flows are uncertain:

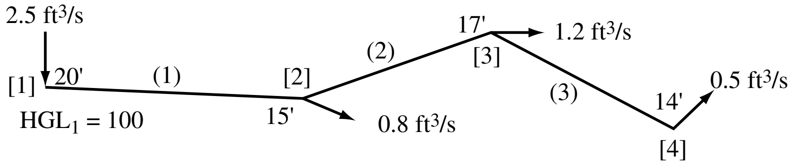
$$
\begin{aligned}
F_1 &= 2.108 Q_1 |Q_1|^{0.852} + 5.722 Q_2 |Q_2|^{0.852} - 19.674 Q_3 |Q_3|^{0.852} = 0 \\
F_2 &= 4.847 Q_4 |Q_4|^{0.852} - 1.009 Q_5 |Q_5|^{0.852} - 5.722 Q_2 |Q_2|^{0.852} = 0
\end{aligned}
\tag{4.10}
$$

These two work-energy equations are obtained by starting at nodes 1 and 2, respectively, and traversing the respective loops I and II in the clockwise direction. If the assumed direction of flow opposes this traverse, a minus sign precedes the head loss term for that pipe. The simultaneous equations, such as those appearing as Eqs. 4.6 and 4.10, are called Q-equations because it is the Q's, the discharges in the pipes, that are the set of primary unknown variables. After the Q's are found (and the head loss in each pipe is therefore also known) for each pipe, the HGL-elevations at the nodes can be found by starting at a known HGL-elevation and repeatedly applying the exponential formula for head loss to each pipe.

If the network is a branched system, then the Q-equations consist of only the junction continuity equations. These can be solved, giving the discharge in every pipe, with a linear algebra solver, i.e. a pocket calculator that implements matrix algebra, a spreadsheet, TK-solver, MathCAD or a solver such as SOLVEQ. Thereafter the individual heads are computed from the head loss equation for each pipe. Methods for solving looped systems are described later.

Example Problem 4.1

The coefficients K and n for the exponential formula are given in the table for each of the three pipes in this branched system. Find the discharge in each pipe and the pressure at each node. The elevation of the HGL at node 1 is $H_1 = 100$ ft.



Pipe	***K***	***n***
1	3.772	1.944
2	5.730	1.926
3	16.29	1.889

A formal method for solving the Q-equations for this network is to use matrix algebra to write the coefficient matrix, an unknown vector, and a known vector in the following way:

$$\begin{bmatrix} \mathbf{1} & \mathbf{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{-1} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix} \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{Bmatrix} = \begin{Bmatrix} 0.8 \\ 1.2 \\ 0.5 \end{Bmatrix}$$

The solution is

$$\begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{Bmatrix} = \begin{Bmatrix} 2.5 \\ 1.7 \\ 0.5 \end{Bmatrix}$$

We should note that it is easy to obtain this solution by inspection. Starting at the downstream end of a branch, the analyst can progressively satisfy each junction continuity equation while working upstream. After finding the discharges, the elevations H of the HGL are determined by starting where the HGL is known, in this case at node 1, and computing the head losses in the pipes that join this node; then the frictional head losses h_f are subtracted from the known values of H, etc. until all of the nodal heads have been determined. The pressures are then determined by subtracting the nodal elevations z from the heads H and multiplying this by the specific weight, i.e., $p = \gamma(H - z)$. The tables which follow present the computed values for this network:

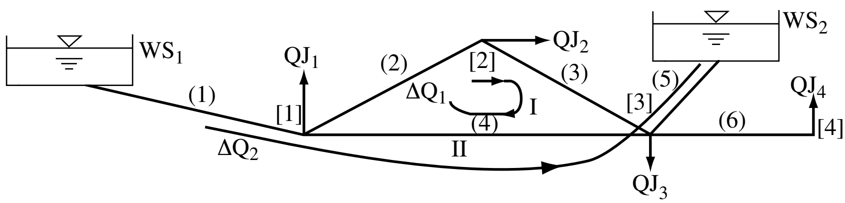
Pipe	Q, ft³/s	$h_f = KQ^n$, ft
1	2.5	22.395
2	1.7	15.922
3	0.5	4.398

Node	$H_{down} = H_{up} - h_f$, ft.	**Pressure**, lb/in^2
1	100.0 Given	34.67
2	100.0 - 22.395 = 77.61	27.13
3	77.61 - 15.922 = 61.69	19.37
4	61.69 - 4.398 = 57.29	18.76

* * *

Example Problem 4.2

Write the system of Q-equations for this network. In these equations use the parameters K_i and n_i, in which i is the pipe number.



Since two supply sources are present, four junction continuity equations are available. They are the following:

$$F_1 = Q_1 - Q_2 - Q_4 - QJ_1 = 0$$

$$F_2 = Q_2 - Q_3 - QJ_2 = 0$$

$$F_3 = Q_3 + Q_4 + Q_5 - Q_6 - QJ_3 = 0$$

$$F_4 = Q_6 - QJ_4 = 0$$

The number of energy loop equations is $NL = NP - NJ = 6 - 4 = 2$ (one pseudo and one real loop). These equations follow:

$$F_5 = K_2 Q_2^{n_2} + K_3 Q_3^{n_3} - K_4 Q_4^{n_4} = 0$$

$$F_6 = K_1 Q_1^{n_1} + K_4 Q_4^{n_4} - K_5 Q_5^{n_5} - WS_1 + WS_2 = 0$$

Since F_4 requires $Q_6 = QJ_4$, this dead end pipe could be removed from the network, and the demand at node 3 would then be changed to $QJ_3 + QJ_4$. These steps would reduce NP to 5 and NJ to 3, and they would eliminate any need for equation F_4. After the HGL elevation, H_3, at node 3 has been determined by solving this equation set, then H_4 can be found by computing h_{f6} and subtracting it from H_3.

* * *

4.2.2. SYSTEM OF H-EQUATIONS

If the elevation of the energy line or hydraulic grade line throughout a network is initially regarded as the primary set of unknown variables, then we develop and solve a system of H-equations. One H-equation is written at each junction (or at $NJ - 1$ junctions if fewer than two supply sources exist). Since looped pipe networks have fewer junctions than pipes, there will be fewer H-equations than Q-equations. Every equation in this smaller set is nonlinear, however, whereas the junction continuity equations are linear in the system of Q-equations.

To develop the system of H-equations, we begin by solving the exponential equation for the discharge in the form

$$Q_{ij} = (h_{f\,ij} / K_{ij})^{1/n_{ij}} = [(H_i - H_j)/K_{ij}]^{1/n_{ij}} \tag{4.11a}$$

Here the frictional head loss has been replaced by the difference in HGL values between the upstream and downstream nodes. In addition, in this equation a double subscript notation has been introduced; the first subscript defines the upstream node of the pipe, and the second subscript defines the downstream node. Thus Q_{ij} and K_{ij} denote the discharge and loss coefficient for the pipe from node i to node j. An alternative way of writing Eq. 4.11a is

$$Q_k = (h_{f\,k} / K_k)^{1/n_k} = [(H_i - H_j)/K_k]^{1/n_k} \tag{4.11b}$$

in which k is the pipe number.

Substituting Eq. 4.11 into the junction continuity equations, Eq. 4.3, yields

$$QJ_j - \sum\{[(H_i - H_j)/K_{ij}]^{1/n_{ij}}\}_{in} + \sum\{[(H_j - H_i)/K_{ij}]^{1/n_{ij}}\}_{out} = 0 \tag{4.12a}$$

in which the summations are over all pipes that flow to and from junction j, respectively. If it is desired to automate the choice of sign, then Eq. 4.12a can be written as

$$\begin{aligned} QJ_j &- \sum\{[(H_i - H_j)/K_{ij}]\left|(H_i - H_j)/K_{ij}\right|^{1/n_{ij}-1}\}_{in} \\ &+ \sum\{[(H_j - H_i)/K_{ij}]\left|(H_j - H_i)/K_{ij}\right|^{1/n_{ij}-1}\}_{out} = 0 \end{aligned} \tag{4.12b}$$

As an application of the H-equations with HGL-elevations at the nodes as the unknowns, consider the one-loop network in Fig. 4.3 which consists of three junctions and three pipes. In this network two independent continuity equations are available, and consequently the head at one of the junctions must be specified. In this case at node [1] the head loss in the pipe that connects the reservoir to the network can first be determined, and then this value can be subtracted from the reservoir water surface elevation to determine H_1.

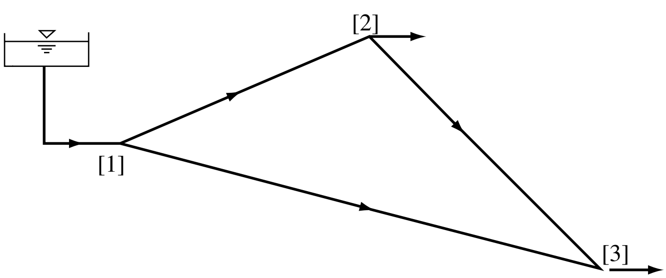


Figure 4.3 A network with three pipes and three junctions.

The two continuity equations are

$$\begin{aligned} Q_{12} + Q_{13} &= QJ_1 = QJ_2 + QJ_3 \\ Q_{21} + Q_{23} &= -\,QJ_2 \quad (or\ -Q_{12} + Q_{23} = -\,QJ_2) \end{aligned} \tag{4.13}$$

Although in the second equation the flow in pipe 1-2 is toward the junction, the discharge Q_{21} is not preceded by a minus sign since the notation 2-1 takes care of this. Alternatively the equations could have been written at junctions 2 and 3 instead of 1 and 2. Substituting Eq. 4.11 into these continuity equations gives the following two equations to determine H_2 and H_3:

$$\left[\frac{H_1 - H_2}{K_{12}}\right]^{1/n_{12}} + \left[\frac{H_1 - H_3}{K_{13}}\right]^{1/n_{13}} = QJ_2 + QJ_3$$
$$-\left[\frac{H_1 - H_2}{K_{12}}\right]^{1/n_{12}} + \left[\frac{H_2 - H_3}{K_{23}}\right]^{1/n_{23}} = -QJ_2 \qquad (4.14)$$

Since a negative value cannot be raised to a power, a minus sign must precede any term in which the subscript notation opposes the direction of flow. Systems of these equations will be called H-equations, since the HGL-elevations are the primary unknowns. After the heads H are found, then the discharge in each pipe can be obtained from Eq. 4.11.

Example Problem 4.3

Write the system of H-equations for the network in Example Problem 4.2.

Refer to the figure in Example Problem 4.2. Only the junction continuity equations are used in forming the H-equations, and each Q_i is replaced by $[(H_{ui} - H_{di})/K_i]^{1/n_i}$, in which subscript u is the upstream node and subscript d is the downstream node. The system is

$$F_1 = \left[\frac{WS_1 - H_1}{K_1}\right]^{1/n_1} - \left[\frac{H_1 - H_2}{K_2}\right]^{1/n_2} - \left[\frac{H_1 - H_3}{K_4}\right]^{1/n_4} - QJ_1 = 0$$

$$F_2 = \left[\frac{H_1 - H_2}{K_2}\right]^{1/n_2} - \left[\frac{H_2 - H_3}{K_3}\right]^{1/n_3} - QJ_2 = 0$$

$$F_3 = \left[\frac{H_2 - H_3}{K_3}\right]^{1/n_3} + \left[\frac{H_1 - H_3}{K_4}\right]^{1/n_4} + \left[\frac{WS_2 - H_3}{K_5}\right]^{1/n_5} - \left[\frac{H_3 - H_4}{K_6}\right]^{1/n_6} - QJ_3 = 0$$

$$F_4 = \left[\frac{H_3 - H_4}{K_6}\right]^{1/n_6} - QJ_4 = 0$$

* * *

4.2.3. SYSTEM OF ΔQ-EQUATIONS

The number of ΔQ-equations is normally about half the number of H-equations for a network. This reduction in number is not necessarily an advantage, since all of the equations are nonlinear and may contain many terms. These equations consider the loop corrective discharges or ΔQ's as the primary unknowns. These corrective discharges or ΔQ's will be determined from the energy equations that are written for NL loops in the network, and thus NL of these corrective discharge equations must be developed. To obtain these equations, we replace the discharge in each pipe of the network by an initial discharge, denoted by Q_{oi}, plus the sum of all of the initially unknown corrective discharges that circulate through pipe i, or

$$Q_i = Q_{oi} + \sum \Delta Q_k \tag{4.15}$$

in which the summation includes all of the corrective discharges passing through pipe i. The initial discharges Q_{oi} must satisfy all of the junction continuity equations. It is not difficult to establish the initial discharge in each pipe so that the junction continuity equations are satisfied. However, these initial discharges usually will not satisfy the energy equations that are written around the loops of the network.

Equation 4.15 is based on the fact that any adjustment can be added (accounting for sign) to the initially assumed flow in each pipe in a loop of the network without violating continuity at the junctions. It is very important to understand the validity of this decompo-sition; it may help to note that any ΔQ entering a junction as it flows around a loop must also leave that junction, and vice versa (See Fig. 4.4). Because of this fact, we decide

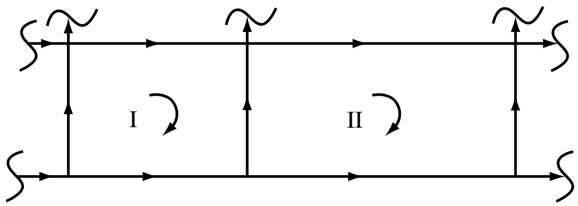


Figure 4.4 A two-loop portion of a network.

to establish NL energy loop equations around the NL loops of the network, in which each initial discharge plus the sum of corrective loop discharges $\Sigma\Delta Q_k$ is used as the discharge. The junction continuity equations are satisfied by the initial discharges Q_{oi} and are not a part of the system of equations. The corrective discharges can be chosen as positive if they circulate around a loop in either the clockwise or counterclockwise direction. It is necessary to be consistent within any one loop, but the sign convention may change from loop to loop, if desired. A corrective discharge adds to the flow Q_{oi} in pipe i if it is in the same direction as the pipe flow, and it subtracts from the initial discharge if it is in the opposite direction.

To summarize how the ΔQ-equations are obtained, replace the Q's in the energy loop equations, Eqs. 4.4 and 4.5, by

$$Q_i = Q_{oi} \pm \sum \Delta Q_k \tag{4.16}$$

Here the summation includes all corrective discharges which pass through pipe i, and the plus sign is used if the net corrective discharge and pipe flow are in the same direction; otherwise the minus sign is used before the summation. Thus Eqs. 4.4 and 4.5 become

$$\sum K_i \left\{Q_{oi} \pm \sum \Delta Q_k\right\}^{n_i} = 0 \quad \text{for real loops} \tag{4.17a}$$

and

$$\sum K_i \left\{Q_{oi} \pm \sum \Delta Q_k\right\}^{n_i} = \Delta WS \quad \text{for pseudo loops} \tag{4.18a}$$

To automate the choice of sign, these equations can be rewritten as

$$\sum K_i \left\{Q_{oi} \pm \sum \Delta Q_k\right\} \left|Q_{oi} \pm \sum \Delta Q_k\right|^{n_i - 1} = 0 \quad \text{for real loops} \tag{4.17b}$$

and $$\sum K_i \left\{ Q_{oi} \pm \sum \Delta Q_k \right\} \left| Q_{oi} \pm \sum \Delta Q_k \right|^{n_i - 1} = \Delta WS \text{ for pseudo loops} \tag{4.18b}$$

To illustrate the development of the system of ΔQ-equations, consider the network in Fig. 4.5. If the Q-equations were used, there would be five junction continuity equations and two loop equations, a total of seven equations. If the H-equations were used, there would be an equation for the HGL-elevation at each of the five nodes where the head is unknown (The head at one node must be known.). But there will be only two

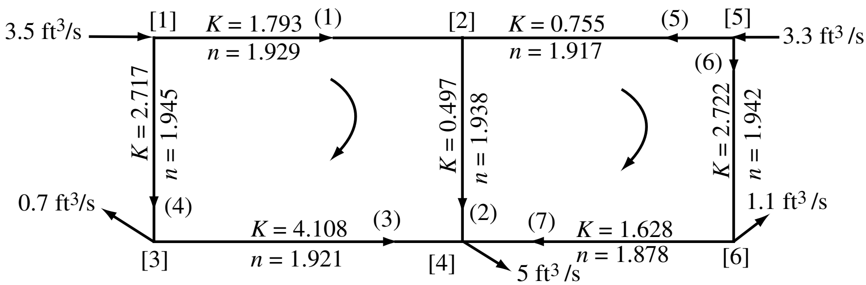


Figure 4.5 A seven-pipe network.

ΔQ-equations, one for each real loop in this network. These two equations are

$$\begin{aligned} F_1 &= K_1\left(Q_{o1} + \Delta Q_1\right)^{n_1} + K_2\left(Q_{o2} + \Delta Q_1 - \Delta Q_2\right)^{n_2} \\ &\quad - K_3\left(Q_{o3} - \Delta Q_1\right)^{n_3} - K_4\left(Q_{o4} - \Delta Q_1\right)^{n_4} = 0 \\ F_2 &= -K_5\left(Q_{o5} - \Delta Q_2\right)^{n_5} + K_6\left(Q_{o6} + \Delta Q_2\right)^{n_6} \\ &\quad + K_7\left(Q_{o7} + \Delta Q_2\right)^{n_7} - K_2\left(Q_{o2} + \Delta Q_1 - \Delta Q_2\right)^{n_2} = 0 \end{aligned} \tag{4.19}$$

The next step is to find an initial estimate for the discharge in each pipe that will satisfy all of the junction continuity equations. One possible set of initial discharges is Q_{o1} = 1.75 ft^3/s, Q_{o2} = 3.55 ft^3/s, Q_{o3} = 1.05 ft^3/s, Q_{o4} = 1.75 ft^3/s, Q_{o5} = 1.8 ft^3/s, Q_{o6} = 1.5 ft^3/s, and Q_{o7} = 0.4 ft^3/s. When these initial discharges and the parameters that are listed on the network sketch are substituted into Eqs. 4.19, the following two equations result; their solution will yield values for the two unknowns ΔQ_1 and ΔQ_2:

$$\begin{aligned} F_1 &= 1.793\left(1.75 + \Delta Q_1\right)^{1.929} + 0.497\left(3.55 + \Delta Q_1 - \Delta Q_2\right)^{1.938} \\ &\quad - 4.108\left(1.05 - \Delta Q_1\right)^{1.921} - 2.717\left(1.75 - \Delta Q_1\right)^{1.945} = 0 \\ F_2 &= -0.755\left(1.8 - \Delta Q_2\right)^{1.917} + 2.722\left(1.5 + \Delta Q_2\right)^{1.942} \\ &\quad + 1.628\left(0.4 + \Delta Q_2\right)^{1.878} - 0.497\left(3.55 + \Delta Q_1 - \Delta Q_2\right)^{1.938} = 0 \end{aligned} \tag{4.20}$$

Upon obtaining the solution to these two equations for the two unknowns, ΔQ_1 and ΔQ_2, the discharge in each pipe can easily be determined by adding these loop corrective discharges to the initial discharges. From these discharges the head loss in each pipe can be determined, and from these values the head at each node can be found.

The nonlinearities in these systems of equations create difficulties when we seek the solution. Later in the chapter we apply the Newton method to overcome this problem.

Example Problem 4.4

Write the system of ΔQ-equations for the network depicted in Example Problem 4.2.

The ΔQ-equations are based on the energy loop equations alone. Therefore these equations can be obtained by taking the equations for F_5 and F_6 directly from Example Problem 4.3 and replacing each discharge Q_i by $Q_{oi} \pm \sum \Delta Q_k$. The ΔQ-equations are

$$F_5 = K_2\left(Q_{o2} + \Delta Q_1\right)^{n_2} + K_3\left(Q_{o3} + \Delta Q_1\right)^{n_3} - K_4\left(Q_{o4} - \Delta Q_1 + \Delta Q_2\right)^{n_4} = 0$$

$$F_6 = K_1\left(Q_{o1} + \Delta Q_2\right)^{n_1} + K_4\left(Q_{o4} - \Delta Q_1 + \Delta Q_2\right)^{n_4} - K_5\left(Q_{o5} - \Delta Q_2\right)^{n_5} - WS_1 + WS_2 = 0$$

With the writing of the ΔQ-equations we must also provide values for Q_{oi} that satisfy all of the junction continuity equations. For this purpose we assume that all four demands are equal to 1.0. Then the following values could serve as an acceptable initialization of the discharges: $Q_{o1} = 3$, $Q_{o2} = 1$, $Q_{o3} = 0$, $Q_{o4} = 1$, $Q_{o5} = 1$, and $Q_{o6} = 1$.

* * *

4.3 PRESSURE REDUCTION AND BACK PRESSURE VALVES

A pressure-reducing valve (PRV) is designed to maintain a constant pressure at its downstream side, independent of the value of the upstream pressure at the device. Only two exceptions to the maintenance of this downstream pressure exist: (1) if the upstream pressure becomes less than the valve setting; or (2) if the downstream pressure exceeds the pressure setting of the valve so that flow would occur in the upstream direction if the PRV were not present. If the first exception occurs, the valve has no effect on flow conditions except to create a local loss; generally its effect is then like a globe valve in dissipating additional head beyond the friction loss in that line. If the second condition occurs, then the PRV acts as a check valve, preventing a reverse flow in the line. Then the PRV allows the pressure immediately downstream from the valve to exceed its pressure setting. In this way PRV's reduce pressures in portions of a pipe distribution system if the pressure would otherwise be excessive, and they may also be used to control the choice of a supply source in response to various demand levels. In the latter applications the PRV acts as a check valve until the pressure is reduced to a critical level by large demands, at which time additional supply sources become available.

A back-pressure valve (BPV) is designed to maintain a constant pressure upstream from it, independent of the value of the downstream pressure. Like a PRV there are exceptions to this normal mode of operation. Should the upstream pressure become less than the pressure setting, the valve can not maintain the pressure setting since it is not an energy-creating device, and the most it can do is shut down the flow in its line. Should the flow want to reverse direction from the positive flow direction through the valve, the valve opens completely and acts as a local loss device. A BPV is used in situations where the pressure would otherwise become too low in elevated portions of the network. Such a situation arises, for example, where a pump is needed to sustain adequate pressures in a higher part of a network but is not needed in the lower portions of the network; without a BPV, or possibly several BPV's, the flow pattern might then lead to discharges through pressure relief valves in the lower portions of the network and possibly create excessively large pressures in the lower region.

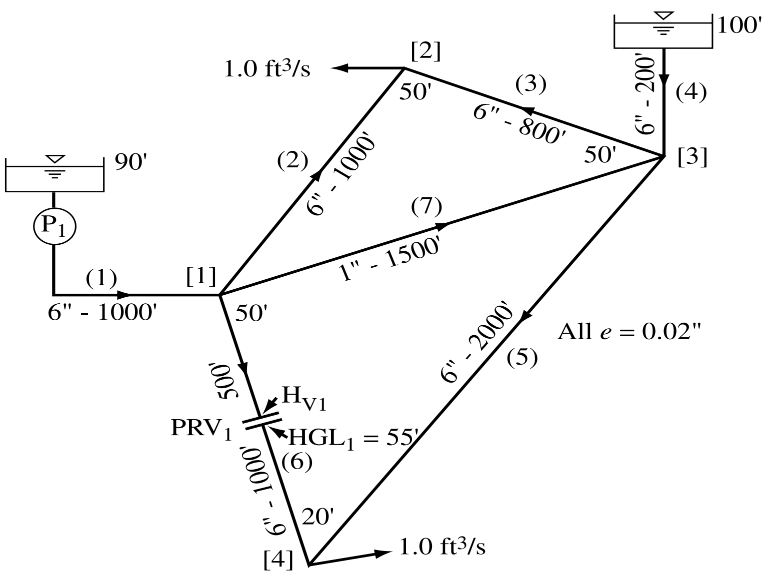
The equations that describe the behavior of a pipe network that contains PRV's or BPV's must include new features to account properly for the effects of these valves on the discharges and pressures throughout the network. Furthermore, the analysis of a pipe network with such devices must be able to determine the correct operational conditions, i.e., determine whether the PRV's and BPV's are operating in their normal modes or in one of their exceptional modes. Methods for adding such devices into network analyses are described in these sections. The discussion begins with pressure reduction valves.

Underlying the writing of the three systems of equations described in Section 4.2 is the basic assumption that a relation exists between the magnitude of the discharge in a pipe and the amount of the head loss, or head difference, between the ends of this pipe. Such a relation does not exist if a PRV (or a BPV) is present in the pipe. Therefore a pipe with a PRV in it should not appear in a normal energy loop equation. However, in the usual mode of operation for a PRV a constant head is maintained at its downstream end; in this way it behaves like a reservoir. Furthermore, regardless of its mode of operation the discharge at the upstream node of a pipe containing a PRV will be the same as the discharge at the downstream node of this pipe. The details of developing a proper system of equations to describe a network containing one or more PRV's are different, depending upon whether a system of Q-equations, H-equations, or ΔQ-equations are desired. Therefore, each of these will be described in a separate section.

4.3.1. *Q*-EQUATIONS FOR NETWORKS WITH PRV'S/BPV'S

The procedure for developing the Q-equation system for a network containing PRV's is as follows: (1) write the junction continuity equations in the usual manner, ignoring the PRV's; (2) replace each PRV with an artificial reservoir which has a water surface elevation equal to the HGL-elevation that is the sum of the pressure head set on the PRV and its elevation in the pipeline; finally (3) write the energy equations around the loops of this modified network. The resulting equations describe the normal mode of operation.

Let's try this procedure on the seven-pipe network shown in Fig. 4.6, in which a PRV exists in pipe 6, located 500 ft. downstream from node 1, the upstream end of this pipe. Since a PRV is a directional device, we must always identify the upstream and downstream



Q ft³/s	Head ft.
1.0	60
1.5	55
2.0	48

Figure 4.6 A seven-pipe network.

ends of the pipe containing it. The system of Q-equations for this network consists of four junction continuity equations and three energy loop equations. According to the usual rules, an independent junction continuity equation can be written for each of the four junctions since there are two supply sources for this network. These junction continuity equations are

$$\begin{aligned} F_1 &= -Q_1 + Q_2 + Q_6 + Q_7 = 0 \\ F_2 &= 1.0 - Q_2 - Q_3 = 0 \\ F_3 &= Q_3 - Q_4 + Q_5 - Q_7 = 0 \\ F_4 &= 1.0 - Q_5 - Q_6 = 0 \end{aligned} \tag{4.21}$$

These continuity equations are unaffected by the presence or absence of a PRV in the network. We next modify the network so the upstream portion of the pipe containing the PRV is removed and the PRV is replaced by a reservoir with a water surface elevation equal to the HGL of the pressure setting of the PRV (see Fig. 4.7). Of the three loops that exist in this modified network, only one is a real loop which traverses pipes 2, 3, and 7. Two pseudo loops also exist. One pseudo loop connects the two original supply sources. This loop can start at the reservoir and end at the source pump so it includes pipes 4, 7, and 1. The second loop must extend from the artificial reservoir created by the PRV to one of the other supply sources (or another artificial reservoir, if two or more PRV's exist in the network). The shortest path for this second pseudo loop will traverse pipes 4, 5, and 6. In writing the head loss in pipe 6, only that portion of the pipe downstream from the PRV is used. A modified loss coefficient K' will be used to denote this change in the exponential formula. The new coefficient K' equals the K for the pipe containing the PRV, multiplied by the ratio of the pipe length from the PRV to the pipe's downstream end divided by the total pipe length, or

$$K' = K(L_d / L) \tag{4.22}$$

or in this example $K_6' = K_6(500/1000) = 0.5K_6$.

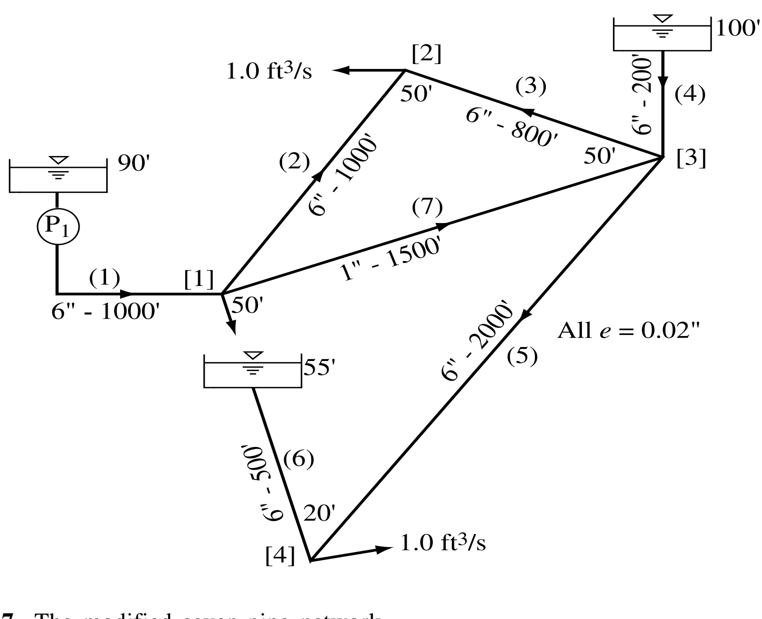


Figure 4.7 The modified seven-pipe network.

The energy equations are

$$F_5 = K_2 Q_2^{n_2} - K_3 Q_3^{n_3} - K_7 Q_7^{n_7} = 0 \ \ (real \ \ loop)$$
$$F_6 = K_4 Q_4^{n_4} - K_7 Q_7^{n_7} - K_1 Q_1^{n_1} + h_{p1} - 100 + 90 = 0 \ \ (pseudo \ \ loop) \qquad (4.23)$$
$$F_7 = K_4 Q_4^{n_4} + K_5 Q_5^{n_5} - K_6' Q_6^{n_6} + 55 - 100 = 0 \ \ (pseudo \ \ loop)$$

The head produced by the pump h_{p1} can be defined by a second-order polynomial passing through three points of the pump curve, or

$$h_{p1} = AQ_1^2 + BQ_1 + C \qquad (4.24)$$

We have now formed seven independent equations that contain the seven unknown discharges $Q_1, Q_2, \ldots, Q_7$. In this example the real loop that was lost by having a PRV in pipe 6 is replaced by an additional pseudo loop. We see that the number of equations again equals the number of unknown discharges.

To obtain a solution for this network by using the computer program NETWK, the input data can be prepared (see the NETWK user manual for input data requirements or the condensed description of this input on the CD) as listed in Fig. 4.8. The solution tables from NETWK are reproduced in Fig. 4.9. A study of this output will show that the PRV is operating in its normal mode of operation.

```
Example of a network containing a PRV
/* 6 1 4 1000               RESER
PIPES                       7 1 3 1500 1              4 100
1 0 1 1000 6 0.02           NODES                     PUMPS
2 1 2                       1 0 50                    1 1 60 1.5 55 2 48 90
3 3 2 800                   2 1                       VALVE
4 0 3 200                   3 0                       6 500 55
5 3 4 2000                  4 1 20                    RUN
```

Figure 4.8 Input data for the network shown in Figs. 4.6 and 4.7.

This solution indicates that the PRV is operating in its normal mode of maintaining the set pressure at its downstream end because the reported downstream HGL-elevation equals the value specified in the input data. If this had not been the case, the solution from NETWK would have indicated either that the PRV had shut off the flow in pipe 6 or that it was completely open and replaced by a minor loss. In solving the network equations, if the discharge in pipe 6 had been negative, then the program would have noted that the PRV would act as a check valve, preventing a reverse flow. If this situation should occur, then the network problem would be altered so it would only have six pipes instead of seven (pipe 6 would not exist in this modified network). The equations describing the flows in

```
LOSSES DUE TO FLUID FRICTION IN ALL PIPES
POWER LOSS  =  11.51 H.P.  =  8.585 KWATTS.
ENERGY LOSS  =  206.0  KWHRS/DAY

PUMPS:
PIPE   HEAD     Q   HORSEPOWER   KILOWATT   KWATT-HRS/DAY
 1     59.1   1.11     7.43        5.54        133.0
ELEVATION OF HGL UPSTREAM AND DOWNSTREAM OF PRVS:
```

Figure 4.9 Output tables from NETWK.

```
   PIPE    UPSTREAM  DOWNSTREAM
             HGL        HGL
    6      121.79     55.00
  UNITS OF SOLUTION ARE:
  DIAMETER,     inch
  LENGTH,       feet
  HEAD,         feet
  ELEVATION,    feet
  PRESSURE,     lb/in2
  DISCHARGE,    ft3/s
  DARCY-WEISBACH FORMULA USED FOR COMPUTING HEAD LOSSES
```

PIPE DATA

PIPE NO.	NODES FROM	TO	L ft.	DIA. in	e x10^3 in	Q ft^3/s	VEL. ft/s	HEAD LOSS ft.	HLOSS/ 1000
1	0	1	1000	6.0	20.0	1.11	5.65	27.28	27.28
2	1	2	1000	6.0	20.0	1.07	5.43	25.26	25.26
*3	2	3	800	6.0	20.0	0.07	0.34	0.10	0.12
4	0	3	200	6.0	20.0	0.89	4.54	3.55	17.74
5	3	4	2000	6.0	20.0	0.96	4.91	41.47	20.74
6	1	4	1000	6.0	20.0	0.04	0.18	0.04	0.04
7	1	3	1500	1.0	20.0	0.01	1.31	25.56	17.04

AVE. VEL. = 3.19 ft/s, AVE. HL/1000 = 15.46, MAX. VEL. = 5.65 ft/s, MIN. VEL. = 0.18 ft/s
*Flow direction is reversed from input data.

NODE DATA

NODE	DEMAND ft^3/s	gal/min	ELEV. ft.	HEAD ft.	PRESSURE lb/in^2	HGL ELEV. ft.
1	0.00	0.0	50.	71.81	31.1	121.81
2	1.00	448.8	50.	46.55	20.2	96.55
3	0.00	0.0	50.	46.45	20.1	96.45
4	1.00	448.8	20.	34.98	15.2	54.98

AVE. HEAD = 49.95 ft., AVE. HGL = 92.45 ft.
MAX. HEAD = 71.81 ft., MIN HEAD = 34.98 ft.

Figure 4.9, concluded. Output tables from NETWK.

this modified network would consist of the original equations with the last one omitted. If the HGL elevation at node 1 had been lower than the HGL setting of the PRV, then it would be known that the PRV would not be able to sustain its pressure setting, and the network problem must then be solved by using equations that replace the PRV with a minor loss device. For this last mode of operation the last energy equation would be replaced by a real loop equation traversing pipes 5, 6, and 7. Pipe 6 would contain a minor loss device to represent the PRV as being fully open.

The procedure for writing the system of Q-equations should now be apparent for back-pressure valves (BPV's) in networks. As with PRV's, the junction continuity equations are written ignoring the presence of BPV's. The junction continuity equations are unaffected by the existence of a BPV. In writing the energy equations, the upstream side of each BPV is replaced by an artificial reservoir; in each case the pipe segment from the downstream end of the BPV to its downstream node is then removed, and the energy equations are written for this revised network.

The writing of a system of Q-equations will be illustrated with the network in Fig. 4.10, which has 9 pipes and 6 nodes, is supplied by a source pump and has two tanks

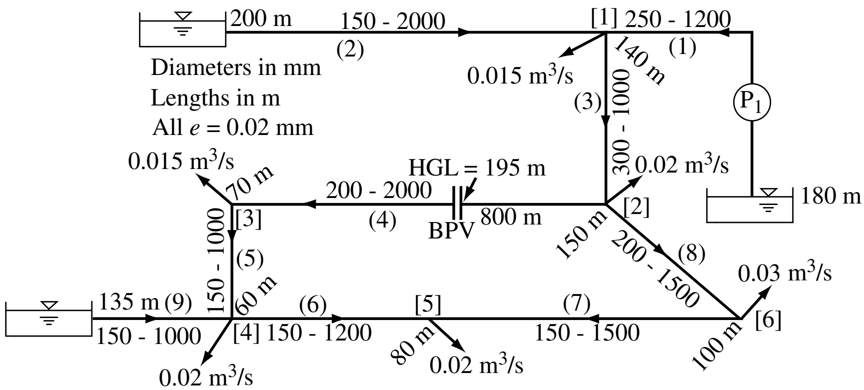


Figure 4.10 A 9-pipe, 6-node network.

(reservoirs) connected to it. Without a BPV (or some other device) this network would cause the lower reservoir at the end of pipe 9 to overflow. There are six junctions in this network. The corresponding six junction continuity equations are

$$
\begin{aligned}
F_1 &= 0.015 - Q_1 - Q_2 + Q_3 = 0 \\
F_2 &= 0.020 - Q_3 + Q_4 + Q_8 = 0 \\
F_3 &= 0.015 - Q_4 + Q_5 = 0 \\
F_4 &= 0.020 - Q_5 + Q_6 - Q_9 = 0 \\
F_5 &= 0.020 - Q_7 - Q_6 = 0 \\
F_6 &= 0.030 - Q_8 + Q_7 = 0
\end{aligned}
\tag{4.25}
$$

Before forming the loops around which the energy equations are written, an artificial reservoir is placed on the upstream side of the BPV with a water surface elevation equal to the HGL resulting from the pressure setting of the valve. The pipe downstream from the BPV is removed. When these changes are completed, the network appears as in Fig. 4.11, and energy equations can next be written around the loops of this modified network. Three loops are needed, since NL = NP - NJ = 9 - 6 = 3. These are all pseudo loops and may be

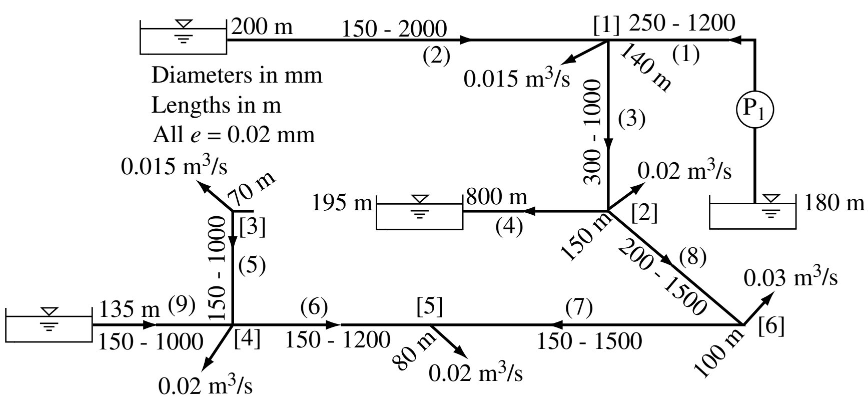


Figure 4.11 The modified 9-pipe, 6-node network.

composed in the following way: the pipes in loop 1 are 1 and 2; the pipes in loop 2 are 1, 3, and 4 (upstream portion); and the pipes in loop 3 are 9, 6, 7, 8, and 4 (upstream portion). It is incorrect to write a loop through pipes 9, 5, and 4 (the upstream portion) because a BPV sets the pressure on its upstream side. Hence the energy equations in the Q-equation system are the following:

$$\begin{aligned} F_7 &= K_1 Q_1^{n_1} - h_{p1} - K_2 Q_2^{n_2} - 180 + 200 = 0 \\ F_8 &= K_1 Q_1^{n_1} - h_{p1} + K_3 Q_3^{n_3} + K_4' Q_4^{n_4} - 180 + 195 = 0 \\ F_9 &= K_9 Q_9^{n_9} + K_6 Q_6^{n_6} - K_7 Q_7^{n_7} - K_8 Q_8^{n_8} + K_4' Q_4^{n_4} - 135 + 195 = 0 \end{aligned} \tag{4.26}$$

One possible input file to NETWK for the solution of this problem is presented in Fig. 4.12, and the resulting solution tables are presented in Fig. 4.13.

```
Network Containing BPV
/* 2 .02 140
$SPECIF NFLOW=3,NPGPM=3,NUNIT=4 $END
PIPES
1 0 1 1200 250 .02
2 0 1 2000 150
3 1 2 1000 300
4 2 3 2000
5 3 4 1000 150
6 4 5 1200
7 6 5 1500
8 2 6 1500 200
9 0 4 1000 150
NODES
1 .015 140

3 .015 70
4 .02 60
5 .02 80
6 .03 100
RESER
2 200
9 135
PUMPS
1 .1 35 .15 32 .2 28 180
BPVALVE
4 1200 195
RUN
```

Figure 4.12 The input data file to NETWK for the 9-pipe, 6-node network.

From this solution we see that the BPV dissipates 65.88 m of head to sustain the upstream HGL setting of 195 m. This value is obtained by subtracting the downstream HGL from the BPV setting. It is a worthwhile exercise to begin with the head losses in the PIPE DATA table and verify the HGL elevations reported in the NODE DATA table; it will lead to a better understanding of the BPV and its effect on pressures and discharges in this network as the BPV operates in its normal mode. If the solution had shown a negative flow through pipe 4, then the downstream pressure would actually be larger than the BPV setting, and the valve would open up completely. For this occurrence the BPV must be re-placed by a minor loss device, and then this modified network problem could be studied. If the HGL at node 2 (the node immediately upstream from the BPV) were less than the HGL established by the BPV setting, then the BPV would close completely. The pipe

```
LOSSES DUE TO FLUID FRICTION IN ALL PIPES
POWER LOSS  =  65.18 H.P.  =  48.63 KWATTS.
ENERGY LOSS  =  1167.0  KWHRS/DAY

PUMPS:
PIPE   HEAD     Q    HORSEPOWER   KILOWATT   KWATT-HRS/DAY
 1     34.88   10.0    46.9        35.0        839.8
HGL DOWNSTREAM AND UPSTREAM FROM BPV
 4     129.12        195.00
```

Figure 4.13 The output tables from NETWK for the 9-pipe, 6-node network.

PIPE DATA

PIPE NO.	NODES FROM	TO	L	DIA.	e x10³	Q	VEL.	HEAD LOSS	HLOSS/ 1000
			m	mm	mm	m^3/s	m/s	m	
1	0	1	1200	250	20.0	0.102	2.09	15.58	12.98
2	0	1	2000	150	20.0	0.004	0.21	0.75	0.37
3	1	2	1000	300.	20.0	0.091	1.29	4.28	4.28
4	2	3	2000	300.	20.0	0.006	0.08	0.06	0.03
5	4	3	1000	150.	20.0	0.009	0.52	1.90	1.89
6	5	4	1200	150.	20.0	0.015	0.86	5.69	4.74
7	6	5	1500	150.	20.0	0.035	2.00	33.11	22.07
8	2	6	1500	200.	20.0	0.065	2.08	25.25	16.83
9	0	4	1000	150.	20.0	0.014	0.79	4.03	4.03

AVE. VEL. = 1.10 m/s, AVE. HL/1000 = 7.47, MAX. VEL. = 2.09 m/s, MIN. VEL. = 0.08 m/s

NODE DATA

NODE	DEMAND m^3/s	DEMAND ft^3/s	ELEV. m	HEAD m	PRESSURE kPa	HGL ELEV. m
1	0.015	0.53	140.0	59.25	580.7	199.25
2	0.020	0.71	140.0	55.02	539.2	195.02
3	0.015	0.53	70.0	59.08	579.0	129.08
4	0.020	0.71	60.0	70.97	695.6	130.97
5	0.020	0.71	80.0	56.66	555.3	136.66
6	0.030	1.06	100.0	69.78	683.8	169.78

AVE. HEAD = 61.79 m, AVE. HGL = 160.13 m
MAX. HEAD = 70.97 m, MIN. HEAD = 55.02 m

Figure 4.13 (Concluded) The output tables from NETWK for the 9-pipe, 6-node network.

containing the BPV should then be removed from the network, and the problem could then be solved by using the equations for this modified network; then the BPV could not maintain the pressure setting, and it would simply prevent any flow from passing through the pipe in which it is installed.

4.3.2. *H*-EQUATIONS FOR NETWORKS WITH PRV'S/BPV'S

The procedure for writing the *H*-equations for a network that contains PRV's and/or BPV's is described here. First, view the HGL resulting from the pressure setting of the device as a reservoir, since under normal operation the HGL is fixed by the device. Second, place an additional unknown variable on the other side of the device to represent the elevation of the HGL there. We will denote this variable by H_{vi}, in which i is the number of the device. For the first PRV or BPV $i = 1$, for the second $i = 2$, etc. For a PRV the value of H_{vi} is the HGL-elevation immediately upstream from the valve, whereas H_{vi} is the HGL-elevation immediately downstream from the valve for a BPV. Third, the junction continuity equations are written in the usual way, with the difference between the upstream and downstream HGL-elevations, divided by K for this pipe, all raised to the reciprocal of the discharge exponent n, i.e., $Q_k = \{(H_i - H_j)/K_k\}^{1/n_k}$. Finally, since an additional unknown is introduced for each PRV or BPV, one additional equation must be added to the system of continuity equations for each device. These additional equations are obtained by noting that the head losses in the upstream and downstream portions of the pipe containing the device are proportional to these two lengths. For a PRV this equation is

$$(HGL - H_d)L_u - (H_u - H_{vi})L_d = 0 \tag{4.27}$$

in which L_u and L_d are the lengths upstream and downstream from the device, respectively, and H_d and H_u are the HGL-elevations at the downstream and upstream ends of pipe i containing the device.

Using again the network depicted previously in Fig. 4.6 to illustrate the formation of the H-equation system, we would first modify the network as shown in Fig. 4.14:

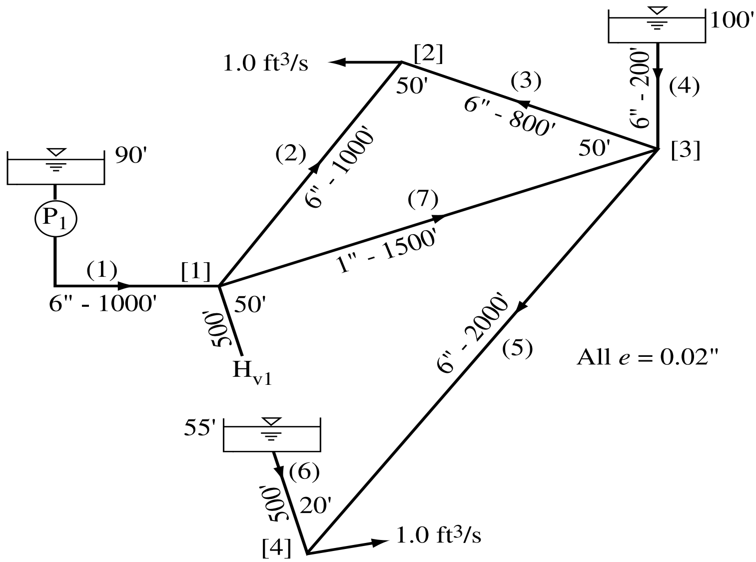


Figure 4.14 A 9-pipe, 6-node network containing a BVP, as modified.

The final H-equations for this network are the following:

$$\begin{aligned}
&-\{(90 + h_{p1} - H_1)/K_1\}^{1/n_1} + \{(H_1 - H_2)/K_2\}^{1/n_2} \\
&\qquad + \{(H_1 - H_{v1})/K_6''\}^{1/n_6} + \{(H_1 - H_3)/K_7\}^{1/n_7} = 0 \\
&1.0 - \{(H_1 - H_2)/K_2\}^{1/n_2} - \{(H_3 - H_2)/K_3\}^{1/n_3} = 0 \\
&\{(H_3 - H_2)/K_3\}^{1/n_3} - \{(100 - H_3)/K_4\}^{1/n_4} \\
&\qquad + \{(H_3 - H_4)/K_5\}^{1/n_5} - \{(H_1 - H_3)/K_7\}^{1/n_7} = 0 \\
&1.0 - \{(H_3 - H_4)/K_5\}^{1/n_5} - \{(HGL_1 - H_4)/K_6'\}^{1/n_6} = 0 \\
&(HGL_1 - H_4)L_u - (H_1 - H_{v1})L_d = (HGL_1 - H_4) - (H_1 - H_{v1}) = 0
\end{aligned} \tag{4.28}$$

in which K_6'' and K_6' are the coefficients for the upstream and downstream portions of pipe 6, respectively.

The network in Fig. 4.10 that contains the BPV should now be viewed as shown in Fig. 4.15, and the H-equation system for this network is presented as Eqs. 4.29.

$$F_1 = 0.015 - \{(180 + h_{p1} - H_1)/K_1\}^{1/n_1}$$
$$- \{(200 - H_1)/K_2\}^{1/n_2} + \{(H_1 - H_2)/K_3\}^{1/n_3} = 0$$
$$F_2 = 0.020 - \{(H_1 - H_2)/K_3\}^{1/n_3}$$
$$+ \{(H_2 - HGL_1)/K_4''\}^{1/n_4} + \{(H_2 - H_6)/K_8\}^{1/n_8} = 0$$
$$F_3 = 0.015 - \{(H_{v1} - H_3)/K_4'\}^{1/n_4} + \{(H_3 - H_4)/K_5\}^{1/n_5} = 0$$
$$F_4 = 0.020 - \{(H_3 - H_4)/K_5\}^{1/n_5}$$
$$+ \{(H_4 - H_5)/K_6\}^{1/n_6} - \{(135 - H_4)/K_9\}^{1/n_9} = 0 \qquad (4.29)$$
$$F_5 = 0.020 - \{(H_4 - H_5)/K_6\}^{1/n_6} - \{(H_6 - H_5)/K_7\}^{1/n_7} = 0$$
$$F_6 = 0.020 - \{(H_2 - H_6)/K_8\}^{1/n_8} + \{(H_6 - H_5)/K_7\}^{1/n_7} = 0$$
$$F_7 = 1200(H_2 - HGL_1) - 800(H_{v1} - H_3) = 0$$

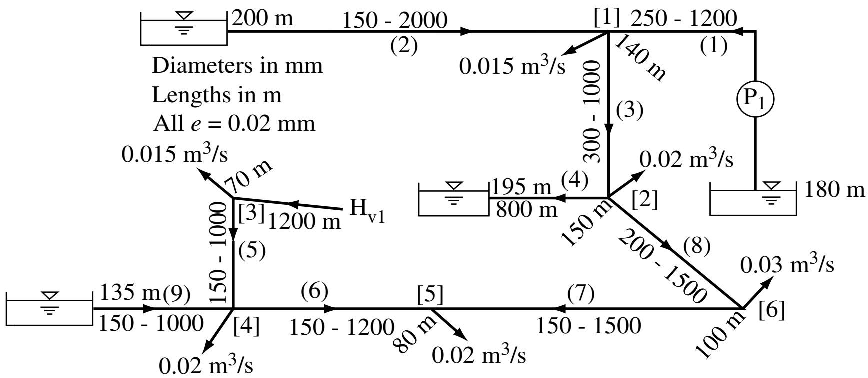


Figure 4.15 A seven-pipe network, modified.

4.3.3. ΔQ-EQUATIONS FOR NETWORKS WITH PRV'S/BPV'S

Let us begin this section by reviewing the underlying concept that is used in writing the ΔQ-equations: if the junction continuity equations are satisfied by the initial discharges Q_{oi}, then a corrective loop discharge, ΔQ, can flow around a loop without violating the principle that the discharge into all junctions will still equal the discharge out of these junctions, regardless of the magnitude of ΔQ. These corrective loop discharges can be regarded as the primary unknowns, and the resulting solution to the system of equations will produce discharges that also satisfy the energy equations around the loops. Therefore the discharges Q_i in the Q-equation loops were replaced by $Q_{oi} \pm \sum \Delta Q_k$. For the junction continuity equations to remain valid for any values of ΔQ_k, these corrective loop discharges must circulate around loops that are formed before any PRV's or BPV's are converted into artificial reservoirs. Thus it is necessary to consider two sets of loops with this method. The first set is the set of loops around which the ΔQ's circulate, and the second set is the set of loops that is used in writing the energy equations. These two sets of loops will be called the corrective discharge or ΔQ loops, and the energy loops. The ΔQ loops are formed while ignoring the existence of PRV's or BPV's. These devices are later replaced by artificial reservoirs, and the energy e quations are written for this modified network. Thus the energy set of loops will always contain more pseudo loops than does the ΔQ set of loops by the number of PRV's and/or BPV's that exist in the network. To track these two separate sets of loops in figures, the ΔQ loops will list ΔQ_i by the arc denoting the loop, and the energy loops will be numbered by roman numerals I, II, etc.

To illustrate the writing of the ΔQ-equations, we examine again the network with a PRV that is in Fig. 4.6. This network is redrawn below in Fig. 4.16 to display both the corrective discharge loops and the energy loops. To emphasize that ΔQ loops are different than energy loops, ΔQ_3 is chosen to pass through pipes 4, 3, 2, and 1. A more effi-cient route for this corrective discharge loop would traverse pipes 4, 7, and 1, coinciding with energy loop II, because one less pipe is in this loop.

To obtain the ΔQ-equations, we replace each Q_i in the energy equation portion of the Q-equations by $Q_i = Q_{oi} \pm \sum \Delta Q_k$, in which the ΔQ's must be those circulating through pipe i, as defined by the ΔQ loops, and the sign before each term in the sum is determined by whether ΔQ agrees with or opposes the direction of the assumed discharge Q_{oi}. If the directions agree, the sign is positive; otherwise the sign is negative. The resulting ΔQ-equations for this network are listed as Eqs. 4.30.

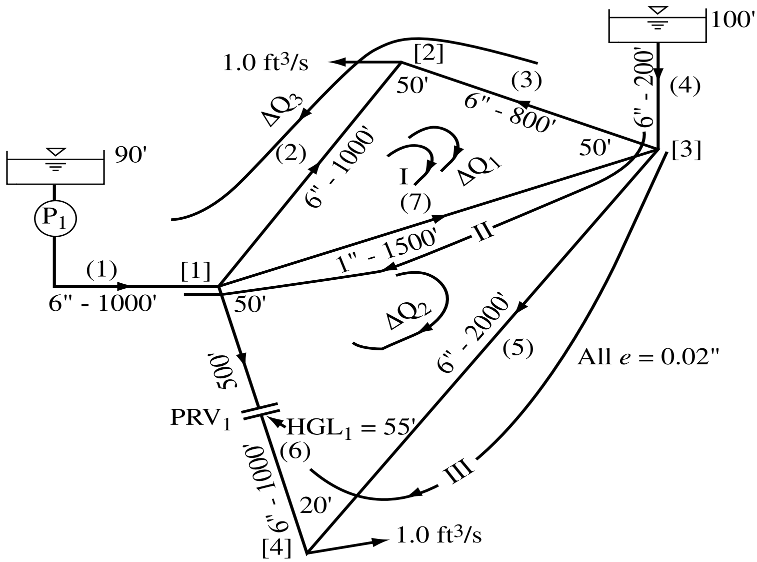


Figure 4.16 The network in Fig. 4.6, modified for solution with the ΔQ-equations.

$$
\begin{aligned}
F_1 &= K_2\left(Q_{o2} + \Delta Q_1 - \Delta Q_3\right)^{n_2} - K_3\left(Q_{o3} - \Delta Q_1 + \Delta Q_3\right)^{n_3} \\
&\qquad\qquad - K_7\left(Q_{o7} - \Delta Q_1 + \Delta Q_2\right)^{n_7} = 0 \\
F_2 &= K_4\left(Q_{o4} + \Delta Q_3\right)^{n_4} - K_7\left(Q_{o7} - \Delta Q_1 + \Delta Q_2\right)^{n_7} \\
&\qquad\qquad - K_1\left(Q_{o1} - \Delta Q_3\right)^{n_1} + h_{p1} - 10 = 0 \\
F_3 &= K_4\left(Q_{o4} + \Delta Q_3\right)^{n_4} + K_5\left(Q_{o5} + \Delta Q_2\right)^{n_5} - K_6'\left(Q_{o6} - \Delta Q_2\right)^{n_6} - 45 = 0
\end{aligned}
\tag{4.30}
$$

These equations are in a sense incomplete until each Q_{oi} is replaced by a value. When this step is completed, then these three equations contain only three unknowns, ΔQ_1, ΔQ_2 and ΔQ_3. In principle each reader could produce a different set of acceptable values for the initial discharges, so long as they do indeed satisfy each and every junction continuity equation. One valid set of Q_{oi}'s is $Q_{o1} = 1.0$, $Q_{o2} = 1.0$, $Q_{o3} = 0.0$, $Q_{o4} = 1.0$, $Q_{o5} = 1.0$, $Q_{o6} = 0.0$, and $Q_{o7} = 0.0$. Finally, the pump head now becomes $h_{p1} = A(Q_{o1} + \Delta Q_1 - \Delta Q_3)^2 + B(Q_{o1} + \Delta Q_1 - \Delta Q_3) + C$ when the pump curve is fitted with a second-order polynomial. If desired, as an alternative either a linear or a higher-order polynomial could be chosen to describe the operating characteristics of this pump.

Now let us revisit the network in Fig. 4.10 that contains a BPV as a second illustration of forming the ΔQ-equations. In this analysis we can visualize the two sets of loops as shown in Fig. 4.17. The ΔQ loops ignore the presence of the BPV in this network, but the energy loops will be written for the modified network with the BPV converted into an artificial reservoir. The resulting ΔQ-equations for this network appear as Eqs. 4.31. In

$$
\begin{aligned}
F_1 &= K_1(Q_{o1} + \Delta Q_2)^{n_1} - h_{p1} - K_2(Q_{o2} - \Delta Q_2 - \Delta Q_3)^{n_2} + 20 = 0 \\
F_2 &= K_1(Q_{o1} + \Delta Q_2)^{n_1} - h_{p1} + K_3(Q_{o3} - \Delta Q_3)^{n_3} \\
&\qquad + K_4'(Q_{o4} - \Delta Q_1 - \Delta Q_3)^{n_4} + 15 = 0 \\
F_3 &= K_9(Q_{o9} + \Delta Q_3)^{n_9} + K_6(Q_{o6} - \Delta Q_1)^{n_6} - K_7(Q_{o7} + \Delta Q_1)^{n_7} \\
&\qquad - K_8(Q_{o8} + \Delta Q_1)^{n_8} + K_4'(Q_{o4} - \Delta Q_1 - \Delta Q_3)^{n_4} + 60 = 0
\end{aligned}
\tag{4.31}
$$

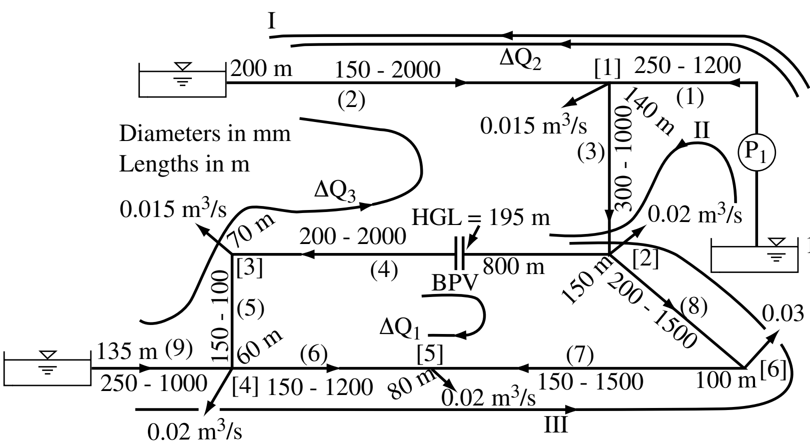
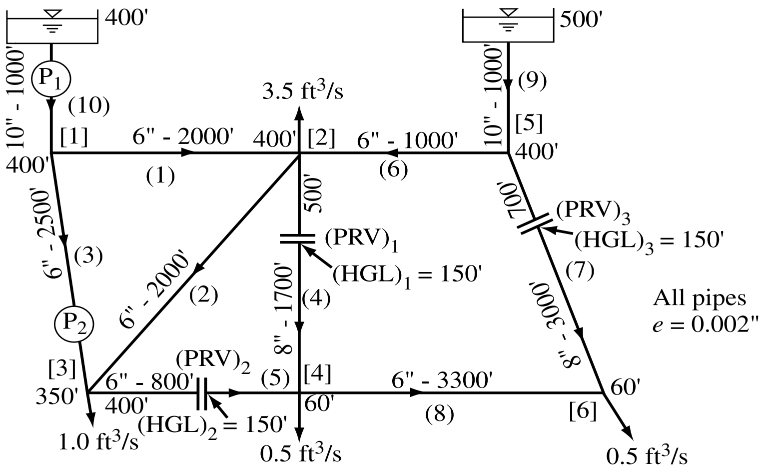


Figure 4.17 The network of Fig. 4.10, modified for the ΔQ-equation system.

these equations $h_{p1} = A(Q_{o1} + \Delta Q_2)^2 + B(Q_{o1} + \Delta Q_2) + C$. The initial flows that satisfy the junction continuity equations are chosen as $Q_{o1} = 0.1$, $Q_{o2} = 0.0$, $Q_{o3} = 0.085$, $Q_{o4} = 0.015$, $Q_{o5} = 0.0$, $Q_{o6} = 0.0$, $Q_{o7} = 0.02$, $Q_{o8} = 0.05$, and $Q_{o9} = 0.02$. The substitution of these values into Eqs. 4.31 yields the final set of ΔQ-equations.

If large differences in ground elevation occur in a network, PRV's are often installed in a sequence of pipes to prevent excessively large pressures in the lower part of the network. Such a series of PRV's may cause pressures in one subregion to be completely independent of the remainder of the network. Such isolation creates what are commonly called separate pressure zones. When separate pressure zones are created, it is normally better to form subnetworks and analyze each one separately, starting with the subnetwork at the lower elevation. The solution from the isolated lower subnetwork can then be used to determine the demands at the nodes of the next higher network, and so on.

The 10-pipe, 6-node network in Fig. 4.18 contains three PRV's in pipes 4, 5, and 7, respectively; it typifies such a situation. In this network the three PRV's cause the pressures at nodes 4 and 6 to be independent of pressures in the remainder of the network. The best analysis, therefore, would begin by studying separately the subnetwork that is composed of pipes 5, 4, 7, and 8 downstream from the PRV's. In this subnetwork the PRV's are modeled as three constant-head reservoirs. The values of Q_4, Q_5, and Q_7 from the solution of the subnetwork are next added to the other demands to determine the demands at nodes 3, 2, and 5, respectively, in organizing the remainder of the network for analysis.



Pump Characteristics

Pump 1		Pump 2	
Q ft³/s	*H* ft.	*Q* ft³/s	*H* ft.
1.5	110	1.5	15.0
2.5	104	2.0	12.0
3.5	92	3.0	7.5

Figure 4.18 A network with two pressure zones.

While it is generally not difficult to determine by visual examination of a map of the piping system whether PRV's isolate a portion of a network into a separate pressure zone, in computer programs a simple test is needed to identify this situation. Such a test can be based on the fact that no series of connected pipes exists between any of the artificial reservoirs created by the PRV's and any of the other reservoirs and source pumps. That no connection exists in the network example can be seen by resketching the network, as shown in Fig. 4.19. As a consequence, if pseudo loops between artificial reservoirs or source pumps cannot be found by a computer program that uses its own internal loop-finding algorithm, then the PRV's isolate a subnetwork into a pressure zone that is separate from the remainder of the network. One difficulty with this kind of test, which relies on the inability to find paths which connect all supply sources, is that errors in the network input data or an ill-defined network itself can also cause this test to be satisfied; network computer programs are supposed to identify such input errors and terminate if any such errors are found. Thus it is desirable to have an independent verification, i.e., a separate test, that can indicate that separate pressure zones exist.

This alternative or verification test could take the form that is described next. The goal in this "test" is to determine whether the sum $NJ + NL$, determined in the usual way, is equal to, or exceeds, the number of pipes in the network.

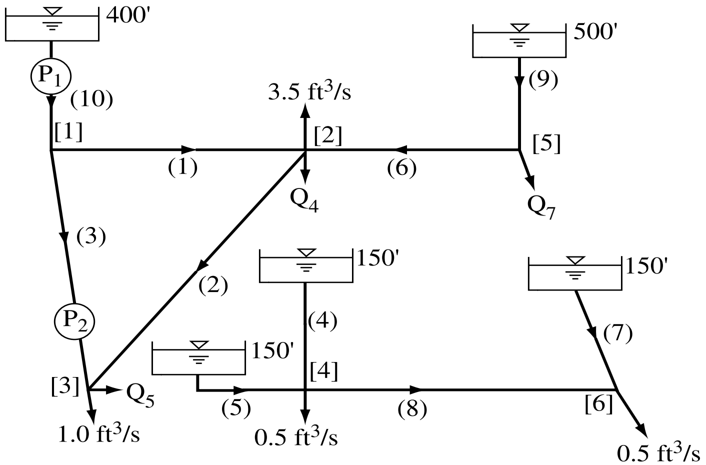


Figure 4.19 PRV's create two separate pressure zones.

To highlight the problem, let us examine closely the network in Fig. 4.19 which is known to have two separate pressure zones. Overall there are six junctions, so $NJ = 6$. There is one real loop, and the usual rule indicates there is $N_{res} - 1 = 5 - 1 = 4$ pseudo loops, or $NL = 5$. Using these values, we obtain $NJ + NL = 6 + 5 = 11$, which is larger than the actual number of pipes, which is $NP = 10$. In this instance the inequality occurs because only four independent loops exist, one real loop and three pseudo loops. These numbers will be found to be correct when we view the overall network as two separate networks. The higher network in Fig. 4.19 has $NP = 6$, one real loop and one pseudo loop, $N_s = N_{res} - 1$ and $NL = 2$. Since there are four junctions in the network with the higher pressure zone, $NJ = 4$, and $NP = NJ + NL = 6$. For the network with the lower pressure zone $NP = 4$, $NJ = 2$, there are no real loops, and the expected number of pseudo loops is $N_{res} - 1 = 2$, giving $NL = 2$. Again $NP = NJ + NL$.

The verification test to determine whether PRV's isolate a portion of a network into a separate pressure zone might therefore be as follows:

1. Find the real loops which exist after pipes containing PRV's have been disconnected from their upstream junctions.
2. Compute NL_s from $NL_s = N_{res} + N_{pump} - 1$.
3. Add the number of loops that were found in steps 1 and 2 to determine NL, and then determine NP from $NP = NJ + NL$.
4. If this computed NP exceeds the number of pipes in the network, then the PRV's isolate a portion of the network. The amount of the difference between the newly computed NP and the actual number of pipes in the network is the number of additional pressure zones that exist in the network; the total number of zones is one more than this number of additional zones.

4.4 SOLVING THE NETWORK EQUATIONS

4.4.1. NEWTON METHOD FOR LARGE SYSTEMS OF EQUATIONS

In Sections 4.2 and 4.3 we explored the writing of systems of algebraic equations to describe the relations between the discharges, pressures, and other variables and parameters in a pipe network. Many of the equations in these systems of equations are nonlinear. A good method for solving nonlinear equations is therefore needed. Numerous methods exist, but the Newton Method is the method of choice here. Its application to the solution of the Q-equations, the H-equations and the ΔQ-equations will be discussed in this section. To treat the unknown discharges (when using the Q-equations), the unknown heads (when using the H-equations), and the unknown corrective loop discharges (when using the ΔQ-

equations) in a uniform way, the primary unknown variable in this section will be called the vector $\{\boldsymbol{x}\}$.

The Newton iterative formula for solving a system of equations can be written as

$$\{x\}^{(m+1)} = \{x\}^{(m)} - [D]^{-1}\{F\}^{(m)} \tag{4.32a}$$

Here $\boldsymbol{x}$ is an entire column vector $\{\boldsymbol{x}\}$ of unknowns, $\{\boldsymbol{F}\}$ is an entire column vector of equations, and $[\boldsymbol{D}]^{-1}$ is the inverse of a matrix $[\boldsymbol{D}]$ which is the Jacobian. The Jacobian occurs in several applications in mathematics, and it represents the following matrix of derivatives:

$$[D] = \begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} & \cdot & \cdot & \frac{\partial F_1}{\partial x_n} \\ \frac{\partial F_2}{\partial x_1} & \frac{\partial F_2}{\partial x_2} & \cdot & \cdot & \frac{\partial F_2}{\partial x_n} \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ \frac{\partial F_n}{\partial x_1} & \frac{\partial F_n}{\partial x_2} & \cdot & \cdot & \frac{\partial F_n}{\partial x_n} \end{bmatrix} \tag{4.33}$$

Likewise $\{\boldsymbol{x}\}$ and $\{\boldsymbol{F}\}$ are actually

$$\{x\} = \begin{Bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ x_n \end{Bmatrix} \qquad \{F\} = \begin{Bmatrix} F_1 \\ F_2 \\ \cdot \\ \cdot \\ F_n \end{Bmatrix} \tag{4.34}$$

Equation 4.32a indicates that the Newton method solves a system of nonlinear equations by iteratively solving a system of linear equations because $[\boldsymbol{D}]^{-1}\{\boldsymbol{F}\}$ represents the solution of the linear system of equations

$$[D]\{z\} = \{F\} \tag{4.32b}$$

That is, the vector that is subtracted from the current estimate of the unknown vector $\{x\}$ in Eq. 4.32a is the solution $\{z\}$ to the linear system of equations that is Eq. 4.32b. In practice we therefore see that the Newton method solves a system of equations by the iterative formula

$$\{x\}^{(m+1)} = \{x\}^{(m)} - \{z\} \tag{4.32c}$$

where $\{z\}$ is the solution vector that is obtained by solving $[\boldsymbol{D}]\{z\} = \{\boldsymbol{F}\}$. If the system should actually contain only linear equations, then the first iteration will produce the exact solution.

The development of Eq. 4.32 follows. We begin by using a multi-dimensional Taylor series expansion to evaluate the individual equations F_i in the neighborhood of an initial solution estimate that we call $\{x\}$ which is presumed to be near the actual solution:

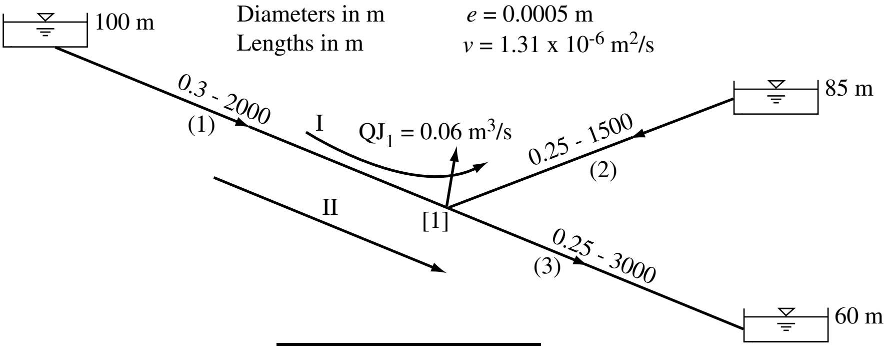
$$
\begin{aligned}
F_1^{(m+1)} &= F_1^{(m)} + \frac{\partial F_1}{\partial x_1}\Delta x_1 + \frac{\partial F_1}{\partial x_2}\Delta x_2 + \cdot\ \ \cdot + \frac{\partial F_1}{\partial x_n}\Delta x_n + \mathrm{O}\left(\Delta x^2\right) = 0 \\
F_2^{(m+1)} &= F_2^{(m)} + \frac{\partial F_2}{\partial x_1}\Delta x_1 + \frac{\partial F_2}{\partial x_2}\Delta x_2 + \cdot\ \ \cdot + \frac{\partial F_2}{\partial x_n}\Delta x_n + \mathrm{O}\left(\Delta x^2\right) = 0 \\
&\cdot \\
&\cdot \\
F_n^{(m+1)} &= F_n^{(m)} + \frac{\partial F_n}{\partial x_1}\Delta x_1 + \frac{\partial F_n}{\partial x_2}\Delta x_2 + \cdot\ \ \cdot + \frac{\partial F_n}{\partial x_n}\Delta x_n + \mathrm{O}\left(\Delta x^2\right) = 0
\end{aligned}
\tag{4.35}
$$

When we use matrix notation and make the substitution $\Delta x_i = x_i^{(m+1)} - x_i^{(m)}$, this system of equations becomes

$$
\begin{Bmatrix} F_1 \\ F_2 \\ \cdot \\ \cdot \\ F_n \end{Bmatrix}^{(m)} + \begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} & \cdot & \cdot & \frac{\partial F_1}{\partial x_n} \\ \frac{\partial F_2}{\partial x_1} & \frac{\partial F_2}{\partial x_2} & \cdot & \cdot & \frac{\partial F_2}{\partial x_n} \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ \frac{\partial F_n}{\partial x_1} & \frac{\partial F_n}{\partial x_2} & \cdot & \cdot & \frac{\partial F_n}{\partial x_n} \end{bmatrix} \begin{Bmatrix} x_1^{(m+1)} - x_1^{(m)} \\ x_2^{(m+1)} - x_2^{(m)} \\ \cdot \\ \cdot \\ x_n^{(m+1)} - x_n^{(m)} \end{Bmatrix} = 0 \tag{4.36}
$$

which can be written compactly as $\{\boldsymbol{F}\}^{(m)} + [\boldsymbol{D}]^{(m)}(\{\boldsymbol{x}\}^{(m+1)} - \{\boldsymbol{x}\}^{(m)}) = \{\boldsymbol{0}\}$ and solved for $\{\boldsymbol{x}\}^{(m+1)}$ to produce Eq. 4.32a.

Now let us see in some detail how the Newton method works in practice. First we shall examine the three-reservoir problem by forming and solving manually the appropriate systems of Q-equations, H-equations and ΔQ-equations. Then we will look at computer programs that could be used to find the solution to the Q-equations; the first program is simpler and more specialized, and the second program is longer but more versatile. Finally we examine a third program that will solve any equation system that is supplied to it in a subroutine.



Pipe	K	n
1	1469	1.974
2	2432	1.927
3	5646	1.971

Figure 4.20 The three-reservoir problem.

Figure 4.20 shows three reservoirs that are connected by three pipes with an external demand at the common junction of the pipes. The highest reservoir has a water surface elevation of 100 m; the middle reservoir water surface elevation is 85 m, and the lowest reservoir has a water surface elevation of 60 m. We will use the data in the figure and table to form and solve the three systems of equations.

The Q-equations are

$$
\begin{aligned}
&F_1 = Q_1 + Q_2 - Q_3 - QJ_1 = 0 && F_1 = Q_1 + Q_2 - Q_3 - 0.06 = 0 \\
&F_2 = K_1 Q_1^{n_1} - K_2 Q_2^{n_2} - WS_1 + WS_2 = 0 && F_2 = 1469 Q_1^{1.974} - 2432 Q_2^{1.927} - 15 = 0 \\
&F_3 = K_1 Q_1^{n_1} + K_3 Q_3^{n_3} - WS_1 + WS_3 = 0 && F_3 = 1469 Q_1^{1.974} + 5646 Q_3^{1.971} - 40 = 0
\end{aligned}
\tag{4.37}
$$

To satisfy the junction continuity equation, equation F_1, and also determine initial values for the Newton method, we can select $Q_1^{(0)} = Q_{o1} = 0.10$ m^3/s, $Q_2^{(0)} = Q_{o2} = 0.05$ m^3/s, and $Q_3^{(0)} = Q_{o3} = 0.09$ m^3/s. The superscript (0) denotes initial values for use by the Newton method in solving the Q-equations, and the subscripts denote initial discharge values for the ΔQ-equations. The initial values for use with the Q-equations are not required to satisfy the junction continuity equations, although this set of values does.

The H-equation is

$$
\begin{aligned}
F_1 &= \left[\frac{WS_1 - H_1}{K_1}\right]^{1/n_1} + \left[\frac{WS_2 - H_1}{K_2}\right]^{1/n_2} - \left[\frac{H_1 - WS_3}{K_3}\right]^{1/n_3} - 0.06 = 0 \\
F_1 &= \left[\frac{100 - H_1}{1469}\right]^{0.507} + \left[\frac{85 - H_1}{2432}\right]^{0.519} - \left[\frac{H_1 - 60}{5646}\right]^{0.507} - 0.06 = 0
\end{aligned}
\tag{4.38}
$$

The ΔQ-equations are

$$
\begin{aligned}
F_1 &= K_1\left(Q_{o1} + \Delta Q_1 + \Delta Q_2\right)^{n_1} - K_2\left(Q_{o2} - \Delta Q_1\right)^{n_2} - WS_1 + WS_2 = 0 \\
F_2 &= K_1\left(Q_{o1} + \Delta Q_1 + \Delta Q_2\right)^{n_1} + K_3\left(Q_{o3} + \Delta Q_2\right)^{n_3} - WS_1 + WS_3 = 0
\end{aligned}
\tag{4.39a}
$$

With the initial discharges that we have chosen, these equations become

$$
\begin{aligned}
F_1 &= 1469\left(0.10 + \Delta Q_1 + \Delta Q_2\right)^{1.974} - 2432\left(0.05 - \Delta Q_1\right)^{1.927} - 15 = 0 \\
F_2 &= 1469\left(0.10 + \Delta Q_1 + \Delta Q_2\right)^{1.974} + 5646\left(0.09 + \Delta Q_2\right)^{1.971} - 40 = 0
\end{aligned}
\tag{4.39b}
$$

We now begin the solution of the Q-equations by the Newton method using the equations $[\boldsymbol{D}]\{z\} = \{\boldsymbol{F}\}$, $\{\boldsymbol{Q}\}^{(m+1)} = \{\boldsymbol{Q}\}^{(m)} - \{z\}$. According to Eq. 4.33, the Jacobian is

$$
[\boldsymbol{D}] = \begin{bmatrix} 1 & 1 & -1 \\ \boldsymbol{n}_1 \boldsymbol{K}_1 \boldsymbol{Q}_1^{\boldsymbol{n}_1 - 1} & -\boldsymbol{n}_2 \boldsymbol{K}_2 \boldsymbol{Q}_2^{\boldsymbol{n}_2 - 1} & 0 \\ \boldsymbol{n}_1 \boldsymbol{K}_1 \boldsymbol{Q}_1^{\boldsymbol{n}_1 - 1} & 0 & \boldsymbol{n}_3 \boldsymbol{K}_3 \boldsymbol{Q}_3^{\boldsymbol{n}_3 - 1} \end{bmatrix}
\tag{4.40a}
$$

or

$$[\boldsymbol{D}] = \begin{bmatrix} 1 & 1 & -1 \\ 2900\boldsymbol{Q}_1^{0.974} & -4686\boldsymbol{Q}_2^{0.927} & 0 \\ 2900\boldsymbol{Q}_1^{0.974} & 0 & 11128\boldsymbol{Q}_3^{0.971} \end{bmatrix} \tag{4.40b}$$

For the first computational cycle we therefore solve the equation set

$$\begin{bmatrix} 1.00 & 1.00 & -1.00 \\ 307.89 & -291.57 & 0.00 \\ 307.89 & 0.00 & 1073.96 \end{bmatrix} \begin{Bmatrix} z_1 \\ z_2 \\ z_3 \end{Bmatrix} = \begin{Bmatrix} 0.00 \\ -6.97 \\ 24.64 \end{Bmatrix} \tag{4.41a}$$

and obtain the results

$$\{z\} = \begin{Bmatrix} -0.0004 \\ 0.0235 \\ 0.0231 \end{Bmatrix} \qquad \{\boldsymbol{Q}\} = \begin{Bmatrix} 0.1004 \\ 0.0265 \\ 0.0669 \end{Bmatrix} \tag{4.42a}$$

We now iterate to obtain the following equation set and updated solution:

$$\begin{bmatrix} 1.00 & 1.00 & -1.00 \\ 309.09 & -161.86 & 0.00 \\ 309.09 & 0.00 & 805.20 \end{bmatrix} \begin{Bmatrix} z_1 \\ z_2 \\ z_3 \end{Bmatrix} = \begin{Bmatrix} 0.000 \\ -1.506 \\ 3.051 \end{Bmatrix} \tag{4.41b}$$

$$\{z\} = \begin{Bmatrix} -0.0017 \\ 0.0061 \\ 0.0044 \end{Bmatrix} \qquad \{\boldsymbol{Q}\} = \begin{Bmatrix} 0.1021 \\ 0.0204 \\ 0.0625 \end{Bmatrix} \tag{4.42b}$$

One more iteration leads to

$$\begin{bmatrix} 1.00 & 1.00 & -1.00 \\ 314.19 & -127.01 & 0.00 \\ 314.19 & 0.00 & 753.42 \end{bmatrix} \begin{Bmatrix} z_1 \\ z_2 \\ z_3 \end{Bmatrix} = \begin{Bmatrix} 0.000 \\ -0.095 \\ 0.151 \end{Bmatrix} \tag{4.41c}$$

$$\{z\} = \begin{Bmatrix} -0.0001 \\ 0.0004 \\ 0.0003 \end{Bmatrix} \qquad \{\boldsymbol{Q}\} = \begin{Bmatrix} 0.1022 \\ 0.0200 \\ 0.0622 \end{Bmatrix} \text{ m}^3\text{/s} \tag{4.42c}$$

This solution is now sufficiently accurate!

The solution of the H-equations by the Newton method uses basically the same equations $[\boldsymbol{D}]\{z\} = \{\boldsymbol{F}\}$ and $\{H\}^{(m+1)} = \{\boldsymbol{H}\}^{(m)} - \{z\}$. These relations lead to a single update equation

$$H_1^{(m+1)} = H_1^{(m)} - F_1/(\frac{dF_1}{dH_1}) \tag{4.43}$$

The derivative is

$$\frac{dF_1}{dH_1} = -\frac{1}{n_1 K_1}\left[\frac{100 - H_1}{K_1}\right]^{1/n_1 - 1} - \frac{1}{n_2 K_2}\left[\frac{85 - H_1}{K_2}\right]^{1/n_2 - 1} - \frac{1}{n_3 K_3}\left[\frac{H_1 - 60}{K_3}\right]^{1/n_3 - 1} \tag{4.44a}$$

or

$$\frac{dF_1}{dH_1} = -\frac{1}{2900}\left[\frac{100 - H_1}{1469}\right]^{-0.493} - \frac{1}{4686}\left[\frac{85 - H_1}{2432}\right]^{-0.481} - \frac{1}{11128}\left[\frac{H_1 - 60}{5646}\right]^{-0.493} \tag{4.44b}$$

If we initiate the solution procedure with the initial estimate of $H_1^{(0)} = 84$, then the first two iterative cycles produce

$$F_1 = -0.00415, \quad \frac{dF_1}{dH_1} = -0.0136, \quad H_1^{(1)} = 84 - 0.00415/0.0136 = 83.70 \tag{4.45a}$$

and

$$F_1 = -0.000247, \quad \frac{dF_1}{dH_1} = -0.01251, \quad H_1^{(2)} = 83.70 - 0.000247/0.01251 = 83.68\,\text{m} \tag{4.45b}$$

which will be regarded as adequate.

Finally, we now solve the ΔQ-equations by the Newton method using again the equations $[\mathbf{D}]\{z\} = \{F\}$ and $\{\boldsymbol{\Delta Q}^{(m+1)}\} = \{\boldsymbol{\Delta Q}^{(m)}\} - \{z\}$. In this case we solve repeatedly the two-equation system for updated correction vectors $\{z\}$ until it is declared to be sufficiently small. Three cycles of computation yield these results:

$$\begin{bmatrix} 599 & 307 \\ 307 & 1382 \end{bmatrix}\begin{Bmatrix} z_1 \\ z_2 \end{Bmatrix} = \begin{Bmatrix} -6.97 \\ 24.6 \end{Bmatrix} \qquad \begin{Bmatrix} z_1 \\ z_2 \end{Bmatrix} = \begin{Bmatrix} -0.0234 \\ 0.0230 \end{Bmatrix} \qquad \begin{Bmatrix} \Delta Q_1 \\ \Delta Q_2 \end{Bmatrix} = \begin{Bmatrix} 0.0234 \\ -0.0230 \end{Bmatrix} \tag{4.46a}$$

$$\begin{bmatrix} 472 & 309 \\ 309 & 1115 \end{bmatrix}\begin{Bmatrix} z_1 \\ z_2 \end{Bmatrix} = \begin{Bmatrix} -1.523 \\ 3.13 \end{Bmatrix} \qquad \begin{Bmatrix} z_1 \\ z_2 \end{Bmatrix} = \begin{Bmatrix} -0.0062 \\ 0.0045 \end{Bmatrix} \qquad \begin{Bmatrix} \Delta Q_1 \\ \Delta Q_2 \end{Bmatrix} = \begin{Bmatrix} 0.0296 \\ -0.0275 \end{Bmatrix} \tag{4.46b}$$

$$\begin{bmatrix} 441 & 314 \\ 314 & 1068 \end{bmatrix}\begin{Bmatrix} z_1 \\ z_2 \end{Bmatrix} = \begin{Bmatrix} -0.0952 \\ 0.1507 \end{Bmatrix} \qquad \begin{Bmatrix} z_1 \\ z_2 \end{Bmatrix} = \begin{Bmatrix} -0.0004 \\ 0.0003 \end{Bmatrix} \qquad \begin{Bmatrix} \Delta Q_1 \\ \Delta Q_2 \end{Bmatrix} = \begin{Bmatrix} 0.0300 \\ -0.0278 \end{Bmatrix} \tag{4.46c}$$

Now the discharges can be computed as $Q_1 = 0.1 + \Delta Q_1 + \Delta Q_2 = 0.1022$ m^3/s, $Q_2 = 0.05 - \Delta Q_1 = 0.0200$ m^3/s, and $Q_3 = 0.09 + \Delta Q_2 = 0.0622$ m^3/s.

Computer programs of differing complexity and generality can also be developed for the solution of these equation systems by application of the Newton method. We will now look at two programs. The first program is relatively simple but must be recoded in part for each application; it will be applied to the solution of the Q-equations for the three-reservoir problem. The second program is more versatile.

Program 4.2, the FORTRAN program listed in Fig. 4.21, is designed to solve three simultaneous equations with the Newton method. It calls a matrix solver that has the coefficient matrix expanded by one column to contain the known vector, and it places the inverse in additional columns beyond the location of the known vector. The first part of the main program is currently written specifically to solve the Q-equations for the three-reservoir problem. However, the portion that numerically evaluates the derivatives in the Jacobian matrix is written more generally, with N giving the size of the matrix problem to be solved. Careful study of this listing will clarify considerably how the various tasks are performed. The subroutine INVERM employs a common method in linear algebra problems by using an expanded matrix. The coefficient matrix is square, here 3 rows by 3 columns. The known vector is placed in the next column, in this case column 4. The subroutine solves the system of equations and provides the inverse matrix. The solution is returned in the same column that initially contained the known vector, here column 4.

```
**************************************************************************
*    PROGRAM NO. 4.2, NEWTON, FORTRAN
*    THIS PROGRAM HAS BEEN INCLUDED FOR THE CONVENIENCE OF THE READER.
*    THE AUTHOR ACCEPTS NO RESPONSIBILITY FOR ITS CORRECTNESS.
*    USERS OF THIS PROGRAM DO SO AT THEIR OWN RISK.
**************************************************************************
*    IMPLEMENTS THE NEWTON METHOD IN SOLVING THREE EQUATIONS
      REAL X(3),F(3),D(3,7),RK(3),RN(3),K1,K2,K3,N1,N2,N3
      DATA N,N1/3,4/,DX/.001/,MAX/15/,ERR/.0001/
      WRITE(5,*)' GIVE: K1,K2,K3,N1,N2,N3,Q1,Q2,Q3'
      READ(5,*) RK,RN,X
      M=0
    1 NT=0
    5 F(1)=X(1)+X(2)-X(3)-0.06
      F(2)=RK(1)*X(1)**RN(1)-RK(2)*X(2)**RN(2)-15.0
      F(3)=RK(1)*X(1)**RN(1)+RK(3)*X(3)**RN(3)-40.0
      IF(NT.NE.0) GO TO 15
      DO 10 I=1,N
   10 D(I,N1)=F(I)
      X(1)=X(1)-DX
      NT=1
      GO TO 5
   15 X(NT)=X(NT)+DX
      DO 20 I=1,N
   20 D(I,NT)=(D(I,N1)-F(I))/DX
      NT=NT+1
      IF(NT.GT.N) GO TO 30
      X(NT)=X(NT)-DX
      GO TO 5
   30 CALL INVERM(D,N)
      DIF=0.
      DO 40 I=1,N
      DIF=DIF+ABS(D(I,N1))
   40 X(I)=X(I)-D(I,N1)
      M=M+1
      IF(DIF.GT. ERR .AND. M.LT.MAX) GO TO 1
      WRITE(5,*)' THE SOLUTION IS ',X
      END
```

Figure 4.21 Program 4.2 to use the Newton method to solve three equations.

The listing of the inverse starts in the next column. In solving three equations the array D(3,7) therefore has 7 as its second subscript, and the last three columns contain the inverse.

Currently the input data to this program includes the coefficients K and n for each pipe and an initial estimate of the discharge in each pipe: 1469 2432 5646 1.974 1.927 1.971 0.10 0.05 0.09. The program produces the same solution as we obtained manually.

The next program, listed in Fig. 4.22, is essentially the same as the previous program, except that it calls the linear equation solver GAUSEL, described in Appendix A, rather than INVERM. GAUSEL is a more versatile subroutine that interchanges rows to minimize truncation error, applies one iterative correction to the solution vector, and returns an estimate of the relative error for each unknown in the array ERRNOR, so the user has parameters to determine the accuracy of the solution. However, the relative error is not printed in this program. This subroutine also illustrates the Microsoft FORTRAN ability to allocate the array sizes that are needed.

```
************************************************************************
*    PROGRAM NO. 4.3, NEWTON, FORTRAN
*    THIS PROGRAM HAS BEEN INCLUDED FOR THE CONVENIENCE OF THE READER.
*    THE AUTHOR ACCEPTS NO RESPONSIBILITY FOR ITS CORRECTNESS.
*    USERS OF THIS PROGRAM DO SO AT THEIR OWN RISK.
************************************************************************
*    IMPLEMENTS THE NEWTON METHOD IN SOLVING THREE EQUATIONS
      REAL X(3),F(3),D(3,3),F1(3),ERRNOR(3),RK(3),RN(3),K1,K2,K3,N1,N2,N3
      DATA N/3/,DX/.001/,MAX/15/,ERR/.0001/
      WRITE(5,*)' GIVE: K1,K2,K3,N1,N2,N3,Q1,Q2,Q3'
      READ(5,*) RK,RN,X
      M=0
    1 NT=0
    5 F(1)=X(1)+X(2)-X(3)-0.06
      F(2)=RK(1)*X(1)**RN(1)-RK(2)*X(2)**RN(2)-15.0
      F(3)=RK(1)*X(1)**RN(1)+RK(3)*X(3)**RN(3)-40.0
      IF(NT.NE.0) GO TO 15
      DO 10 I=1,N
   10 F1(I)=F(I)
      X(1)=X(1)-DX
      NT=1
      GO TO 5
   15 X(NT)=X(NT)+DX
      DO 20 I=1,N
   20 D(I,NT)=(F1(I)-F(I))/DX
      NT=NT+1
      IF(NT.GT.N) GO TO 30
      X(NT)=X(NT)-DX
      GO TO 5
   30 CALL GAUSEL(3,3,D,F1,DET,ERRNOR)
      DIF=0.
      DO 40 I=1,N
      DIF=DIF+ABS(F1(I))
   40 X(I)=X(I)-F1(I)
      M=M+1
      IF(DIF.GT. ERR .AND. M.LT.MAX) GO TO 1
      WRITE(5,*)' THE SOLUTION IS ',X
      END
```

Figure 4.22 Program 4.3, a more versatile implementation of the Newton method.

With this introduction to the Newton method, let us look further at the structure of a computer program that would solve any system of equations. This program should consist of two primary elements: First, a main or driver program that accomplishes the following tasks: (a) it allows the user to assign values to known variables and initial estimates for unknown variables; (b) it creates the Jacobian matrix and the known vector and supplies values to these arrays; (c) it calls a linear algebra solver; (d) it implements the Newton iteration; and (e) it prints the solution. Second, it must contain a subroutine (or function subprogram) that defines the system of equations to be solved. The equations in this subroutine will change, depending upon the nature of the problem that is being solved, and therefore the statements in this subroutine would be changed as different types of problems are to be solved. A listing of such a general purpose program EQUSOL1.FOR will be found in Fig. 4.23. The subroutine FUNCT provides the equations for this program. The main program calls on the linear algebra solver SOLVEQ that is described in Appendix B.

```
***********************************************************************
*    PROGRAM NO. 4.4, EQUSOL1, FORTRAN
*    THIS PROGRAM HAS BEEN INCLUDED FOR THE CONVENIENCE OF THE READER.
*    THE AUTHOR ACCEPTS NO RESPONSIBILITY FOR ITS CORRECTNESS.
*    USERS OF THIS PROGRAM DO SO AT THEIR OWN RISK.
***********************************************************************
*    THIS EQUATION SOLVER IMPLEMENTS THE NEWTON METHOD
      LOGICAL IV[ALLOCATABLE](:)
      INTEGER*2 INDX[ALLOCATABLE](:)
      CHARACTER*3 SYMB[ALLOCATABLE](:),CH
      REAL X[ALLOCATABLE](:),F[ALLOCATABLE](:),F1[ALLOCATABLE](:),
     &D[ALLOCATABLE](:,:)
*
      WRITE(*,*)' GIVE (1) NO. OF EQS., (2) NO. OF VARIABLES,',
     &' (3) INPUT UNIT, (4) OUTPUT UNIT'
      READ(*,*) N,NV,IN,IOUT
      ALLOCATE(X(NV),F(N),F1(N),D(N,N),SYMB(NV),IV(NV),INDX(N))
      IF(IN.EQ.0 .OR. IN.EQ.5) WRITE(IN,100) NV
  100 FORMAT(' GIVE',I3,' LINES WITH:',/,3X,'(1) SYMBOL FOR VAR.(3 CH)',
     &/,3X,'(2) K OR U FOR KNOWN OR UNKNOWN AND',/,3X,'(3) VALUE')
      J=0
      DO 10 I=1,NV
      READ(IN,110) SYMB(I),CH,X(I)
  110 FORMAT(A3,1X,A1,1X,F10.0)
      IF(CH.EQ.'U' .OR.  CH.EQ.'u') THEN
      IV(I)=.TRUE.
      J=J+1
      ELSE IF(CH.EQ.'K' .OR. CH.EQ.'k') THEN
      IV(I)=.FALSE.
      ELSE
      WRITE(*,*)' ERROR IN INPUT FOR VARIABLE', I
      STOP
      ENDIF
   10 CONTINUE
      IF(J.EQ.N) GO TO 12
      WRITE(*,*)' YOU GAVE',N,' EQS. BUT',J,' UNKNOWNS'
      STOP
   12 NCT=0
   15 CALL FUNCT(X,F)
      J=0
      DO 30 JJ=1,NV
      IF(IV(JJ)) THEN
      XX=X(JJ)
      X(JJ)=1.005*X(JJ)
      J=J+1
      CALL FUNCT(X,F1)
      DO 20 I=1,N
   20 D(I,J)=(F1(I)-F(I))/(X(JJ)-XX)
      X(JJ)=XX
      ENDIF
   30 CONTINUE
      CALL SOLVEQ(N,1,N,D,F,1,DD,INDX)
      NCT=NCT+1
      SUM=0.
```

Figure 4.23 A listing of the program EQUSOL1.FOR.

```
      J=0
      DO 40 I=1,NV
       IF(IV(I)) THEN
      J=J+1
       X(I)=X(I)-F(J)
       SUM=SUM+ABS(F(J))
      ENDIF
   40 CONTINUE
       WRITE(*,*)' NCT =',NCT,' SUM =',SUM
       IF(NCT.LT.20 .AND. SUM.GT. 0.0001) GO TO 15
      WRITE(IOUT,120)(I,SYMB(I),X(I),I=1,NV)
  120 FORMAT(I5,1X,A3,' =',F10.3)
      END
*
      SUBROUTINE FUNCT(X,F)
      REAL F(22),X(41)
      DATA E,G2,P,AP/0.005,64.4,8.6714174E-6,5.4541539E-3/
      F(1)=X(41)-X(1)-X(3)-X(35) !Unknowns
      F(2)=X(1)-X(2)-X(36)       !  1=Q2   15=H4   29=L1
      F(3)=X(3)-X(4)-X(37)       !  2=Q3   16=H5   30=L2
      F(4)=X(2)+X(4)-X(5)-X(38)  !  3=Q4   17=f1   31=L3
      F(5)=X(12)-X(40)+X(6)      !  4=Q5   18=f2   32=L4
      F(6)=X(13)-X(12)+X(7)      !  5=Q6   19=f3   33=L5
      F(7)=X(14)-X(12)+X(9)      !  6=hf1  20=f4   34=L6
      F(8)=X(15)-X(13)+X(8)      !  7=hf2  21=f5   35=QJ1
      F(9)=X(16)-X(15)+X(11)     !  8=hf3  22=f6   36=QJ2
      F(10)=X(7)+X(8)-X(10)-X(9) !  9=hf4 Knowns   37=QJ3
      RF1=1./SQRT(X(17))         ! 10=hf5  23=D1   38=QJ4
      RF2=1./SQRT(X(18))         ! 11=hf6  24=D2   39=QJ5
      RF3=1./SQRT(X(19))         ! 12=H1   25=D3   40=WS1
      RF4=1./SQRT(X(20))         ! 13=H2   26=D4   41=Q1
      RF5=1./SQRT(X(21))         ! 14=H3   27=D5
      RF6=1./SQRT(X(22))         ! 15=H4   28=D6
      F(11)=X(6)-X(17)*X(29)*12./X(23)*(X(41)/(AP*X(23)**2))**2/G2
      F(12)=X(7)-X(18)*X(30)*12./X(24)*(X(1)/(AP*X(24)**2))**2/G2
      F(13)=X(8)-X(19)*X(31)*12./X(25)*(X(2)/(AP*X(25)**2))**2/G2
      F(14)=X(9)-X(20)*X(32)*12./X(26)*(X(3)/(AP*X(26)**2))**2/G2
      F(15)=X(10)-X(21)*X(33)*12./X(27)*(X(4)/(AP*X(27)**2))**2/G2
      F(16)=X(11)-X(22)*X(34)*12./X(28)*(X(5)/(AP*X(28)**2))**2/G2
      F(17)=RF1-1.14+2.*ALOG10(E/X(23)+P*X(23)*RF1/X(41))
      F(18)=RF2-1.14+2.*ALOG10(E/X(24)+P*X(24)*RF2/X(1))
      F(19)=RF3-1.14+2.*ALOG10(E/X(25)+P*X(25)*RF3/X(2))
      F(20)=RF4-1.14+2.*ALOG10(E/X(26)+P*X(26)*RF4/X(3))
      F(21)=RF5-1.14+2.*ALOG10(E/X(27)+P*X(27)*RF5/X(4))
      F(22)=RF6-1.14+2.*ALOG10(E/X(28)+P*X(28)*RF6/X(5))
      RETURN
      END
```

Figure 4.23 (Concluded) A listing of the program EQUSOL1.FOR.

Let's examine how the main program does its tasks of providing values to the Jacobian matrix $[\boldsymbol{D}]$ and equation vector $\{\boldsymbol{F}\}$ and then carrying out a Newton solution. The key portion of the main program that implements the Newton method appears in **bold** characters in Fig. 4.23. Three tasks are accomplished by these statements: (1) defining the equation vector; (2) numerically evaluating the elements of the Jacobian matrix; and (3) solving the resulting linear system of equations and subtracting this solution from the current vector of unknowns, as described by Eq. 4.32a.

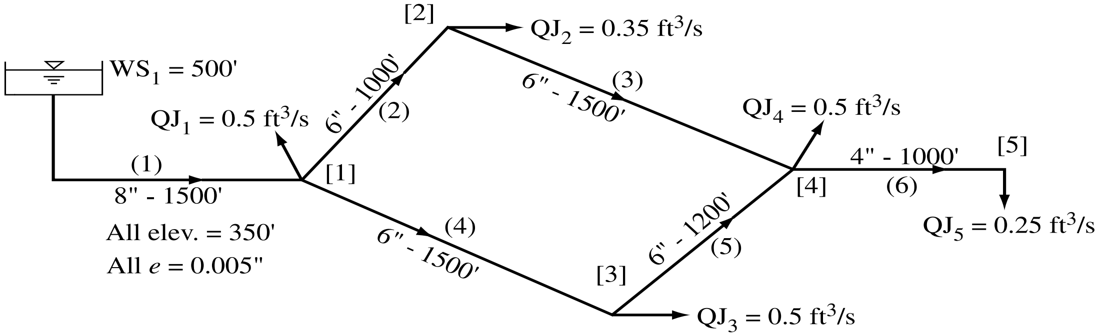
The FORTRAN integer NCT is the iteration counter; it is set to 0 before beginning the Newton iteration. Statement `15 CALL FUNCT(X,F)` has two arguments, an array `X` for the variables and an array `F` for the equations. The array `X` includes both the known and unknown variables of the problem. Upon returning from `CALL FUNCT`, the array `F` contains a set of equations that have been evaluated by using the initial estimates of the unknowns. Since the initial estimates are incorrect, the individual elements of $\{\boldsymbol{F}\}$ will not be zero, but subsequent Newton iterations will drive these elements progressively closer to zero. Statement `DO 30 JJ=1,NV`, in which `NV` is the total of all variables, evaluates individual columns of the Jacobian matrix `D(I,J)` by using a first-order numerical evaluation of the derivatives. Since `IV(JJ)` is `.FALSE.` for known variables and `.TRUE.` for unknown variables, we note that nothing happens in loop `30` if `IV` is `.FALSE.` Hence `J`, which identifies the column in which the Jacobian derivatives are entered, is incremented only for unknown variables. When an unknown is encountered, x_j, which is `X(JJ)`, is incremented by multiplying its current value by 1.005 before the equation is evaluated again by calling `FUNCT`. Upon returning from `FUNCT`, the array `F1` now contains equation values based on $1.005x_j$, and then the statement `D(I,J) = (F1(I) - F(I))/(X(JJ) - XX)` numerically evaluates the derivatives of the equations by using a first-order approximation. The statement `DO 20 I=1,N` fills all row entries for column `J` of the Jacobian matrix $[\boldsymbol{D}]$.

Upon completing the `DO 30` loop, the equation vector $\{\boldsymbol{F}\}$ and the Jacobian matrix $[\boldsymbol{D}]$ have been fully evaluated. The next statement `CALL SOLVEQ` calls a linear equation solver, which upon return has replaced the elements of the array `F` with the solution vector $\{z\}$ found in Eq. 4.32b. The statement `DO 40 I=1,NV` implements Eq. 4.32c with `SUM` accumulating the absolute sum of the corrections applied to the unknown vector $\{\boldsymbol{x}\}$. If this `SUM` is larger than the allowable error and fewer than 20 iterations have been completed, then `GO TO 15` at the end of this code segment will begin another Newton iteration.

In our example it would be relatively easy to derive the actual partial derivatives of each equation with respect to each unknown, and the elements of the Jacobian could be evaluated by using these derivatives. The length of the program would be longer if these derivative expressions were used. The numerical approximation of the derivatives requires extra arithmetic, particularly since many derivatives are zero, but the advantage of a shorter code makes the numerical approximation of the derivatives attractive.

Example Problem 4.5

Use program EQUSOL1 to solve the 6-pipe, 5-node network shown below. Obtain this solution in four ways: (1) use the program as it now exists with subroutine FUNCT; (2) use the Q-equations; (3) use the H-equations; (4) use the ΔQ-equations.



1. The existing subroutine FUNCT explicitly defines the equations that we want to solve; there are 22 equations, `F(1)` through `F(22)`. There are 41 variables associated with the solution; therefore 41 - 22 = 19 of these variables are known. These equations are as follows:

Junction continuity equations:

$$Q_1 - Q_2 - Q_4 - QJ_1 = 0 \quad (1)$$
$$Q_2 - Q_3 - QJ_2 = 0 \quad (2)$$
$$Q_4 - Q_5 - QJ_3 = 0 \quad (3)$$
$$Q_3 + Q_5 - Q_6 - QJ_4 = 0 \quad (4)$$

(The junction continuity equation at node 5 is not included here, but this simple equation $Q_6 - QJ_5 = 0$ establishes the discharge in pipe 6 as 0.25 ft^3/s.)

Head loss equations giving the HGL at a downstream node relative to the upstream node:

$$H_1 = WS_1 - h_{f1} \quad (5)$$
$$H_2 = H_1 - h_{f2} \quad (6)$$
$$H_3 = H_1 - h_{f4} \quad (7)$$
$$H_4 = H_2 - h_{f3} \qquad (\text{or} \quad H_4 = H_3 - h_{f5}) \quad (8)$$
$$H_5 = H_4 - h_{f6} \quad (9)$$

Energy equation around a loop:

$$h_{f2} + h_{f3} - h_{f5} - h_{f4} = 0 \quad (10)$$

Darcy-Weisbach equations to define the frictional head losses (pipe numbers $i = 1, 6$):

$$h_{fi} = f_i \frac{L_i}{D_i} \frac{Q_i^2}{2gA_i^2} \quad (11\text{-}16)$$

Colebrook-White equations (pipe numbers $i = 1, 6$):

$$\frac{1}{\sqrt{f_i}} = 1.14 - 2\,log_{10}\left\{\frac{e_i}{D_i} + \frac{9.35\nu D_i}{(4/\pi)Q_i\sqrt{f_i}}\right\} \quad (17\text{-}22)$$

In the program listing the integer within `X( )` identifies the variable in the array, as is seen by the comments following the exclamation points there. We note there are 22 equations: a continuity equation for $NJ - 1 = 4$ junctions, a separate head difference equation for each pipe, a Darcy-Weisbach equation for each pipe, a companion Colebrook-White equation for each pipe, and finally an energy loop equation, for a total of $3NP + NJ = 18 + 5 = 22$ equations. Since the entire system demand must come from pipe 1, its discharge must be $Q_1 = 2.1$ ft^3/s, and the unknowns are 5 unknown discharges Q_2 .. Q_6, 6 unknown head losses h_{f1} .. h_{f6}, 5 unknown heads H_1 .. H_5, and 6 unknown friction factors f_1 .. f_6, for a total of 22. The input and solution files for the program now follow:

Input File		**Output File**	
Q2 U 0.8	f6 U 0.020	1 Q2 = 0.820	22 f6 = 0.024
Q3 U 0.5	D1 K 8.0	2 Q3 = 0.470	23 D1 = 8.000
Q4 U 0.8	D2 K 6.0	3 Q4 = 0.780	24 D2 = 6.000
Q5 U 0.3	D3 K 6.0	4 Q5 = 0.280	25 D3 = 6.000
Q6 U 0.3	D4 K 6.0	5 Q6 = 0.250	26 D4 = 6.000
hf1 U 24.0	D5 K 6.0	6 hf1 = 23.952	27 D5 = 6.000
hf2 U 11.0	D6 K 4.0	7 hf2 = 11.279	28 D6 = 4.000
hf3 U 0.2	L1 K 1500.0	8 hf3 = 5.872	29 L1 = 1500.0
hf4 U 0.15	L2 K 1000.0	9 hf4 = 15.368	30 L2 = 1000.0
hf5 U 2.0	L3 K 1500.0	10 hf5 = 1.783	31 L3 = 1500.0
hf6 U 6.0	L4 K 1500.0	11 hf6 = 9.117	32 L4 = 1500.0
H1 U 476.0	L5 K 1200.0	12 H1 = 476.048	33 L5 = 1200.0
H2 U 465.0	L6 K 1000.0	13 H2 = 464.769	34 L6 = 1000.0
H3 U 460.0	QJ1 K 0.50	14 H3 = 460.680	35 QJ1 = 0.50
H4 U 458.0	QJ2 K 0.35	15 H4 = 458.897	36 QJ2 = 0.35
H5 U 450.0	QJ3 K 0.50	16 H5 = 449.780	37 QJ3 = 0.50
f1 U 0.020	QJ4 K 0.50	17 f1 = 0.019	38 QJ4 = 0.50
f2 U 0.020	QJ5 K 0.25	18 f2 = 0.021	39 QJ5 = 0.25
f3 U 0.020	WS1 K 500.0	19 f3 = 0.022	40 WS1 = 500.0
f4 U 0.020	Q1 K 2.1	20 f4 = 0.021	41 Q1 = 2.100
f5 U 0.020		21 f5 = 0.024	

2. The Q-equations are

$$F_1 = Q_1 - Q_2 - Q_4 - QJ_1 = 0$$
$$F_2 = Q_2 - Q_3 - QJ_2 = 0$$
$$F_3 = Q_4 - Q_5 - QJ_3 = 0$$
$$F_4 = Q_3 + Q_5 - Q_6 - QJ_4 = 0$$
$$F_5 = K_2 Q_2^{n_2} + K_3 Q_3^{n_3} - K_5 Q_5^{n_5} - K_4 Q_4^{n_4} = 0$$

The K and n for each pipe must now be determined. Program 2.1, PIPK_N, or some other means will provide these values:

Pipe	K	n
1	5.6845	1.9381
2	16.4967	1.9185
3	24.3685	1.8858
4	24.7450	1.9185
5	19.0411	1.8611
6	126.3843	1.8970

To compute the five unknown discharges Q_i $(i = 2, 6)$ (with $Q_1 = 2.1$ ft^3/s known), the subroutine FUNCT must be modified as follows:

```
SUBROUTINE FUNCT(X,F)
REAL F(5),X(11)
REAL K2/16.4967/,K3/24.3685/,K4/24.745/,K5/19.0411/
REAL N2/1.9185/,N3/1.8858/,N4/1.9185/,N5/1.8611/
F(1)=X(6)-X(1)-X(3)-X(7)     ! Unknowns          Knowns
F(2)=X(1)-X(2)-X(8)          ! 1 = Q2, 4 = Q5, 6 =  Q1,  9 = QJ3
F(3)=X(3)-X(4)-X(9)          ! 2 = Q3, 5 = Q6, 7 = QJ1, 10 = QJ4
F(4)=X(2)+X(4)-X(5)-X(10)    ! 3 = Q4,         8 = QJ2, 11 = QJ5
F(5)=K2*X(1)**N2+K3*X(2)**N3-K5*X(4)**N5-K4*X(3)**N4
RETURN
END
```

The input data (all in ft^3/s) that were used to solve this problem (with 5 and 11 plus 2 and 3 for I/O units from the keyboard) and the solution are listed now:

Input Data | **Solution**

Variable	Type	Initial value		Index	Value
Q2	U	0.80		1 Q2	0.82
Q3	U	0.50		2 Q3	0.47
Q4	U	0.80		3 Q4	0.78
Q5	U	0.30		4 Q5	0.28
Q6	U	0.25		5 Q6	0.25
Q1	K	2.10		6 Q1	2.10
QJ1	K	0.50		7 QJ1	0.50
QJ2	K	0.35		8 QJ2	0.35
QJ3	K	0.50		9 QJ3	0.50
QJ4	K	0.50		10 QJ4	0.50
QJ5	K	0.25		11 QJ5	0.25

Since X(11) = QJ5 is not used in the equations, the keyboard input could have been changed to 5 10 2 3, and the last line of input could then be deleted.

3. The number of H-equations could be reduced below five, but we will use five head equations to determine the head at the five nodes. These equations are

$$F_1 = \left[\frac{500 - H_1}{K_1}\right]^{1/n_1} - \left[\frac{H_1 - H_2}{K_2}\right]^{1/n_2} - \left[\frac{H_1 - H_3}{K_4}\right]^{1/n_4} - QJ_1 = 0$$

$$F_2 = \left[\frac{H_1 - H_2}{K_2}\right]^{1/n_2} - \left[\frac{H_2 - H_4}{K_3}\right]^{1/n_3} - QJ_2 = 0$$

$$F_3 = \left[\frac{H_1 - H_3}{K_4}\right]^{1/n_4} - \left[\frac{H_3 - H_4}{K_5}\right]^{1/n_5} - QJ_3 = 0$$

$$F_4 = \left[\frac{H_2 - H_4}{K_3}\right]^{1/n_3} + \left[\frac{H_3 - H_4}{K_5}\right]^{1/n_5} - \left[\frac{H_4 - H_5}{K_6}\right]^{1/n_6} - QJ_4 = 0$$

$$F_5 = \left[\frac{H_4 - H_5}{K_6}\right]^{1/n_6} - QJ_5 = 0$$

These equations are arranged to allow us to find H_i $(i = 1, 5)$ with a demand at each of the five nodes as additional variables, so there are 5 unknowns and 10 variables. The appropriate modifications of subroutine FUNCT are as follows:

```
      SUBROUTINE FUNCT(X,F)
      REAL F(5),X(10)
      REAL K1/5.6845/,K2/16.4967/,K3/24.3685/,K4/24.745/,K5/19.0411/
     &,K6/126.3843/,R1/.515969/,R2/.52124/,R3/.53028/,R4/.52124/
     &,R5/.53732/,R6/.52715/
C UNKNOWNS: 1 = H1, 2 = H2, 3 = H3, 4 = H4, 5 = H5;
```

```
C KNOWNS: 6 = QJ1, 7 = QJ2, 8 = QJ3, 9 = QJ4, 10 = QJ5
      F(1)=(ABS(500-X(1))/K1)**R1-(ABS(X(1)-X(2))/K2)**R2
     &-(ABS(X(1)-X(3))/K4)**R4-X(6)
      F(2)=(ABS(X(1)-X(2))/K2)**R2-(ABS(X(2)-X(4))/K3)**R3-X(7)
      F(3)=(ABS(X(1)-X(3))/K4)**R4-(ABS(X(3)-X(4))/K5)**R5-X(8)
      F(4)=(ABS(X(2)-X(4))/K3)**R3+(ABS(X(3)-X(4))/K5)**R5
     &-(ABS(X(4)-X(5))/K6)**R6-X(9)
      F(5)=(ABS(X(4)-X(5))/K6)**R6-X(10)
      RETURN
      END
```

The input data (all in ft or ft^3/s) for this problem (with an additional 5 and 10 plus 2 and 3 for I/O units from the keyboard) and the solution follow:

Input Data			Solution	
Variable	Type	Initial value	Index	Value
H1	U	476.0	1 H1	476.1
H2	U	465.0	2 H2	464.8
H3	U	460.0	3 H3	460.7
H4	U	458.0	4 H4	458.9
H5	U	450.0	5 H5	449.8
QJ1	K	0.50	6 QJ1	0.50
QJ2	K	0.35	7 QJ2	0.35
QJ3	K	0.50	8 QJ3	0.50
QJ4	K	0.50	9 QJ4	0.50
QJ5	K	0.25	10 QJ5	0.25

4. For this problem there is only one ΔQ-equation, which is

$$F_1 = K_2\left(Q_{o2} + \Delta Q_1\right)^{n_1} + K_3\left(Q_{o3} + \Delta Q_1\right)^{n_3} - K_5\left(Q_{o5} - \Delta Q_1\right)^{n_5} - K_4\left(Q_{o4} - \Delta Q_1\right)^{n_4} = 0$$

The input, since it is only two lines, can be given directly from the keyboard as

```
1 1 5 6
DQ1 U 0.1
```

The estimate of 0.1 ft^3/s is used because the main program uses 1.005 times the current value. This might be changed with an IF statement that adds 0.001 to the variable if its value is zero. The solution is DQ1 = 0.020 ft^3/s. The subroutine FUNCT can be modified, with the initial discharge in each pipe chosen to be $Q_{o2} = 0.8$ ft^3/s, $Q_{o3} = 0.45$ ft^3/s, $Q_{o4} = 0.80$ ft^3/s, and $Q_{o5} = 0.30$ ft^3/s to satisfy continuity, as shown:

```
      SUBROUTINE FUNCT(X,F)
      REAL F(1),X(1)
      REAL K2/16.4967/,K3/24.3685/,K4/24.745/,K5/19.0411/,N2/1.9185/
     &,N3/1.8858/,N4/1.9185/,N5/1.8611/
      REAL QO2/0.80/,QO3/0.45/,QO4/0.80/,QO5/0.30/
C UNKNOWN: DQ1
      F(1)=K2*(QO2+X(1))**N2+K3*(QO3+X(1))**N3-
     &K5*(QO5-X(1))**N5-K4*(QO4-X(1))**N4
      RETURN
      END
```

Writing every equation, as was done in the listing of EQUSOL1, makes it easy to follow the computational sequence in subroutine FUNCT. However, since there are as many Darcy-Weisbach equations as there are pipes and the Gauss-Seidel iteration could be used to solve the Colebrook-White equations internally within the system of equations, FUNCT can be simplified. A separate function can be written to evaluate the Colebrook-White equation, and the Darcy-Weisbach equations are in a DO loop. Now the equations to be solved are the four junction continuity equations and the six head loss equations that indicate the difference in head along a pipe is equal to the frictional head loss between the pipe ends. For variety, Q_1 is now unknown, and in its place Q_6 is assumed to be known.

The listing of the modified subroutine FUNCT follows:

```
      SUBROUTINE FUNCT(X,F)
      INTEGER*2 ID(6)/7,8,10,9,10,11/       ! 1 = Q1, 10 = H4,  19 = D3
     &,IU(6)/16,7,8,7,9,10/                 ! 2 = Q2, 11 = H5,  20 = D4
      REAL F(10),X(28)                      ! 3 = Q3, 12 = QJ1, 21 = D5
      DATA G2/64.4/,P4/0.7853982/           ! 4 = Q4, 13 = QJ2, 22 = D6
      F(1)=X(1)-X(2)-X(4)-X(12)             ! 5 = Q5, 14 = QJ3, 23 = L1
      F(2)=X(2)-X(3)-X(13)                  ! 6 = Q6, 15 = QJ4, 24 = L2
      F(3)=X(4)-X(5)-X(14)                  ! 7 = H1, 16 = WS1, 25 = L3
      F(4)=X(3)+X(5)-X(6)-X(15)             ! 8 = H2, 17 = D1,  26 = L4
      DO 10 I=1,6                           ! 9 = H3, 18 = D2,  27 = L5
      J=I+16                                !                   28 = L6
   10 F(I+4)=X(ID(I))-X(IU(I))+FR(I,J,X)*X(I+22)
     & /X(J)*(X(I)/(P4*X(J)**2))**2/G2
      RETURN
      END
*
      FUNCTION FR(I,J,X)
      REAL X(28)
      REAL FI(6)/6*.02/
      DATA E/0.0004166667/,CCVISC/1.03543E-4/
      F1=1./SQRT(FI(I))
   10 F2=F1
      F1=1.14-2.*ALOG10(E/X(J)+CCVISC*X(J)*F2/X(I))
      IF(ABS(F1-F2).GT. 1.E-6) GO TO 10
      FR=1./F1/F1
      FI(I)=FR
      RETURN
      END
```

The input data file (all in ft or ft^3/s) and the solution are given next:

Input Data			**Solution**	
Variable	**Type**	**Initial value**	**Index**	**Value**
Q1	U	2.10	1 Q1	2.000
Q2	U	0.80	2 Q2	0.770
Q3	U	0.50	3 Q3	0.420
Q4	U	0.80	4 Q4	0.730
Q5	U	0.30	5 Q5	0.230
Q6	K	0.25	6 Q6	0.250
H1	U	476.0	7 H1	478.2
H2	U	465.0	8 H2	468.2
H3	U	460.0	9 H3	464.7

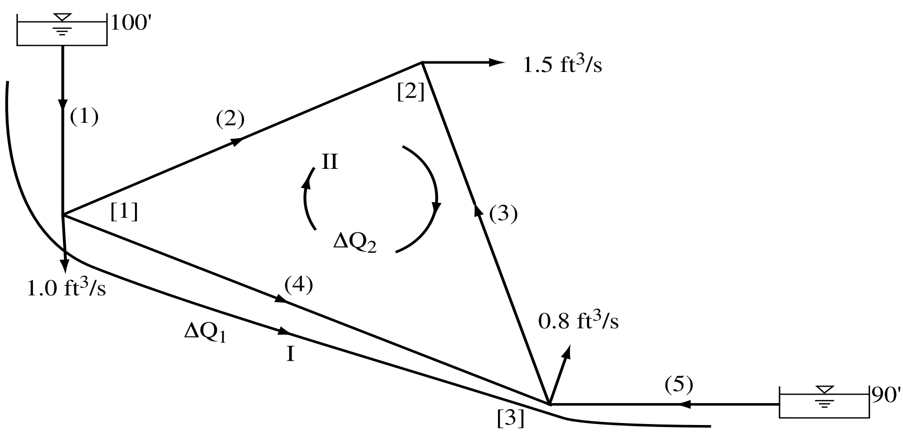
Continued:

Input Data			Solution	
Variable	**Type**	**Initial value**	**Index**	**Value**
H4	U	458.0	10 H4	463.5
H5	U	450.0	11 H5	454.3
QJ1	K	0.50	12 QJ1	0.50
QJ2	K	0.35	13 QJ2	0.35
QJ3	K	0.50	14 QJ3	0.50
QJ4	K	0.50	15 QJ4	0.50
WS1	K	500.0	16 WS1	500.0
D1	K	0.667	17 D1	0.667
D2	K	0.50	18 D2	0.50
D3	K	0.50	19 D3	0.50
D4	K	0.50	20 D4	0.50
D5	K	0.50	21 D5	0.50
D6	K	0.333	22 D6	0.333
L1	K	1500	23 L1	1500
L2	K	1000	24 L2	1000
L3	K	1500	25 L3	1500
L4	K	1500	26 L4	1500
L5	K	1200	27 L5	1200
L6	K	1000	28 L6	1000

* * *

4.4.2. SOLVING THE THREE EQUATION SYSTEMS VIA NEWTON

The Newton method will now be applied in turn to the solution of the *Q*-equations, the *H*-equations and the ΔQ-equations for network shown in Fig. 4.24. Considerable detail will be presented in these solutions so the details of applying the Newton method can be



Pipe	K	n
1	7.59	1.936
2	9.63	1.901
3	48.6	1.882
4	39.7	1.768
5	16.5	1.935

Figure 4.24 A 5-pipe, 3-node network.

examined. The reader is encouraged to check numerically some of these steps. In the Q-equations the elements of the Jacobian will either be $\partial F_i / \partial Q_j = \pm 1$ or zero in row i for a junction continuity equation row. The Jacobian terms for the energy loop equation rows will either be $\partial F_i / \partial Q_j = \pm\ n_j K_j Q_j^{n_j - 1}$ or zero. The non-zero elements of the Jacobian for the H-equations are $\partial F_i / \partial H_j = \pm\ \{1/(n_m K_m)\}\{(H_j - H_k)/K_m\}^{1/n_m - 1}$ in which the sign is determined by the sign in front of this term in the equation and the sign before H_j within the parentheses. Non-zero terms in the Jacobian for the ΔQ-equations will be of the form $\partial F_i / \partial \Delta Q_j = \pm\ n_k K_k (Q_{ok} \pm \sum \Delta Q_m)^{n_k - 1}$.

The **Q-equations** are

$$
\begin{aligned}
F_1 &= Q_1 - Q_2 - Q_4 - 1.0 &&= 0 \\
F_2 &= Q_2 + Q_3 \qquad - 1.5 &&= 0 \\
F_3 &= Q_4 - Q_3 + Q_5 - 0.8 &&= 0 \\
F_4 &= K_1 Q_1^{n_1} + K_4 Q_4^{n_4} - K_5 Q_5^{n_5} - 10 &&= 0 \\
F_5 &= K_2 Q_2^{n_2} - K_3 Q_3^{n_3} - K_4 Q_4^{n_4} &&= 0
\end{aligned}
\tag{4.47}
$$

The Newton method is described by $[\boldsymbol{D}]\{z\}=\{\boldsymbol{F}\}$ and $\{\boldsymbol{Q}\}^{(m+1)} = \{\boldsymbol{Q}\}^{(m)} - \{z\}$ with

$$
[D] = \begin{bmatrix}
1 & -1 & 0 & -1 & 0 \\
0 & 1 & 1 & 0 & 0 \\
0 & 0 & -1 & 1 & 1 \\
n_1 K_1 Q_1^{n_1 - 1} & 0 & 0 & n_4 K_4 Q_4^{n_4 - 1} & -n_5 K_5 Q_5^{n_5 - 1} \\
0 & n_2 K_2 Q_2^{n_2 - 1} & -n_3 K_3 Q_3^{n_3 - 1} & -n_4 K_4 Q_4^{n_4 - 1} & 0
\end{bmatrix}
\tag{4.48}
$$

If we choose the initial estimate of the solution vector to be

$$
\{\boldsymbol{Q}\}^{(0)} = \begin{Bmatrix} 2.0 \\ 0.9 \\ 0.6 \\ 0.1 \\ 1.3 \end{Bmatrix}
\tag{4.49}
$$

with which we have been careful to satisfy the junction continuity equations, so these discharges can be used in the ΔQ-equations, the first evaluation of the Jacobian matrix and right-hand side leads to

$$
\begin{bmatrix}
1.0000 & -1.0000 & 0.0000 & -1.0000 & 0.0000 \\
0.0000 & 1.0000 & 1.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & -1.0000 & 1.0000 & 1.0000 \\
28.1133 & 0.0000 & 0.0000 & 11.9749 & -40.8039 \\
0.0000 & 16.6487 & -58.2888 & -11.9749 & 0.0000
\end{bmatrix}
\begin{Bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \end{Bmatrix}
=
\begin{Bmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \\ -7.6935 \\ -11.3783 \end{Bmatrix}
\tag{4.50a}
$$

with the solution

$$\{z\} = \begin{Bmatrix} -0.1169 \\ -0.1470 \\ 0.1470 \\ 0.0301 \\ 0.1169 \end{Bmatrix} \qquad \{Q\}^{(1)} = \begin{Bmatrix} 2.1169 \\ 1.0470 \\ 0.4530 \\ 0.0699 \\ 1.1831 \end{Bmatrix} \tag{4.51a}$$

The Newton equations for the next cycle are

$$\begin{bmatrix} 1.0000 & -1.0000 & 0.0000 & -1.0000 & 0.0000 \\ 0.0000 & 1.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & -1.0000 & 1.0000 & 1.0000 \\ 29.6481 & 0.0000 & 0.0000 & 9.0910 & -37.3636 \\ 0.0000 & 19.0804 & -45.4902 & -9.0910 & 0.0000 \end{bmatrix} \begin{Bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \end{Bmatrix} = \begin{Bmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \\ -0.0682 \\ -0.7993 \end{Bmatrix} \tag{4.50b}$$

with the solution

$$\{z\} = \begin{Bmatrix} -0.0022 \\ -0.0111 \\ 0.0111 \\ 0.0089 \\ 0.0022 \end{Bmatrix} \qquad \{Q\}^{(2)} = \begin{Bmatrix} 2.1191 \\ 1.0581 \\ 0.4419 \\ 0.0610 \\ 1.1809 \end{Bmatrix} \tag{4.51b}$$

One more cycle would yield the final solution

$$\{Q\}^{(3)} = \begin{Bmatrix} 2.1191 \\ 1.0583 \\ 0.4417 \\ 0.0608 \\ 1.1809 \end{Bmatrix} \tag{4.51c}$$

Referring again to Fig. 4.24, since we have only three nodes, we must construct three ***H*-equations**. They are

$$\begin{aligned} F_1 &= \left[\frac{100 - H_1}{K_1}\right]^{1/n_1} - \left[\frac{H_1 - H_2}{K_2}\right]^{1/n_2} - \left[\frac{H_1 - H_3}{K_4}\right]^{1/n_4} - 1.0 = 0 \\ F_2 &= \left[\frac{H_1 - H_2}{K_2}\right]^{1/n_2} + \left[\frac{H_3 - H_2}{K_3}\right]^{1/n_3} - 1.5 = 0 \\ F_3 &= \left[\frac{H_1 - H_3}{K_4}\right]^{1/n_4} - \left[\frac{H_3 - H_2}{K_3}\right]^{1/n_3} + \left[\frac{90 - H_3}{K_5}\right]^{1/n_5} - 0.8 = 0 \end{aligned} \tag{4.52}$$

Using $[\mathbf{D}]\{z\} = \{\boldsymbol{F}\}$ and $\{\boldsymbol{H}\}^{(m+1)} = \{\boldsymbol{H}\}^{(m)} - \{z\}$ to implement the Newton method with an initial estimate of the nodal heads as

$$\{\boldsymbol{H}\}^{(0)} = \begin{Bmatrix} 93 \\ 85 \\ 88 \end{Bmatrix} \tag{4.53}$$

successive computational cycles produce

$$\begin{bmatrix} -0.166 & 0.060 & 0.035 \\ 0.060 & -0.100 & 0.040 \\ 0.035 & 0.040 & -0.162 \end{bmatrix} \begin{Bmatrix} z_1 \\ z_2 \\ z_3 \end{Bmatrix} = \begin{Bmatrix} -1.258 \\ -0.365 \\ -0.382 \end{Bmatrix} \quad \{z\} = \begin{Bmatrix} 15.953 \\ 17.241 \\ 10.088 \end{Bmatrix} \quad \{\boldsymbol{H}\}^{(1)} = \begin{Bmatrix} 77.047 \\ 67.759 \\ 77.912 \end{Bmatrix} \tag{4.54a}$$

$$\begin{bmatrix} -0.171 & 0.056 & 0.075 \\ 0.056 & -0.078 & 0.023 \\ 0.075 & 0.023 & -0.134 \end{bmatrix} \begin{Bmatrix} z_1 \\ z_2 \\ z_3 \end{Bmatrix} = \begin{Bmatrix} -0.095 \\ -0.084 \\ -0.499 \end{Bmatrix} \quad \{z\} = \begin{Bmatrix} 8.019 \\ 9.614 \\ 9.830 \end{Bmatrix} \quad \{\boldsymbol{H}\}^{(2)} = \begin{Bmatrix} 69.028 \\ 58.145 \\ 68.083 \end{Bmatrix} \tag{4.54b}$$

$$\begin{bmatrix} -0.158 & 0.052 & 0.072 \\ 0.052 & -0.075 & 0.023 \\ 0.072 & 0.023 & -0.123 \end{bmatrix} \begin{Bmatrix} z_1 \\ z_2 \\ z_3 \end{Bmatrix} = \begin{Bmatrix} -0.120 \\ -0.003 \\ -0.049 \end{Bmatrix} \quad \{z\} = \begin{Bmatrix} 1.547 \\ 1.353 \\ 0.770 \end{Bmatrix} \quad \{\boldsymbol{H}\}^{(3)} = \begin{Bmatrix} 67.480 \\ 56.793 \\ 67.312 \end{Bmatrix} \tag{4.54c}$$

$$\begin{bmatrix} -0.239 & 0.052 & 0.153 \\ 0.052 & -0.075 & 0.022 \\ 0.153 & 0.022 & -0.202 \end{bmatrix} \begin{Bmatrix} z_1 \\ z_2 \\ z_3 \end{Bmatrix} = \begin{Bmatrix} 0.019 \\ 0.000 \\ -0.019 \end{Bmatrix} \quad \{z\} = \begin{Bmatrix} -0.032 \\ 0.002 \\ 0.070 \end{Bmatrix} \quad \{\boldsymbol{H}\}^{(4)} = \begin{Bmatrix} 67.513 \\ 56.791 \\ 67.242 \end{Bmatrix} \tag{4.54d}$$

$$\begin{bmatrix} -0.210 & 0.052 & 0.124 \\ 0.052 & -0.075 & 0.023 \\ 0.124 & 0.023 & -0.174 \end{bmatrix} \begin{Bmatrix} z_1 \\ z_2 \\ z_3 \end{Bmatrix} = \begin{Bmatrix} 0.002 \\ -0.001 \\ 0.002 \end{Bmatrix} \quad \{z\} = \begin{Bmatrix} -0.005 \\ -0.001 \\ 0.006 \end{Bmatrix} \quad \{\boldsymbol{H}\}^{(5)} = \begin{Bmatrix} 67.517 \\ 56.793 \\ 67.236 \end{Bmatrix} \tag{4.54e}$$

Depending on the desired accuracy of this solution, the process might have been terminated one or two cycles sooner. One of the best features of the Newton method is a quadratic convergence rate as the solution is approached; here we can easily see the rapid reduction in the size of the corrections $\{z\}$ in successive cycles.

When this problem is solved by using the ***ΔQ*-equations**, we need only one energy loop and one pseudo loop, as is indicated on Fig. 4.24. The ΔQ-equations are

$$\begin{aligned} F_1 &= K_1\left(Q_{o1} + \Delta Q_1\right)^{n_1} + K_4\left(Q_{o4} - \Delta Q_2 + \Delta Q_1\right)^{n_4} - K_5\left(Q_{o5} - \Delta Q_1\right)^{n_5} - 10 = 0 \\ F_2 &= K_2\left(Q_{o2} + \Delta Q_2\right)^{n_2} - K_3\left(Q_{o3} - \Delta Q_2\right)^{n_3} - K_4\left(Q_{o4} - \Delta Q_2 + \Delta Q_1\right)^{n_4} = 0 \end{aligned} \tag{4.55}$$

The equations for the Newton method are $[\mathbf{D}]\{z\}=\{\boldsymbol{F}\}$ and $\{\boldsymbol{\Delta Q}\}^{(m+1)} = \{\boldsymbol{\Delta Q}\}^{(m)} - \{z\}$ in which the Jacobian and vector of initial discharges are

$$[\boldsymbol{D}] = \begin{bmatrix} \dfrac{\partial \boldsymbol{F}_1}{\partial \Delta \boldsymbol{Q}_1} & \dfrac{\partial \boldsymbol{F}_1}{\partial \Delta \boldsymbol{Q}_2} \\ \dfrac{\partial \boldsymbol{F}_2}{\partial \Delta \boldsymbol{Q}_1} & \dfrac{\partial \boldsymbol{F}_2}{\partial \Delta \boldsymbol{Q}_2} \end{bmatrix} \qquad \{\boldsymbol{Q_o}\}^{(0)} = \begin{Bmatrix} 2.0 \\ 0.9 \\ 0.6 \\ 0.1 \\ 1.3 \end{Bmatrix} \tag{4.56a,b}$$

Three successive solution cycles produce

$$\begin{bmatrix} 80.892 & -11.975 \\ -11.975 & 86.913 \end{bmatrix} \begin{Bmatrix} z_1 \\ z_2 \end{Bmatrix} = \begin{Bmatrix} -7.694 \\ -11.378 \end{Bmatrix} \quad \{z\} = \begin{Bmatrix} -0.117 \\ -0.147 \end{Bmatrix} \quad \{\Delta \boldsymbol{Q}\}^{(1)} = \begin{Bmatrix} 0.117 \\ 0.147 \end{Bmatrix} \tag{4.57a}$$

$$\begin{bmatrix} 76.103 & -9.091 \\ -9.091 & 73.662 \end{bmatrix} \begin{Bmatrix} z_1 \\ z_2 \end{Bmatrix} = \begin{Bmatrix} -0.068 \\ -0.799 \end{Bmatrix} \quad \{z\} = \begin{Bmatrix} -0.002 \\ -0.011 \end{Bmatrix} \quad \{\Delta \boldsymbol{Q}\}^{(2)} = \begin{Bmatrix} 0.119 \\ 0.158 \end{Bmatrix} \tag{4.57b}$$

$$\begin{bmatrix} 75.163 & -8.188 \\ -8.188 & 71.954 \end{bmatrix} \begin{Bmatrix} z_1 \\ z_2 \end{Bmatrix} = \begin{Bmatrix} 0.004 \\ -0.009 \end{Bmatrix} \quad \{z\} = \begin{Bmatrix} 0.000 \\ 0.000 \end{Bmatrix} \quad \{\Delta \boldsymbol{Q}\}^{(3)} = \begin{Bmatrix} 0.119 \\ 0.158 \end{Bmatrix} \tag{4.57c}$$

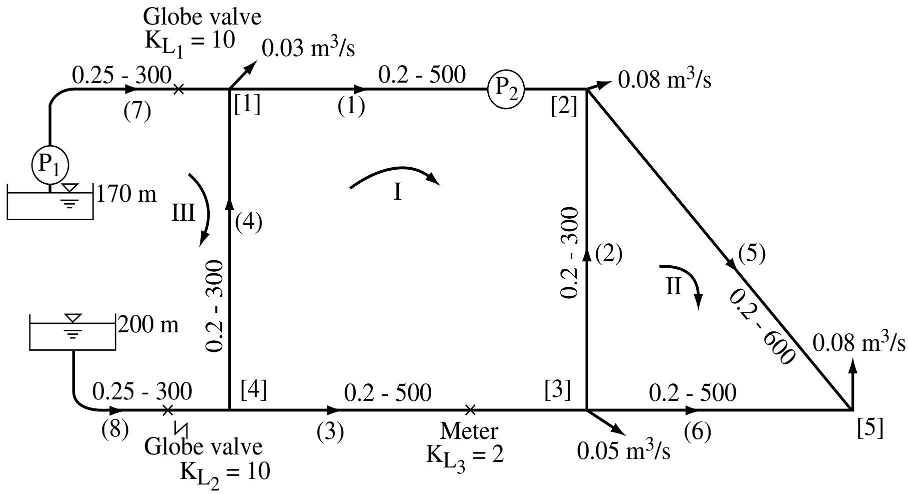
From these results we can compute the discharges themselves in ft^3/s as

$$\begin{aligned} \boldsymbol{Q}_1 &= \boldsymbol{Q_o}_1 + \Delta \boldsymbol{Q}_1 &&= 2.119 \\ \boldsymbol{Q}_2 &= \boldsymbol{Q_o}_2 + \Delta \boldsymbol{Q}_2 &&= 1.058 \\ \boldsymbol{Q}_3 &= \boldsymbol{Q_o}_3 - \Delta \boldsymbol{Q}_2 &&= 0.442 \\ \boldsymbol{Q}_4 &= \boldsymbol{Q_o}_4 + \Delta \boldsymbol{Q}_1 - \Delta \boldsymbol{Q}_2 &&= 0.061 \\ \boldsymbol{Q}_5 &= \boldsymbol{Q_o}_5 - \Delta \boldsymbol{Q}_1 &&= 1.181 \end{aligned} \tag{4.58}$$

Readers will find it instructive to solve this same problem by modifying subroutine FUNCT in program EQUSOL1. For still more experience the reader should consider the use of an equation-solving software program such as MathCAD, TK-Solver, or MathLab.

4.4.3. COMPUTER SOLUTIONS TO NETWORKS

In this section we concentrate on the implementation of solutions to networks using computers, and how pumps, local losses and/or PRV's are readily included. To begin this process consider first the eight-pipe network in Fig. 4.25 that includes a source pump that supplies some of the system demand and a booster pump in pipe 1. There are also local loss devices in pipes 7, 8, and 3, the first two of which have a loss coefficient of 10, and the third has a loss coefficient of 2. (The Roman numeral loop notation will be used in Example Problem 4.6.) After developing and solving the equations for this network, we will place a PRV in pipe 5.



Pump Characteristics
(Q in m³/s and h_p in meters)

Pump No.	Point 1		Point 2		Point 3	
	Q	h_p	Q	h_p	Q	h_p
1	0.025	12.0	0.040	10.5	0.055	8.0
2	0.060	4.0	0.090	3.8	0.120	3.5

Figure 4.25 An eight-pipe network with pumps and local losses.

For this network there are five junction continuity equations and three energy loop equations. The Q-equations are

$$
\begin{aligned}
F_1 &= -Q_1 + Q_4 + Q_7 - 0.03 = 0 \\
F_2 &= Q_1 + Q_2 - Q_5 - 0.08 = 0 \\
F_3 &= -Q_2 + Q_3 - Q_6 - 0.05 = 0 \\
F_4 &= -Q_3 - Q_4 + Q_8 - 0.00 = 0 \\
F_5 &= Q_5 + Q_6 - 0.08 = 0 \\
F_6 &= K_1 Q_1^{n_1} - h_{p2} - K_2 Q_2^{n_2} - K_3 Q_3^{n_3} - 2Q_3^2/(2gA_3^2) + K_4 Q_4^{n_4} = 0 \\
F_7 &= K_5 Q_5^{n_5} - K_6 Q_6^{n_6} + K_2 Q_2^{n_2} = 0 \\
F_8 &= K_7 Q_7^{n_7} - h_{p1} + 10Q_7^2/(2gA_7^2) - K_4 Q_4^{n_4} - K_8 Q_8^{n_8} - 10Q_8^2/(2gA_8^2) + 30 = 0
\end{aligned}
\tag{4.59}
$$

In these equations the local loss coefficients 10, 10, and 2 have been inserted in the equations, but the pump heads are written as h_{p1} and h_{p2}. These pump heads can be expressed as a function of discharge by fitting a second-order polynomial through three points on the pump characteristic curve over the range of expected operation. Alternatively, if the power supplied by the pump to the fluid is assumed to be constant, then these pump heads can be defined by $h_p = \text{Power}/(\gamma Q)$.

Example Problem 4.6

Solve the 8-equation system, Eqs. 4.59, for the network shown in Fig. 4.25 by using hand methods. Then verify this solution by using program EQUSOL1 and/or an equation-solving software package such as MathCAD or TK-Solver.

This and the next few Example Problems will be solved by rewriting the subroutine FUNCT for use with EQUSOL1 in each case. MathCAD and TK-Solver models of these problems will be found on the CD that accompanies this book.

The first step is to estimate the pipe discharges; based on these values, we then compute K and n for the 8 pipes. The listed discharges produce the K and n values in the table:

Pipe No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Q **(m³/s)**	0.100	0.015	0.100	0.080	0.030	0.050	0.070	0.170
K	1160	613	1160	690	1292	1115	322	239
n	1.827	1.788	1.827	1.824	1.801	1.812	1.772	1.832
f	0.0134	0.0314	0.0134	0.0212	0.0168	0.0152	0.0223	0.0127

The next step is to fit the three pairs of points for the two pumps to obtain the coefficients for the polynomials: A_1 = - 2220, B_1 = 44.4, C_1 = 12.28 and A_2 = - 55.6, B_2 = 1.667, C_2 = 4.10. These values can now be substituted into the equations, the equations can be differentiated to produce the Jacobian matrix and equation vector, and all of the terms can be evaluated by using the data in the table and figure. (The reader should evaluate some terms to verify that the process is fully understood.) The results are

$$[D] = \begin{bmatrix} -1.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 1.0 & 0.0 \\ 1.0 & 1.0 & 0.0 & 0.0 & -1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & -1.0 & 1.0 & 0.0 & 0.0 & -1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & -1.0 & -1.0 & 0.0 & 0.0 & 0.0 & 1.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 1.0 & 0.0 & 0.0 \\ 325 & -40.2 & -337 & 158 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 40.2 & 0.0 & 0.0 & 140 & -178 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & -158 & 0.0 & 0.0 & 370 & -173 \end{bmatrix}$$

and

$$\{F\}^T = \{0.020 \;\; 0.005 \;\; -0.015 \;\; -0.010 \;\; 0.000 \;\; 1.560 \;\; -2.226 \;\; 4.903\}$$

with the superscript T indicating the transpose of the right-side equation vector. The solution to this linear system of equations produces

$$\{z\}^T = \{-0.0029 \;\; 0.0008 \;\; -0.0071 \;\; 0.0021 \;\; -0.0071 \;\; 0.0071 \;\; 0.0150 \;\; -0.0150\}$$

so that the discharges after the first iteration are

$$\{Q\}^T = \{0.1029 \;\; 0.0142 \;\; 0.1071 \;\; 0.0779 \;\; 0.0371 \;\; 0.0429 \;\; 0.0550 \;\; 0.1850\}$$

After two additional iterations, the following solution, in m^3/s, is obtained:

$$\{Q\}^T = \{0.1028 \;\; 0.0142 \;\; 0.1072 \;\; 0.0780 \;\; 0.0370 \;\; 0.0430 \;\; 0.0548 \;\; 0.1852\}$$

The subroutine FUNCT is on the CD under EPRB4_6Q.FOR for use with EQUSOL1, and the TK-Solver model is listed on the CD under EPRB4_6Q.TK. Upon supplying the input file in column one below to EQUSOL1, the solution in the second column is obtained:

From the keyboard: 8 8 2 3	Solution (m^3/s)
Q1 U 0.100	1 Q1 = 0.103
Q2 U 0.015	2 Q2 = 0.014
Q3 U 0.100	3 Q3 = 0.107
Q4 U 0.080	4 Q4 = 0.078
Q5 U 0.030	5 Q5 = 0.037
Q6 U 0.050	6 Q6 = 0.043
Q7 U 0.070	7 Q7 = 0.055
Q8 U 0.170	8 Q8 = 0.185

* * *

Next let us examine the formulation and the solution of the H-equations for the network in Example Problem 4.6. The H-equations are presented as Eqs. 4.60. The pump heads have been added to the upstream HGL-elevations by indicating these heads as h_p. In a similar way the local losses, denoted simply as h_L, have been subtracted from the HGL-elevations in pipes 3, 7, and 8. By using second-order polynomials for the pump characteristics and noting that each Q in these equations can again be replaced by similar

$$\begin{aligned}
F_1 &= \left[\frac{H_1 + h_{p2} - H_2}{K_1}\right]^{1/n_1} - \left[\frac{H_4 - H_1}{K_4}\right]^{1/n_4} - \left[\frac{170 + h_{p1} - h_{L1} - H_1}{K_7}\right]^{1/n_7} + 0.03 = 0 \\
F_2 &= \left[\frac{H_2 - H_5}{K_5}\right]^{1/n_5} - \left[\frac{H_3 - H_2}{K_2}\right]^{1/n_2} - \left[\frac{H_1 + h_{p2} - H_2}{K_1}\right]^{1/n_1} + 0.08 = 0 \\
F_3 &= \left[\frac{H_3 - H_5}{K_6}\right]^{1/n_6} + \left[\frac{H_3 - H_2}{K_2}\right]^{1/n_2} - \left[\frac{H_4 - h_{L3} - H_3}{K_3}\right]^{1/n_3} + 0.05 = 0 \\
F_4 &= \left[\frac{H_4 - h_{L3} - H_3}{K_3}\right]^{1/n_3} + \left[\frac{H_4 - H_1}{K_4}\right]^{1/n_4} - \left[\frac{200 - h_{L2} - H_4}{K_8}\right]^{1/n_8} + 0.00 = 0 \\
F_5 &= -\left[\frac{H_2 - H_5}{K_5}\right]^{1/n_5} - \left[\frac{H_3 - H_5}{K_6}\right]^{1/n_6} + 0.08 = 0
\end{aligned} \tag{4.60}$$

head-difference terms, we find that we cannot create an equation that does not contain the pump discharge in the pipes that contain the pumps. The same is true for pipes that contain local losses because again the magnitude of the loss is a function of the discharge through the pipe. If an iterative approach were used to approximate the discharge in terms of the upstream and downstream nodal heads, the quadratic convergence rate of the Newton method would be sacrificed. This dilemma highlights a deficiency in using the H-equations when pumps, whose heads are strongly dependent upon the discharges passing through them, are present. If many pumps exist in a network and the H-equations are to be used, especially if equation-solving software such as MathCAD or TK-Solver is used, it might be advantageous to write the continuity equations in terms of the discharges, then add additional equations to describe these discharges in terms of nodal heads, and finally solve simultaneously for the heads and discharges. This approach will be taken in

Example Problem 4.7. A successful technique that solves the H-equations in a computer program will be described in a subsequent section. This technique obtains the current value of the discharge in every pipe from the heads that exist during an iteration by calculating $Q = [(H_i - H_j)/K]^{1/n}$, and when a pump exists in a pipe, the Newton method can find the discharge Q from the single equation for that pipe $Q = [(H_i + h_p - H_j)/K]^{1/n}$ in which the pump head is $h_p = AQ^2 + BQ + C$.

Example Problem 4.7

Solve the H-equations for the 8-pipe network shown in Fig. 4.25 that was the subject of study in Example Problem 4.6.

The form of the subroutine FUNCT that is needed in EQUSOL1 to solve the H-equations is on the CD in EPRB4_7H.FOR with input data in EPRB4_7H.DAT; the corresponding TK-Solver model will be found there as EPRB4_7H.TK.

* * *

Finally, we now turn our attention to the ΔQ-equations, which are given in Eq. 4.61 for this same pipe network. In addition, we must select an appropriate set of initial discharges.

$$
\begin{aligned}
F_1 &= K_1\left(Q_{o1} + \Delta Q_1\right)^{n_1} - h_{p2} - K_2\left(Q_{o2} - \Delta Q_1 + \Delta Q_2\right)^{n_2} - K_3\left(Q_{o3} - \Delta Q_1\right)^{n_3} \\
&\quad - 2\left(Q_{o3} - \Delta Q_1\right)^2 / \left(2gA_3^2\right) + K_4\left(Q_{o4} + \Delta Q_1 - \Delta Q_3\right)^{n_4} = 0 \\
F_2 &= K_5\left(Q_{o5} + \Delta Q_2\right)^{n_5} - K_6\left(Q_{o6} - \Delta Q_2\right)^{n_6} + K_2\left(Q_{o2} - \Delta Q_1 + \Delta Q_2\right)^{n_2} = 0 \\
F_3 &= K_7\left(Q_{o7} + \Delta Q_3\right)^{n_7} + 10\left(Q_{o7} + \Delta Q_3\right)^2 / \left(2gA_7^2\right) - K_4\left(Q_{o4} + \Delta Q_1 - \Delta Q_3\right)^{n_4} \\
&\quad - h_{p1} - K_8\left(Q_{o8} - \Delta Q_3\right)^{n_8} - 10\left(Q_{o8} - \Delta Q_3\right)^2 / \left(2gA_8^2\right) + 30 = 0
\end{aligned}
\qquad (4.61)
$$

In these ΔQ-equations the pump heads will be replaced by second-order polynomial equations in the forms

$$
\begin{aligned}
h_{p1} &= A_1(Q_{o7} + \Delta Q_3)^2 + B_1(Q_{o7} + \Delta Q_3) + C_1 \\
h_{p2} &= A_2(Q_{o1} + \Delta Q_1)^2 + B_2(Q_{o1} + \Delta Q_1) + C_2
\end{aligned}
\qquad (4.62)
$$

The local losses are replaced by $h_L = K_L Q^2/(2gA^2)$, in which each discharge is written as the algebraic sum of Q_{oi} and the corrective discharges in that pipe. The derivatives of these terms that contribute to elements of the Jacobian are then easily evaluated.

Example Problem 4.8

Solve the ΔQ-equations for the 8-pipe network depicted in Fig. 4.25.

To solve the ΔQ-equations using EQUSOL1, download EPRB4_8D.FOR from the CD; the appropriate input is found in EPRB4_8D.DAT. A TK-Solver model will be found as EPRB4_8D.TK. A set of initial discharges might be selected as follows: Q_{o1} = 0.100 m^3/s, Q_{o2} = 0.015 m^3/s, Q_{o3} = 0.110 m^3/s, Q_{o4} = 0.060 m^3/s, Q_{o5} = 0.035 m^3/s, Q_{o6} = 0.045 m^3/s, Q_{o7} = 0.070 m^3/s, and Q_{o8} = 0.170 m^3/s.

* * *

4.4.4. INCLUDING PRESSURE REDUCING VALVES

To acquire experience in analyzing networks containing pressure reducing valves (and similar appurtenances such as back pressure valves), let us assume that pipe 5 in our 8-pipe network contains a PRV that is 200 m downstream from junction 2, and the pressure setting of this valve is equivalent to a reservoir water surface elevation of 149 m. The five junction continuity equations are unchanged for this network. The energy equations now consist of one real loop and two pseudo loops, as the revised diagram of this network shows in Fig. 4.26. The pump data in Fig. 4.25 are unchanged. In the

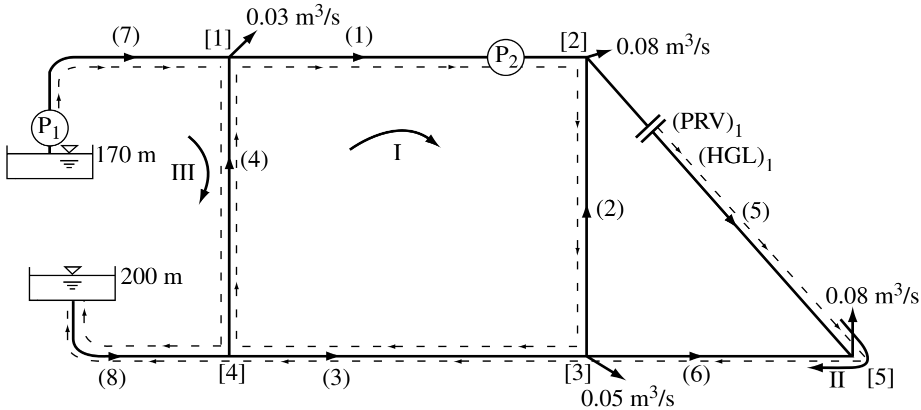


Figure 4.26 An eight-pipe network with pumps and local losses, now including a PRV.

Q-equations, Eqs. 4.72, equations F_6 and F_8 are unchanged, but equation F_7 must now be written around the new pseudo loop; it becomes

$$F_7 = K_5'Q_5^{n_5} - K_6Q_6^{n_6} - K_3Q_3^{n_3} - 2Q_3^2/(2gA_3^2) - K_8Q_8^{n_8} - 10Q_8^2/(2gA_8^2) + 200 - HGL_1 = 0 \tag{4.63}$$

in which K_5' represents the portion of pipe 5 downstream from the PRV. Eight independent equations therefore exist, which may be solved for the discharges Q_i, $i = 1,8$. If the PRV does not close completely, the solution is obtained with HGL_1 in F_7 equal to the head that is set at the valve, i.e., 149 m in this example. If the Newton solution process that is based on this assumption should produce a negative value for Q_5, the PRV will close completely. Then the discharge in the pipe containing the PRV is no longer an unknown but is zero, i.e., $Q_5 = 0$, and the value of the HGL immediately downstream from the PRV becomes the unknown. In other words, when this PRV closes, the same system of equations still describes the network operation, but the set of unknowns changes to $\{Q_1, Q_2, Q_3, Q_4, HGL_1, Q_6, Q_7, Q_8\}$. In a computer program this change can be accommodated by dropping the pipe number containing the PRV from the integer arrays that define the junction continuity equations and the energy loop equations. Also, a flag is set in the program to indicate in the solution array that the HGL of the PRV is stored in place of the discharge in that pipe.

Subroutine FUNCT for use with EQUSOL1 for this problem is on the CD under the name EPRB4_VQ.FOR. The reader should study a listing of this file to understand the logic that will model the PRV when it closes. The input from the keyboard for this problem is 8 8 2 3 (Thus there are 8 unknowns and 8 variables, and the 2 and 3 are the input and output units.). The input file that was used in Example Problem 4.6 still applies. There are only two basic changes in the program that was used to solve this network problem in the absence of the PRV: (1) element X(5) is now either Q_5 or *HGL*, depending upon whether it is negative, and (2) now F(7) has become a pseudo loop from the artificial reservoir at the PRV to the reservoir at the end of pipe 5. The computer provides the following solution (units are m^3/s or m):

$$Q_1 = 0.102,\ Q_2 = -\ 0.022,\ Q_3 = 0.108,\ Q_4 = 0.077,$$
$$Q_5 = 149.227,\ Q_6 = 0.080,\ Q_7 = 0.055,\ Q_8 = 0.185.$$

From this solution we see that the flow has tried to reverse direction in pipe 5, indicating that the PRV must then close. Thus $Q_5 = 0$, and the reported value of 149.227 is the HGL on the downstream side of the PRV, which is slightly above its pressure setting. Instead of operating in its normal mode, the PRV has acted as a check valve, not permitting the flow to reverse its direction.

Let's increase the demand at node 5 to 0.100 m^3/s. To obtain a solution for this demand, we must change the line that defines the continuity equation at node 5 in subroutine FUNCT to `F(5)=Q5+X(6)-0.100`. Now the execution of the program produces the following solution (units are m^3/s):

$$Q_1 = 0.113,\ Q_2 = 0.002,\ Q_3 = 0.117,\ Q_4 = 0.079$$
$$Q_5 = 0.034,\ Q_6 = 0.066,\ Q_7 = 0.063,\ Q_8 = 0.197$$

With this slightly larger demand at node 5 the PRV operates normally, maintaining an HGL = 149 m on its downstream side. From these discharges the pipe head losses, the pump heads, and other quantities can be evaluated, and from these the head at each node can be determined. Upon carrying out such computations, we find that the heads are H_1 = 173.6 m, H_2 = 155.7 m, H_3 = 155.7 m, H_4 = 180.3 m, and H_5 = 147.2 m. The head on the upstream side of the PRV is 154.8 m, so the PRV loss is 154.8 - 149 = 5.8 m.

If the demand at node 5 is $QJ_5 = 0.16$ m^3/s, the continuity statement in FUNCT for node 5 is changed to `F(5)=Q5+X(6)-0.16`, then the solution (units are m^3/s) becomes

$$Q_1 = 0.145,\ Q_2 = 0.050,\ Q_3 = 0.145,\ Q_4 = 0.088$$
$$Q_5 = 0.115,\ Q_6 = 0.045,\ Q_7 = 0.087,\ Q_8 = 0.233$$

Without a more complete examination of these solutions, we might be inclined to accept all of them as valid. However, upon computing some head losses and pump heads, the following are found: h_{f7} = 6.98 m, h_{p1} = - 28.0 m, h_{p2} = 4.04 m, and h_{f1} = 4.92 m, so the HGL at node 2 is H_2 = 134.2 m. Obviously the negative head for pump 1 is unrealistic; it is caused by operating the pump with a discharge that is outside the range of the three pairs of points that were used to define this pump characteristic curve. Also H_2 = 134.2 m is much smaller than the HGL setting of the PRV, and since this device can not act as a pump to increase the head across it, the most it can do is open completely. If a PRV opens completely, it typically still acts as a local loss device in a way that is similar to a globe valve with a loss coefficient of about 10. Thus the last solution is not valid, and the problem must be solved again with another local loss device in place of the PRV. Currently there exists no simple *a priori* test to learn that the PRV should open fully and that it is unable to maintain its pressure setting; a solution must first be obtained when we use the *Q*-equations, because the nodal heads are determined as a secondary step after the discharges are found. The same statements apply to the use of the ΔQ-equations.

The three modes in which a PRV may operate are treated most conveniently with the *H*-equations, since it is then possible to check directly, as the solution is obtained, whether the head at the upstream side of the PRV is less than H_v (the HGL upstream of the PRV) and whether the head at the downstream pipe node is greater than the set HGL. If the head H at the downstream node becomes larger than the HGL setting of the PRV, then it should shut off the flow in the pipe, and if H_v becomes smaller than HGL, then the PRV should open fully.

When the ΔQ-equations are used to analyze networks that contain PRV's, we must work with two different sets of loops, one around which the ΔQ's circulate, and one around which the energy equations are written. For our 8-pipe example, Fig. 4.27, the first and

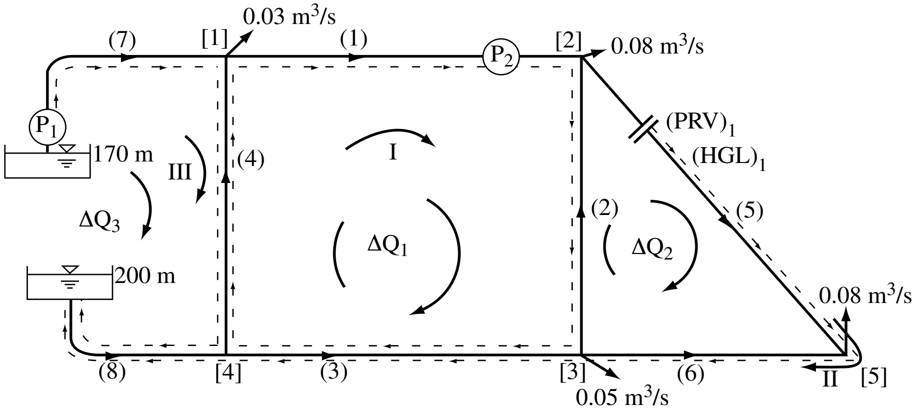


Figure 4.27 The eight-pipe network with a PRV, with loop notation shown.

third equations match the corresponding equations, Eqs. 4.61, for this network without a PRV in pipe 5. The second equation is replaced by

$$
\begin{aligned}
F_2 = K_5'(Q_{o5} + \Delta Q_2)^{n_5} - K_6(Q_{o6} - \Delta Q_2)^{n_6} - K_3(Q_{o3} - \Delta Q_1)^{n_3} \\
- 2(Q_{o3} - \Delta Q_1)^2/(2gA_3^2) - K_8(Q_{o8} - \Delta Q_3)^{n_8} \\
- 10(Q_{o8} - \Delta Q_3)^2/(2gA_8^2) - HGL + 200 = 0
\end{aligned}
\tag{4.64}
$$

It is notable that this equation does not contain ΔQ_2 in every term and that the system of equations does not produce a symmetric Jacobian. To determine the correct operational mode for a PRV when using the ΔQ-equations is much the same as with the Q-equations. Should the flow in a pipe reverse direction, then the PRV should close, and if the HGL at the upstream end of the PRV is less than its setting, then the PRV should open fully; otherwise the PRV is operating in its normal mode. Logic can easily be included in the computer program to check whether the flow is negative in pipes containing PRV's and then change the nature of the problem being solved. The fully-open mode of operation can not be determined until the nodal heads are computed, as with the Q-equations. Should a PRV close, then the discharge in that pipe becomes zero, and the HGL becomes unknown and larger than the setting. If a pipe containing a PRV has only one ΔQ flowing through it, then that corrective discharge becomes known and is $\Delta Q_j = \pm Q_{oi}$, in which the minus sign applies if the assumed directions for Q_{oi} and ΔQ_j coincide, and the plus sign applies if these directions are opposed. In place of ΔQ_j as the unknown, the HGL is unknown, and the number of unknowns still equals the number of equations. Should a PRV close that is internal, with two or more corrective discharges circulating through it, then one of these corrective discharges must be expressed in terms of the others, and the HGL of the PRV replaces this ΔQ in the list of unknowns. In our example, if the PRV were in pipe 2 instead of pipe 5, as shown in Fig. 4.28, then $\Delta Q_2 = Q_{o2} + \Delta Q_1$, and it is replaced by this quantity wherever else ΔQ_2 appears, such as in the discharges for pipes 6 and 7.

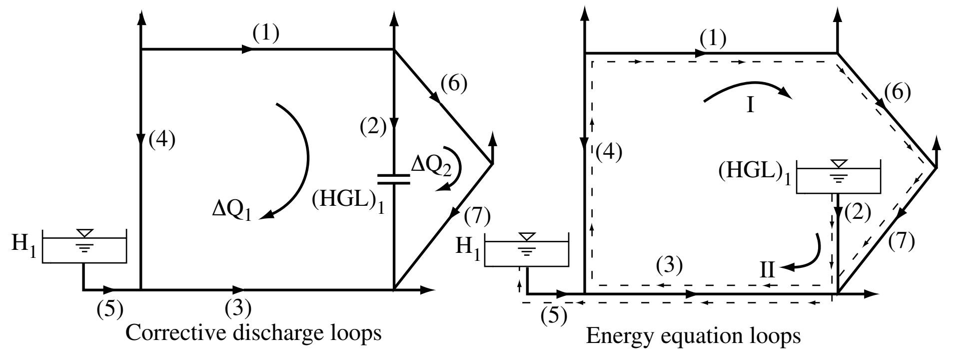


Figure 4.28 The modeling of a PRV in pipe 2 of the 8-pipe network.

To study this problem further, the reader should obtain a listing of FUNCT under the name EPRB4_9.FOR. It can be used to solve this problem. One additional integer variable IDOO has been added to the list of arguments in FUNCT; it is given a value of 0 in the calling program when the equation vector is evaluated and 1 when the Jacobian is evaluated. This variable is needed because we do not want to close the PRV when we evaluate the derivatives. It is instructive to trace the logic that sets Q_5 to zero when the PRV is closed, fixes the value of $\Delta Q_2 = Q_{o5}$ and replaces ΔQ_2 by the HGL as the unknown represented by X(2). These changes are made when Q_5 becomes negative. Subsequent checks might determine whether the HGL becomes less than the PRV setting; if this occurs, the PRV should be reopened. Another modification of this subroutine allows the initial discharges Q_{oi}, i = 1-8, to enter FUNCT through X(i), i = 4-11, thus making it possible to change the demands without changing Q_{oi} within the subroutine.

Example Problem 4.9

Solve the eight-pipe network shown in Fig. 4.27 by using the ΔQ-equations. Obtain this solution first for a demand at node 5 of $QJ_5 = 0.100$ m^3/s and then for $QJ_5 =$ 0.080 m^3/s.

The input data (EPRB4_9.DAT) to solve this problem with a demand of 0.100 m^3/s at node 5 is listed below with the solution:

Input Data				**Solution**	
DQ1	U	0.00	1	DQ1 =	- 0.00749
DQ2	U	0.00	2	DQ2 =	- 0.00570
DQ3	U	0.00	3	DQ3 =	- 0.01668
Qo1	K	0.12	4	Qo1 =	0.12
Qo2	K	0.00	5	Qo2 =	0.00
Qo3	K	0.11	6	Qo3 =	0.11
Qo4	K	0.07	7	Qo4 =	0.07
Qo5	K	0.04	8	Qo5 =	0.04
Qo6	K	0.06	9	Qo6 =	0.06
Qo7	K	0.08	10	Qo7 =	0.08
Qo8	K	0.18	11	Qo8 =	0.18

Applying these solution values for the ΔQ_i, as appropriate, to the initial discharges gives the final discharges, and then the pipe head losses can be computed, using the proper K and n for each pipe, as listed in the table:

Pipe	1	2	3	4	5	6	7	8
Q, m^3/s	0.1125	- 0.0018	0.1175	0.0792	0.0343	0.0657	0.0633	0.1967
h_L, m	21.40	0.0075	23.20	6.77	2.97	8.03	2.43	12.14

From these discharges the pump heads and local losses are $h_{p1} = 6.18$ m, $h_{p2} = 3.58$ m, $h_{L1} = 0.848$ m, $h_{L2} = 8.18$ m, and $h_{L3} = 1.426$ m. From these the nodal heads can be found as $H_1 = 175.3$ m, $H_2 = 157.5$ m, $H_3 = 155.1$ m, $H_4 = 179.7$ m, $H_5 = 147.0$ m, and $H_{v1} = 156.5$ m. We see that the head upstream from the PRV is 156.5 m which is less than $H_2 = 157.5$ m, so the PRV has not opened fully. The head at node 5 downstream, $H_5 = 147.0$ m, is less than the HGL setting of the PRV (149 m) so the PRV has not closed but operates normally.

When the demand at node 5 is $QJ_5 = 0.080$ m^3/s, then the input data and solution are

Input Data

DQ1	U	0.00
DQ2	U	0.00
DQ3	U	0.00
Qo1	K	0.12
Qo2	K	0.00
Qo3	K	0.09
Qo4	K	0.07
Qo5	K	0.04
Qo6	K	0.04
Qo7	K	0.08
Qo8	K	0.16

Solution

1	DQ1 =	- 0.01827
2	DQ2 =	149.2
3	DQ3 =	- 0.02508
4	Qo1 =	0.12
5	Qo2 =	0.00
6	Qo3 =	0.09
7	Qo4 =	0.07
8	Qo5 =	0.04
9	Qo6 =	0.04
10	Qo7 =	0.08
11	Qo8 =	0.16

In this input file the initial discharge estimates Qoi have been altered from previous values so that all continuity equations remain satisfied with $QJ_5 = 0.080$ m^3/s. The solution values remind us that ΔQ_2 is actually the HGL at the downstream end of the PRV since it has closed, and FUNCT has set $\Delta Q_2 = - Q_{o5}$ and then used X(2) as the position for HGL. The table lists the discharges and head losses for this situation:

Pipe	1	2	3	4	5	6	7	8
Q, m^3/s	0.1017	0.0217	0.1083	0.0768	0.0	0.0800	0.0549	0.1851
h_L, m	17.83	0.652	19.98	6.40	0.0	11.47	1.887	10.86

The pump heads and local losses are $h_{p1} = 8.02$ m, $h_{p2} = 3.69$ m, $h_{L1} = 0.638$ m, $h_{L2} = 7.25$ m and $h_{L3} = 1.211$ m, with nodal heads of $H_1 = 177.4$ m, $H_2 = 163.2$ m, $H_3 = 160.7$ m, $H_4 = 181.9$ m, and $H_5 = 149.2$ m. Now the PRV has closed entirely so the flow in pipe 5 is zero, and the HGL at its downstream end is above its set point.

* * *

4.4.5. SYSTEMATIC SOLUTION OF THE Q-EQUATIONS

In earlier sections we have developed the three systems of equations that can be used to analyze pipe networks. We have written these equation systems for several small networks, seen how the Newton method can be applied to any system of nonlinear equations and how to solve a problem by using a general purpose program that implements the Newton method for all three equation systems, and finally we have carried out detailed computations by hand to obtain some solutions by iteration. In this section we will see how this knowledge can be used in developing computer programs that will analyze any pipe network by using the Q-equations, and the programs will require only enough information from the user to describe adequately the network and its connectivity. In the next two sections similar programs will be developed for the solution of the H-equations and the ΔQ-equations.

Let us begin by assuming that there are no local losses. If they exist, they can be modeled simply by assigning a larger equivalent sand roughness, or Hazen-Williams C_{HW}, to the pipes containing minor losses. Here we ignore the Manning equation.

In describing any network of pipes, we need two types of information: (1) Pipe information consisting of the diameter, length, roughness coefficient, and end nodes for each pipe. This information can be called pipe-oriented data, since we assemble it by going though a list of the pipes in the network; (2) Junction information, including the demand at the junction, its elevation, and possibly the pipes that join at the junction. This information is called node-oriented data, since it is assembled by moving through the nodes of the network. Actually the connectivity of the network can be defined either by giving the nodes at the ends of each pipe, or by giving the pipe numbers that join at each node. We shall use this duplicative information to check that the user has not erred in defining the network.

Now we shall describe the input data that are required. Details on the form of this input will be provided subsequently. Prior to study of this section the reader should obtain a listing of the program SOLQEQS.FOR (or C if you prefer) from the text CD. The information that is required from the user is the following:

1. A line that gives (a) the number of pipes, (b) the number of nodes, (c) the number of reservoirs that supply the network, (d) the number of pumps, and (e) the options which you wish to change from the default values. (The default options and how these are changed will be described later.)
2. For each pipe, list its (a) number, (b) upstream node, (c) downstream node, (d) length, (e) diameter, and (f) roughness coefficient.
3. For each node, list its (a) number, (b) demand, and (c) elevation, and (d) a list of pipes that join this node.
4. For each reservoir, list (a) the pipe number that connects this reservoir to the network, and (b) the water surface elevation of the reservoir.
5. For each pump, list (a) the number of the pipe that contains the pump, and (b) three (Q, h_p) pairs of discharge vs. pump head that will allow its operating characteristics to be defined.
6. Finally, because the algorithms that could be used to determine the minimum set of independent loops for the energy equations are relatively complex, we require a list of pipe numbers around each loop (with a minus sign before a pipe number if the movement around the loop opposes the assumed direction of flow in that pipe). We require that pseudo loops be provided first, and then the real loops.

The information in item one is used to dimension the arrays that will store the remaining information about the pipe network and to determine how many lines of each information type will be read from the input data file. The information must be provided in the sequence that is listed above.

The program must perform five major tasks:

1. Read the input data that defines the network.
2. Develop from this information the system of Q-equations, i.e., the junction continuity equations and the energy equations around pseudo and real loops of the network. This task defines the equations and also forms each element of the Jacobian matrix.
3. Solve the system of equations. Here we will simply call a standard linear algebra solver. However, a program for larger network problems should have a special

linear algebra solver that takes advantage of the special properties of a sparse Jacobian matrix.

4. Obtain the head H at each node after the pipe discharges have been found.
5. Write the solution results in tables that can be readily understood. We choose to provide these results in two tables: a pipe data table and a node data table.

In reading the pipe numbers that connect at a node and the pipe numbers that define a loop, a pointer is used to separate data for consecutive nodes, and a second pointer separates data for consecutive loops. The pipes that join at nodes are placed consecutively in a one-dimensional array *JN*, with *NN* pointing to the position in this array that separates data for consecutive nodes. A similar one-dimensional array *IK* contains the pipe numbers that form the loops, with *LP* pointing to the first pipe number in each loop. The use of one-dimensional arrays with pointers is a more efficient use of storage than the use of two-dimensional arrays, because the second subscript of a two-dimensional array must then be at least as large as the maximum number of pipes that may exist in a loop.

When solving the Q-equations (or ΔQ-equations), we compute the nodal heads after obtaining the solution for the discharges. These heads are found by starting at all reservoirs and computing each H at the node at the other end of a pipe from (to) the reservoir by subtracting (adding) the pipe head loss from (to) the reservoir water surface elevation. After these heads have been determined, the nodes one pipe away from these can be determined next, and so on. This process continues until the head at every node has been determined.

In program SOLQEQS the computation of heads begins after the PIPE DATA table is written by the `DO 130` loop. This loop finds each head at the other end of a pipe that is connected to a reservoir, and upon computing H the integer array INDX, with its argument equal to this node number, is set to 1 to show that nodal head has been computed. Now loop `DO 160` passes through the nodes, but nothing is done if INDX(I) for node I is zero. Otherwise INDX(I) = 1, and then the pipes that join this node are accessed through array JN; if H at the other end of a pipe is not known, it is computed. Since not all nodal heads will be found from the first pass through the nodes, the integer IJ also accumulates the number of nodes whose head has been found. One way to learn if another pass is needed is to check whether IJ is less than NJ, the total number of nodes. Actually we see whether IJ has increased from the previous pass. If so, we pass through the nodes again. This method may result in passing through the nodes one extra time, but it prevents the creation of an infinite loop if there is an error in the network description so that fewer than NJ heads can be found. After finding every head, the NODE DATA table is written. The program then allows the user to solve another problem whose data is in a different file, or to change the peaking factor for the same network.

Detailed instructions on the preparation of input data to SOLQEQS follows:

Line 1: No. of Pipes (NP), No. of Nodes (NJ), No. of Reservoirs (NRES), No. of Pumps (NPUMP), No. of Options (NOPT), Option Pairs.

The options consist of a letter in quotes followed by a value, as follows:

Letter	Controls What	Value	Default
U or u	ES or SI units	0 = ES, 1 = SI	0
Q or q	Discharge units	0 = ft^3/s, 1 = gal/min, 2 = mgd, 3 = m^3/s, 4 = l/s	0 or 3
D or d	Pipe diameter and roughness units	0 = in, 1 = ft, 2 = m, 3 = cm, 4 = mm	0 if ES 4 if SI
F or f	Peaking factor	Multiplier of demands	1.0
V or v	Kinematic viscosity	ν = value	ES, 1.217E-5 SI, 1.310E-6
G or g	Specific weight	γ = value	ES, 62.4 SI, 9806.0
C or c	Network check	1 = yes, 0 = no	1

Here is an example of specifying options: 2 'U' 1 'F' 1.5. The 2 indicates two options are being changed, to specify SI units and to specify a peaking factor of 1.5. In giving the options, the units (ES or SI) option should appear first if it is to be elected, but otherwise the options can be given in any order.

Next group: Pipe data consisting of NP lines, each giving pipe number, node 1, node 2, length, diameter, and roughness. Pipes must be numbered consecutively, starting with 1, but they need not be entered consecutively. The roughness may be either the equivalent sand roughness e (in the same units as the diameter) for use in the Colebrook-White and Darcy-Weisbach equations, or a Hazen-Williams C_{HW}, and these may be mixed. The program decides which equation to use, based on the roughness size.

Next group: Node data consisting of NJ lines, each giving node number, demand, elevation, number of pipes at the node, and a list of these pipe numbers with a minus if the flow is from the junction. This information is used to define the junction continuity equations.

Next group: Reservoir data consisting of NRES lines, each giving the number of the pipe connecting the reservoir to the network and the water surface elevation.

Next group: Pump data consisting of NPUMP lines, each with the number of the pipe containing the pump, followed by three (Q, h_p) pairs to define the pump curve.

Next group: Loop data consisting of NL = NP - NJ lines, one loop on each line with the number of pipes in the loop and a list of these pipes. A negative sign must precede the pipe number if the direction around the loop opposes the assumed direction of flow in this pipe. Pseudo loops connecting reservoirs must appear first in this list, and the real loops follow.

Example Problem 4.10

Use program SOLQEQS to solve the network of Example Problem 4.5. Obtain two solutions: (1) for the given demands and (2) with these demands multiplied by 2.0.

The input data takes the form

```
6 5 1 0 1 'D' 1
1  0  1  1500  0.667  0.000417
2  1  2  1000  0.5  0.000417
3  2  4  1500  0.5  0.000417
4  1  3  1500  0.5  0.000417
5  3  4  1200  0.5  0.000417
6  4  5  1000  0.333  0.000417
1 0.50 350.  3  1  - 2  - 4
2 0.35 350.  2  2  - 3
3 0.50 350.  2  4  - 5
4 0.50 350.  3  3  5  - 6
5 0.25 350.  1  6
1  500
4  2  3  - 5  - 4
```

or, if the diameters and e are given in inches (Inches is the default; either giving 0 options as the last 0 in the first line below, or giving 1 'D' 0, tells SOLQEQS to use inches), then the input would be

```
6 5 1 0 0
1  0  1  1500  8.0  0.005
2  1  2  1000  6.0  0.005
3  2  4  1500  6.0  0.005
4  1  3  1500  6.0  0.005
5  3  4  1200  6.0  0.005
6  4  5  1000  4.0  0.005
1 0.50  350.  3  1  - 2  - 4
2 0.35  350.  2  2  - 3
3 0.50  350.  2  4  - 5
4 0.50  350.  3  3  5  - 6
5 0.25  350.  1  6
1  500
4  2  3  - 5  - 4
```

When prompted after the first solution, we give the peaking factor 2.0. The solution tables follow. In the last NODE DATA table we see that some heads are negative, so the network is unable to supply demands that are double the initial values.

PIPE DATA

PIPE NO.	NODES FROM	TO	L ft.	DIA. ft	e x10^4 ft	Q ft³/s	VEL. ft/s	HEAD LOSS ft.	HLOSS/ 1000
1	0	1	1500	0.667	4.17	2.10	6.02	23.50	15.67
2	1	2	1000	0.500	4.17	0.82	4.18	11.00	11.00
3	2	4	1500	0.500	4.17	0.47	2.39	5.67	3.78
4	1	3	1500	0.500	4.17	0.78	3.97	14.97	9.98
5	3	4	1200	0.500	4.17	0.28	1.43	1.70	1.42
6	4	5	1000	0.333	4.17	0.25	2.87	8.83	8.83

NODE DATA

NODE	DEMAND Estimate	DEMAND ft³/s	ELEV. ft.	HEAD ft.	PRESSURE lb/in²	HGL ELEV. ft.
1	0.5	0.500	350.0	126.50	54.82	476.50
2	0.3	0.350	350.0	115.50	50.05	465.50
3	0.5	0.500	350.0	111.53	48.33	461.53
4	0.5	0.500	350.0	109.82	47.59	459.82
5	0.3	0.250	350.0	101.00	43.77	451.00

For peaking factor = 2.0: **PIPE DATA**

PIPE NO.	NODES FROM	TO	L ft.	DIA. ft	e x10^4 ft	Q ft³/s	VEL. ft/s	HEAD LOSS ft.	HLOSS/ 1000
1	0	1	1500	0.667	4.17	4.20	12.03	91.53	61.01
2	1	2	1000	0.500	4.17	1.64	8.36	42.48	42.48
3	2	4	1500	0.500	4.17	0.94	4.79	21.53	14.35
4	1	3	1500	0.500	4.17	1.56	7.94	57.69	38.46
5	3	4	1200	0.500	4.17	0.56	2.85	6.32	5.27
6	4	5	1000	0.333	4.17	0.50	5.73	33.66	33.66

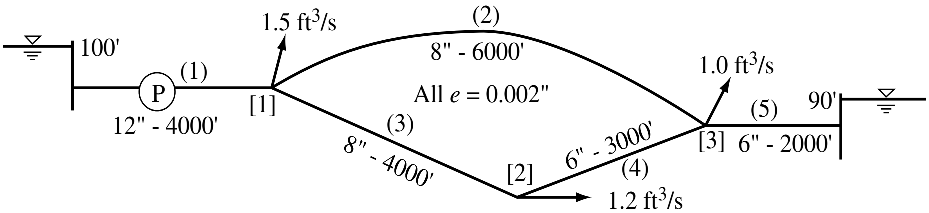
NODE DATA

NODE	DEMAND Estimate	DEMAND ft³/s	ELEV. ft.	HEAD ft.	PRESSURE lb/in²	HGL ELEV. ft.
1	1.0	1.000	350.0	58.48	25.34	408.48
2	0.7	0.700	350.0	15.99	6.93	365.99
3	1.0	1.000	350.0	0.79	0.34	350.79
4	1.0	1.000	350.0	- 5.53	- 2.40	344.47
5	0.5	0.500	350.0	- 39.20	- 16.99	310.80

* * *

Example Problem 4.11

Use program SOLQEQS to analyze the 5-pipe, 3-node network in the figure. In pipe 1 is a pump, with the characteristics given in the table, which is connected to a reservoir. Let $\nu = 1.417 \times 10^{-5}$ ft^2/sec. The elevation of all nodes is zero.



The pump curve is described by data in the following table:

Q, ft^3/s	H, ft
4.5	54
4.0	50
3.5	44

The input data this problem are listed first in two columns, followed by the solution tables.

```
5 3 2 1 1 'V' 1.417E-5          2 1.2 0  2  3  - 4
1  0  1  4000  12  0.002        3 1.0 0  3  2  4  - 5
2  1  3  6000  8  0.002         1  100
3  1  2  4000  8  0.002         5  90
4  2  3  3000  6  0.002         1   4.5  54  4.0  50  3.5  44
5  3  0  2000  6  0.002         3  1  2  5
1 1.5 0  3  1  - 2  - 3         3  2  - 4  - 3
```

PIPE DATA

PIPE NO.	NODES FROM	TO	L ft.	DIA. in	e x10^3 in	Q ft^3/s	VEL. ft/s	HEAD LOSS ft.	HLOSS/ 1000
1	0	1	4000	12.0	2.0	4.13	5.26	26.24	6.56
2	1	3	6000	8.0	2.0	1.21	3.45	29.13	4.85
3	1	2	4000	8.0	2.0	1.42	4.08	26.48	6.62
4	2	3	3000	6.0	2.0	0.22	1.14	2.64	0.88
5	3	0	2000	6.0	2.0	0.43	2.18	5.85	2.93

Pump 1 in pipe 1: Head = 52.21 ft, Q = 4.13 ft^3/s

NODE DATA

NODE	DEMAND Estimate	ft^3/s	ELEV. ft.	HEAD ft.	PRESSURE lb/in^2	HGL ELEV. ft.
1	1.5	1.500	0.0	124.98	54.16	124.98
2	1.2	1.200	0.0	98.50	42.68	98.50
3	1.0	1.000	0.0	95.85	41.54	95.85

* * *

Example Problem 4.12
Solve the 7-pipe, 4-node network shown in Fig. 4.6, which contains a PRV in pipe 6, by using program SOLQEQS.

The input data for this problem are listed after this paragraph. Then the two solution tables follow. Several observations should be made here: In the lines of nodal data the information after the nodal demand that lists the pipes that join at a node is used to define the junction continuity equations; therefore the list of pipes that join at node 1 must include pipe 6 with the PRV in it. The input that lists the pipes that define the loops of the network are used to define the energy loop equations; this group should therefore define a loop that starts (or ends) at the artificial reservoir created by the PRV, so for this network there will be two pseudo loops and one real loop. Also, since the downstream part of pipe 6 defines K', its length is 500 ft, and its upstream node is given as 0 (a reservoir).

```
7 4 3 1 1 'C' 0
1 0 1 1000 6 0.02
2 1 2 1000 6 0.02
3 3 2 800 6 0.02
4 0 3 200 6 0.02
5 3 4 2000 6 0.02
6 0 4 500 6 0.02
7 1 3 1500 1 0.02
1 0 50 4 1 - 2 - 6 - 7
2 1 50 2 2 3
3 0 50 4 4 7 -3 -5
4 20 1 2 5 6
1 90
4 100
6 55
1 1.0 60 1.5 55 2.0 48
3 1 7 - 4
3 6 - 5 - 4
3 2 - 3 - 7
```

PIPE DATA

PIPE NO.	NODES FROM	TO	L ft.	DIA. in	e x10^3 in	Q ft^3/s	VEL. ft/s	HEAD LOSS ft.	HLOSS/ 1000
1	0	1	1000	6.0	20.0	1.11	5.65	27.28	27.28
2	1	2	1000	6.0	20.0	1.07	5.43	25.26	25.26
3	3	2	800	6.0	20.0	- 0.07	- 0.34	0.10	0.12
4	0	3	200	6.0	20.0	0.89	4.54	3.53	17.74
5	3	4	2000	6.0	20.0	0.96	4.91	41.47	20.74
6	0	4	500	6.0	20.0	0.04	0.18	0.04	0.04
7	1	3	1500	1.0	20.0	0.01	1.31	25.56	17.04

Pump 1 in pipe 1: Head = 59.09 ft, Q = 1.11 ft^3/s

NODE DATA

NODE	DEMAND Estimate	DEMAND ft^3/s	ELEV. ft.	HEAD ft.	PRESSURE lb/in^2	HGL ELEV. ft.
1	0.0	0.000	50.0	71.81	31.1	121.81
2	1.0	1.000	50.0	46.55	20.2	96.55
3	0.0	0.000	50.0	46.45	20.1	96.45
4	1.0	1.000	20.0	34.98	15.2	54.98

* * *

4.4.6. SYSTEMATIC SOLUTION OF THE *H*-EQUATIONS

This section is similar to Section 4.4.5, but now the objective is to describe a program that analyzes a network by solving the H-equations. This program will be restricted to the solution of the H-equations for networks that do not contain a PRV or a BPV and in which minor losses can be neglected. (Exercises to include these devices can be found in the end-of-chapter problems.) With these restrictions the Jacobian matrix of the equation system is symmetric. Symmetry occurs because the partial derivatives of terms which describe the discharge in pipe k between nodes i and j, such as $\{(H_i - H_j)/K_k\}^{1/n_k}$, will be the same in the equations where this discharge occurs, so long as neither i nor j are the node for which this junction continuity equation is written. With the sign convention that flow to a junction is positive and flow from a junction is negative, this term will be preceded by a plus sign when j is the junction for which the equation is written. The derivative with respect to H_i will be positive. The derivative with respect to H_j will be negative. If the term describes a pipe whose flow leaves the junction, a negative sign will precede the term and i will be the junction for which the equation is written, and the derivative for the other node j will be positive. Thus the off-diagonal elements of the Jacobian matrix are positive, and the diagonal elements are negative, as we have already seen in an example. We will utilize this symmetry property in developing an algorithm to generate the Jacobian. However, we first examine alternative means for evaluating the derivatives.

The direct way to differentiate $\{(H_i - H_j)/K\}^{1/n}$, in which the pipe number subscript k has been omitted, is to use the power rule of calculus to obtain

$$\pm[\{(H_i - H_j)/K\}^{(1-n)/n}]/(nK) \tag{4.65}$$

in which the minus sign applies when differentiating with respect to H_j. When a pump is present in the pipe, however, it is no longer a straightforward process to differentiate this term, as it now is $\{(H_i + h_p - H_j)/K\}^{1/n}$, in which $h_p = h_p(Q)$ is normally expressed as $h_p = AQ^2 + BQ + C$.

Another way to obtain these derivatives is to start with

$$Q = \left[\frac{H_i - H_j}{K}\right]^{1/n} \tag{4.66}$$

and compute the differential of this formula as

$$dQ = \{H_i - H_j\}^{1/n-1} dH/(nK^{1/n}) = Q^{1-n} dH/(nK) \tag{4.67}$$

The partial derivative with respect to H_i is then

$$\partial Q/\partial H_i = Q^{1-n}/(nK) \tag{4.68}$$

and the partial derivative with respect to H_j is identical, except for a minus sign. So Jacobian matrix elements can be obtained quickly via Eq. 4.68. With this approach we can compute the Jacobian terms for a pipe with a pump in it. Also write $\{(H_i - H_j)/K\}^{1/n}$ as $|(H_i - H_j)/K|^{1/n-1}(H_i - H_j)/K$ to allow a sign change for flows that oppose the assumed direction, which may occur during an intermediate iteration even if the assumed direction is correct for the final solution.

When a pump is present in a pipe, then we can write

$$H_i - H_j + AQ^2 + BQ + C - KQ^n = 0 \tag{4.69}$$

Following the procedure of computing the differential of this equation, we find

$$\partial Q/\partial H = \pm 1/(nKQ^{n-1} - 2AQ - B) \tag{4.70}$$

If the derivative is with respect to H_i, choose the plus sign; otherwise choose the minus sign for H_j. Thus, for a pipe containing a pump, Eq. 4.69 is first solved for Q, and this Q is then used in Eq. 4.70 to evaluate the derivatives for the Jacobian matrix.

Now we can modify SOLQEQS to solve a system of H-equations. Now please obtain a listing from the CD of SOLHEQS and refer to it as you read this section. Here a

NAMELIST (actually an extension of standard Fortran 77, but implemented in many Fortran compilers) will be used to handle options. The NAMELIST will also be used in programs in later chapters. The options that may be in the &OPTIONS list are the following: IU (to set ES or SI units), IQ (to set the discharge units), ID (to give the units for diameters and roughnesses), IC (= 0 to omit checking the dual network connectivity description), VIS (kinematic viscosity), PF (peaking factor), GAMMA (specific weight) and ERR (Newton error criterion). With the H-equations there are no loop energy equations, so the input for loops is eliminated, as is the program segment that generates the loop equations. The section that creates the system of equations will include the junction continuity equations, but this section is modified to implement the new method of obtaining the system Jacobian and the H-equations. In SOLHEQS the length of array H, which stores the nodal heads, has been augmented to include the reservoir heads, so that H_i and H_j now provide the nodal heads at the ends of each pipe, including those that supply the network from a reservoir. So we can easily detect a pump in a pipe, its upstream node number is changed to a negative value. The function subprogram COMPK_N now supplies n , but $(1 - 1/n)$ is stored in array N for later use.

In this program we must have access to discharge values during any iteration for any pipe containing a pump. We do this by computing the discharge from the heads that exist during any iteration by letting ARG = $[(K/(H_i - H_j)]$ and DD = ARG$^{(1 - 1/n)}$; then we find that Q = ARG/DD. Statements following label 146 in the program listing carry out this step. When a pump exists in the pipe, then the Newton method is used to solve Eq. 4.69 by statements found in the `DO 145` loop.

Example Problem 4.13

Prepare suitable input data to analyze the network of Example Problem 4.11 by using program SOLHEQS.

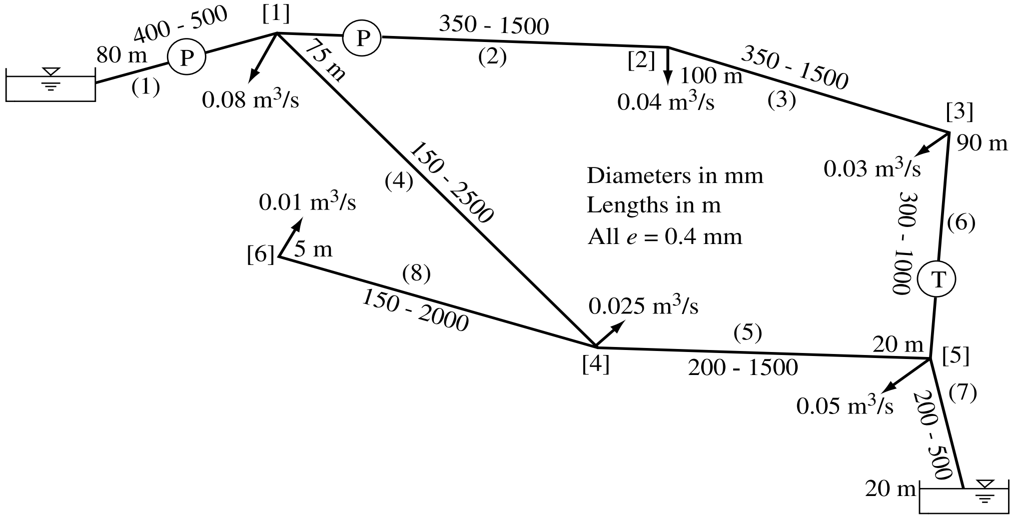
Only minor modifications to the input data in Example Problem 4.11 are needed. First, because the options are entered via a NAMELIST in program SOLHEQS, the first line of the input data now should contain only four values: the numbers of pipes, nodes, reservoirs, and pumps. The second line of input data begins with &OPTIONS, and the next entries contain the namelist variables that differ from the default values, each followed by an equals sign and the value of that variable. This list is ended with a /. Since no loop data are needed, the two lines of loop data are deleted from the input data for the solution to Example Problem 4.11. Since IQ = 0 is the default value, it need not be included in the list after &OPTIONS. With these changes, the input data are now as follows:

```
5 3 2 1
&OPTIONS IQ=0,VIS=1.417E-5/
1 0 1 4000 12 0.002
2 1 3 6000 8 0.002
3 1 2 4000 8 0.002
4 2 3 3000 6 0.002
5 3 0 2000 6 0.002
1 1.5 0 3 1 -2 -3
2 1.2 0 2 3 -4
3 1.0 0 3 2 4 -5
1 100
5 90
1 4.5 54 4.0 50 3.5 44
```

* * *

Example Problem 4.14

The network below is supplied by the source pump in pipe 1, and a booster pump is needed to get the water over the hill below nodes 2 and 3. A turbine is placed in pipe 6 to extract the extra head after the water is moved over the hill crest. Analyze this network using program SOLHEQS. Diameters are in mm, and lengths in m. The kinematic viscosity is $\nu = 1.31 \times 10^{-6}$ m^2/s.



Source Pump

Q m^3/s	**h_p** m
0.20	50
0.30	47
0.50	43

Booster Pump

Q m^3/s	**h_p** m
0.20	15
0.25	14
0.30	12

Turbine

Q m^3/s	**h_t** m
0.15	- 30
0.25	- 25
0.35	- 18

The turbine can be modeled as a pump; the heads are recorded as negative values in preparing its operating characteristics. Since this network is described in SI units, the options for units, discharges and diameters must all be specified. The input data file for this problem, listed in two columns, is therefore

```
8 6 2 3
&OPTIONS IU=1,IQ=3,ID=4/
1 0 1 500 400 0.4
2 1 2 1500 350 0.4
3 2 3 1500 350 0.4
4 1 4 2500 150 0.4
5 5 4 1500 200 0.4
6 3 5 1000 300 0.4
7 5 0 500  200 0.4
8 4 6 2000 150 0.4
1 75 0.08 3 1 - 2 - 4
2 100 0.04 2 2 - 3
3  90 0.03 2 3 - 6
4  10 0.05 3 4 5 - 8
5  20 0.025 3 6 - 5 - 7
6   5 0.01  1 8
1 80
7 20
1  0.2  50  0.3  47  0.5  43
2  0.20  15  0.25  14  0.30  12
6  0.15  - 30  0.25  - 25  0.35  - 18
```

The solution tables from SOLHEQS are

PIPE DATA

PIPE NO.	**NODES FROM**	**TO**	**L** m	**DIA.** mm	**e x10^2** mm	**Q** m^3/s	**VEL.** m/s	**HEAD LOSS** m	**HLOSS/ 1000**
1	0	1	500	400	40.0	0.330	2.63	8.78	17.55
2	- 1	2	1500	350	40.0	0.217	2.26	23.02	15.35
3	2	3	1500	350	40.0	0.177	1.84	15.39	10.26
4	1	4	2500	150	40.0	0.033	1.87	76.55	30.62
5	5	4	1500	200	40.0	0.027	0.86	6.93	4.62
6	- 3	5	1000	300	40.0	0.147	2.08	15.87	15.87
7	5	8	500	200	40.0	0.095	3.03	27.83	55.66
8	4	6	2000	150	40.0	0.010	0.57	5.89	2.95

Pump 1 in pipe 1: Head = 46.22 m, Q = 0.330 m^3/s
Pump 2 in pipe 2: Head = 14.77 m, Q = 0.217 m^3/s
Pump 3 in pipe 6: Head = - 30.11 m, Q = 0.147 m^3/s

NODE DATA

NODE	DEMAND Estimate	DEMAND m^3/s	ELEV. m	HEAD M	PRESSURE kPa	HGL ELEV. m
1	0.1	0.080	75.0	42.45	416.2	117.45
2	0.0	0.040	100.0	9.19	90.1	109.19
3	0.0	0.030	90.0	3.80	37.3	93.80
4	0.1	0.050	10.0	30.90	303.0	40.90
5	0.0	0.025	20.0	27.83	272.9	47.83
6	0.0	0.010	5.0	30.01	294.3	35.01

* * *

4.4.7. SYSTEMATIC SOLUTION OF THE ΔQ-EQUATIONS

In this section we describe the development of the computer program SOLDQEQS that is based on the ΔQ-equations and analyzes pipe networks. This program requires the user to specify the initial discharges, Q_{oi}, so they satisfy all of the junction continuity equations, because algorithms that do this automatically involve considerable logic. We will also omit the input that provides the dual description of the network connectivity; instead we will generate the pipe numbers that interconnect the network nodes from the data on the nodes at the ends of the pipes. This generated data will be used to verify that the input Q_{oi} do satisfy the junction continuity equations. Finally, this program will not allow a PRV or any similar device in the network. With this restriction the Jacobian matrix will be symmetric and positive definite, thereby allowing a special linear algebra solver that requires only the upper (or lower) triangular and diagonal elements of the Jacobian to be available during the solution process. This approach provides us a solution variant that could also be used in solving the H-equations by the Newton method.

To describe the computer program logic that forms the ΔQ-equations and the derivatives that form the Jacobian elements, it will be convenient to be able to refer to the equations and the nonzero derivatives with respect to ΔQ from an example. At this time obtain a listing of SOLDQEQS from the CD so it can be studied while you read the rest of this section. The network in Example Problem 4.14 will serve the purpose of illustrating the logic of this program. Since this network contains several pumps, one of which produces a negative head as a turbine, this example will help us incorporate pumps correctly into the code. The two ΔQ-equations for this network are

$$F_1 = K_1(Q_{o1} + \Delta Q_1)^{n_1} - h_{p1} + K_4(Q_{o4} + \Delta Q_1 - \Delta Q_2)^{n_4} - K_5(Q_{o5} - \Delta Q_1 + \Delta Q_2)^{n_5} + K_7(Q_{o7} + \Delta Q_1)^{n_7} - 80 + 20 = 0 \quad (4.71)$$

and

$$F_2 = K_2(Q_{o2} + \Delta Q_2)^{n_2} - h_{p2} + K_3(Q_{o3} + \Delta Q_2)^{n_3} + K_6(Q_{o6} + \Delta Q_2)^{n_6} - h_{p3} + K_5(Q_{o5} - \Delta Q_1 + \Delta Q_2)^{n_5} - K_4(Q_{o4} + \Delta Q_1 - \Delta Q_2)^{n_4} = 0 \quad (4.72)$$

In these equations the head h_{pj} of pump j is described by $h_{pj} = A_j(Q_{oi} \pm \sum \Delta Q_k)^2 + B_j(Q_{oi} \pm \sum \Delta Q_k) + C_j$, and the coefficients A, B, and C are chosen to fit three pairs of points along the pump curve, as before. These energy equations are written around the same loops in which the corrective discharges ΔQ_1 and ΔQ_2 circulate. Therefore, every term in Eq. 4.71 contains a ΔQ_1, and every term in Eq. 4.72 contains a ΔQ_2.

The Jacobian $[\boldsymbol{D}]$ will have two rows, one for each of the two equations, and two columns corresponding to the two unknowns ΔQ_1 and ΔQ_2, or

$$[D] = \begin{bmatrix} \dfrac{\partial F_1}{\partial \Delta Q_1} & \dfrac{\partial F_1}{\partial \Delta Q_2} \\ \dfrac{\partial F_2}{\partial \Delta Q_1} & \dfrac{\partial F_2}{\partial \Delta Q_2} \end{bmatrix} \tag{4.73}$$

in which the individual elements are

$$\begin{aligned} \frac{\partial F_1}{\partial \Delta Q_1} &= n_4 K_4 (Q_{o4} + \Delta Q_1 - \Delta Q_2)^{n_4 - 1} + n_5 K_5 (Q_{o5} - \Delta Q_1 + \Delta Q_2)^{n_5 - 1} \\ &\quad + n_7 K_7 (Q_{o7} + \Delta Q_1)^{n_7 - 1} + n_1 K_1 (Q_{o1} + \Delta Q_1)^{n_1 - 1} - 2A_1 (Q_{o1} + \Delta Q_1) - B_1 \end{aligned} \tag{4.74}$$

$$\frac{\partial F_1}{\partial \Delta Q_2} = \frac{\partial F_2}{\partial \Delta Q_1} = - n_4 K_4 (Q_{o4} + \Delta Q_1 - \Delta Q_2)^{n_4 - 1} - n_5 K_5 (Q_{o5} - \Delta Q_1 + \Delta Q_2)^{n_5 - 1} \tag{4.75}$$

$$\begin{aligned} \frac{\partial F_2}{\partial \Delta Q_2} &= n_2 K_2 (Q_{o2} + \Delta Q_2)^{n_2 - 1} + n_3 K_3 (Q_{o3} + \Delta Q_2)^{n_3 - 1} + n_6 K_6 (Q_{o6} + \Delta Q_2)^{n_6 - 1} \\ &\quad + n_5 K_5 (Q_{o5} - \Delta Q_1 + \Delta Q_2)^{n_5 - 1} + n_4 K_4 (Q_{o4} + \Delta Q_1 - \Delta Q_2)^{n_4 - 1} \\ &\quad - 2A_2 (Q_{o2} + \Delta Q_2) - B_2 - 2A_3 (Q_{o6} + \Delta Q_2) - B_3 \end{aligned} \tag{4.76}$$

To allow for the possibility that one of more flows might change direction and $Q_{oi} \pm \sum \Delta Q_k$ would become negative, the quantities $K_i (Q_{oi} \pm \sum \Delta Q_k)^{n_i}$ will be rewritten as $K_i |(Q_{oi} \pm \sum \Delta Q_k)|^{n_i - 1} (Q_{oi} \pm \sum \Delta Q_k)$. Doing this will be convenient since all factors but the last are also needed to evaluate terms in the derivatives.

For this program we must define the loops around which (1) the energy equations will be written, and (2) each ΔQ circulates. Thus the user must supply the pipe numbers which define each energy loop, with a negative pipe number whenever the direction around the loop opposes the assumed direction of flow in that pipe. This information was also required as input to SOLQEQS. These loop data determine the terms in each equation and the sign of each term. As in SOLQEQS, this data resides in a one-dimensional integer array *IK*, with a pointer *LP* to separate individual loops. The corrective loop discharge data for each pipe is in a similar array *IK1*, with a pointer *LP1* to separate entries between individual pipes. Thus the positions in array *IK1* that will contain information on a corrective loop discharge through pipe *I* will start with subscript (argument of the array) *LP1*(*I*) + 1 and end with subscript *LP1*(*I*+1). Thus *LP1* must have dimension *NP* + 1. In a similar way *LP* must have dimension *NL* + 1 = *NP* - *NJ* + 1.

Let us now examine an algorithm to obtain the corrective loop discharges in each pipe from the loop information. The pipes around the two loops in the example network are

Loop 1: 1 4 - 5 7
Loop 2: 2 3 6 5 - 4

and this data will be stored in *IK* as follows:

IK(1) = 1, *IK*(2) = 4, *IK*(3) = - 5, *IK*(4) = 7,

$IK(5) = 2,\ IK(6) = 3,\ IK(7) = 6,\ IK(8) = 5\ \ IK(9) = -4,$
with $LP(1) = 0,\ LP(2) = 4,$ and $LP(3) = 9.$

Since ΔQ_1 circulates through loop 1 and ΔQ_2 circulates through loop 2, we see that the loop number (the argument of LP) gives the corrective loop discharge through a pipe when the pipe number occurs in the list of IK's for that loop. For example, since pipe 4 is a pipe number in loops 1 and 2, the corrective loop discharges ΔQ_1 and ΔQ_2 both circulate through it, and also ΔQ_1 is in the same direction as the assumed flow in pipe 4 since it is positive in loop 1, whereas ΔQ_2 opposes the assumed flow since it is negative in the list of pipes in loop 2. The number of corrective loop discharges through a pipe is not known in advance, so it is simpler to use a two-dimensional array initially, with the pipe number as the first subscript and the number of corrective loop discharges through that pipe as the second subscript. Hence the second subscript of this array must equal or exceed the maximum number of ΔQ's passing through any pipe, so most of this array space will be unused; once these numbers are known, the information can be transferred into the one-dimensional array *IK1*. Then the two-dimensional array can be deallocated and the memory used for other purposes. An alternative for this array is to EQUIVALENCE it to another array that is not used until later, such as the array for the Jacobian matrix. Figure 4.29 lists Fortran statements that could be used to generate these arrays, with the array *LP1* zeroed before beginning this algorithm.

A very similar algorithm can be used to generate the pipe numbers that join at each node. The essential difference is that the upstream and downstream nodes in the arrays *L1* and *L2* identify the node to which the pipes attach. In program SOLDQEQS this started in the `DO 24` loop. Since we want to verify that the user-supplied initial discharges Q_{oi}

```
      DO 50 I=1,NL
      DO 50 J=LP(I)+1,LP(I+1)
      II=IABS(IK(J))
      NI=LP1(II)+1
      IK2(II,NI)=ISIGN(I,IK(J))
   50 LP1(II)=NI
      NI=0
      NCT=NI
      DO 54 I=1,NP
      DO 53 J=1,LP1(I)
      NI=NI+1
   53 IK1(NI)=IK2(I,J)
      LP1(I)=NCT
   54 NCT=NI
      LP1(NP+1)=NI
```

Figure 4.29 Listing of Fortran code to generate arrays *IK1* and *LP1*.

do satisfy all of the junction continuity equations, this check is performed immediately after the pipes that join at each node are identified. This information makes it easy to obtain the heads H at the nodes after the discharges and head losses in the pipes are computed by using essentially the algorithm that is in SOLQEQS for this purpose.

Now let's see how to obtain the system of ΔQ-equations and the Jacobian that are needed to implement the Newton method. The symmetry that occurs in the Jacobian, if devices such as PRV's do not exist, will be advantageously used, and a one-dimensional array will store the banded portion of the Jacobian. In SOLDQEQS these tasks are accomplished within the outer `DO 90` loop. The index I in this loop tracks the NL loop equations, and the equation values are generated and stored in the array F. The process begins with F(I)=F(I)+FI* ... The columns of the Jacobian matrix are each related to a ΔQ, and these values are placed in the one-dimensional array IK1. The pipe numbers in each loop, which identify the terms that are needed to evaluate the equations and the Jacobian elements, are stored in the one-dimensional array IK. The array LP is a pointer that separates consecutive equations, e.g. loops, in array IK.

In a banded matrix all elements which are displaced more than the band width from the diagonal are zero. If i is the row number and j is the column number, then the band width NBAND is the maximum difference between a nonzero element in any column and its row number, plus one, or

$$\text{NBAND} = |j - i|_{max} + 1 \tag{4.77}$$

In some literature this definition is the half band width since, if the matrix is not symmetric, as many elements must to be stored to the left of the diagonal as to its right. In any symmetric matrix $[\boldsymbol{A}]$ each element $A_{ij} = A_{ji}$. If a two-dimensional array is used in a computer program to store the elements of a banded matrix, the first subscript (for rows) must be at least as large as the number of equations to be solved, and the second subscript must be as least as large as 2NBAND - 1. A special algorithm that properly accounts for the matrix properties is needed to solve a banded matrix problem. If the banded matrix is symmetric, it is not necessary to store all of the elements above and below the diagonal if the solution algorithm accounts for this symmetry. Either the elements above and on the diagonal, or those below and on the diagonal, are all that must be stored.

Program SOLDQEQS uses a one-dimensional array to store the banded elements of the Jacobian and calls a linear algebra subroutine SYMMAT to return the solution to the linear equation system in the array F. Before calling SYMMAT, the upper triangular portion of a banded symmetric Jacobian matrix is stored in a one-dimensional array DJ. In the declaration portion of SOLDQEQS we will find that DJ is a one-dimensional allocatable array with DJ[ALLOCATABLE](:) and that the number of real positions to store values in DJ is allocated with ALLOCATE(DJ(NL*NBAND-MM)), in which NBAND is the band width and MM = NBAND - 1. Thus a preliminary task is to determine the band width before allocating DJ and storing the nonzero derivative values in it. The listing in Fig. 4.30 determines the required band width.

```
C      FINDS BAND WIDTH
       MM=0
       DO 56 I=1,NL
       DO 56 J=LP(I)+1,LP(I+1)
       IJ=IABS(IK(J))
       DO 56 JJ=LP1(IJ)+1,LP1(IJ+1)
       II=IABS(IK1(JJ))-I
       IF(II.GT.MM) MM=II
    56 CONTINUE
       NBAND=MM+1
```

Figure 4.30 Band width algorithm.

The first position in array DJ is the diagonal element in the first row. The diagonal element of the second row is in position (2 - 1)NBAND + 1, the position of the diagonal element in the third row is (3 - 1)NBAND + 1, and in general the diagonal element in the ith row is in position id = (i - 1)NBAND + 1. An alternative formula for locating the diagonal position is id = i NBAND - MM in which MM = NBAND - 1, the number of elements beyond the diagonal. Thus we see that the storage that is needed by DJ is NL*NBAND - MM (NL is the number of equations), as given in the ALLOCATE statement. The position of off-diagonal elements in this one-dimensional array will be the diagonal position id plus the difference between the column number and the row number for the element. In any equation this position is iu = id + (j - i) = id - i + j. Thus in SOLDQEQS the statement after `DO 90 I=1,NL` that is used to define the NL equations is ID=NBAND*I-MM, which locates the position of the diagonal element for each row, and the statement NI = ID - I is an integer which locates the nonzero off-diagonal positions in DJ when the column position is added. Thus the statements that store the values in the proper locations of DJ are

```
      DJ(NI+JJ1)=DJ(NI+JJ1)+FI*FLOAT(IK1(JJ)/JJ1)*DD1
   87 CONTINUE
   90 DJ(ID)=DJ(ID)+DD1
```

The portion of the code, within the `DO 90` loop in program SOLDQEQS, that generates the system of equations and the values for the Jacobian and then obtains the solution that is used as the Newton correction, consists of the lines listed in Fig. 4.31:

```
      DO 90 I=1,NL
      IB=NBAND*I-MM
      NI=IB-I
      II=LP(I)+1
      II1=LP(I+1)
      DO 90 J=II,II1
      IJ=IABS(IK(J))
      IF(I.GE.NRES.OR.J.GT.II) GO TO 83
      IJ1=IABS(IK(II1))
      DO 80 JJ=1,NRES
      IF(IRES(JJ).EQ.IJ) F(I)=F(I)-ELE(JJ)
      IF(IRES(JJ).EQ.IJ1) F(I)=F(I)+ELE(JJ)
   80 CONTINUE
   83 FI=IK(J)/IJ
      QQ=Q(IJ)
      DO 85 JJ=LP1(IJ)+1,LP1(IJ+1)
      JJ1=IABS(IK1(JJ))
   85 QQ=QQ+FLOAT(IK1(JJ)/JJ1)*DQ(JJ1)
      DD=K(IJ)*ABS(QQ)**N(IJ)
      DD1=DD*(N(IJ)+1.)
      IF(L1(IJ).LT.0) THEN
      JJ=1
      DO 86 WHILE (IPUMP(JJ).NE.IJ)
   86 JJ=JJ+1
      DD1=DD1-2.*AP(JJ)*QQ-BP(JJ)
      F(I)=F(I)+FI*(DD*QQ-(AP(JJ)*QQ+BP(JJ))*QQ-CP(JJ))
      ELSE
      F(I)=F(I)+FI*DD*QQ
      ENDIF
      DO 87 JJ=LP1(IJ)+1,LP1(IJ+1)
      JJ1=IABS(IK1(JJ))
      IF(JJ1.LE.I) GO TO 87
      DJ(NI+JJ1)=DJ(NI+JJ1)+FI*FLOAT(IK1(JJ)/JJ1)*DD1
   87 CONTINUE
   90 DJ(IB)=DJ(IB)+DD1
C     SOLVES LINEAR EQUATIONS
      CALL SYMMAT(NL,NBAND,DJ,F)
```

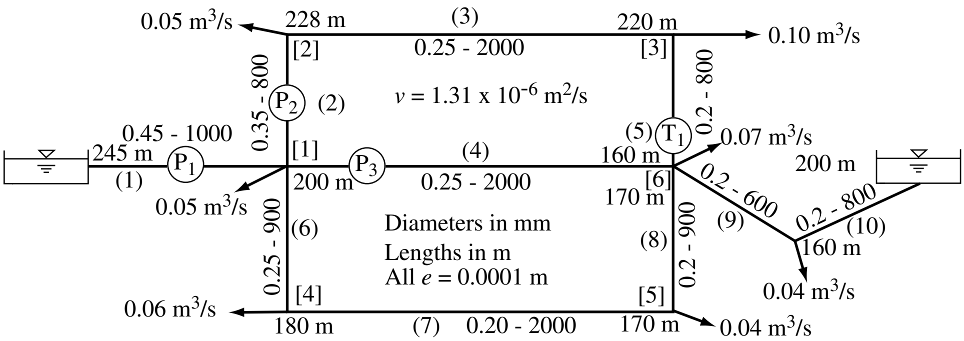
Figure 4.31 The solution algorithm.

To enhance solution efficiency we might try to arrange the equations to reduce the band width as much as possible. Not only will a smaller band width reduce the required amount of computer memory for a solution, but it also reduces the computational effort in solving the linear equation system. As the loop data are developed, the user can reduce the band width of the Jacobian matrix by trying to arrange the ΔQ numbering to be as close as possible to the equation numbering . The band width will equal the maximum difference in any equation between the equation number and the ΔQ number, plus 1.

However, placing this burden on the user is not desirable, especially since a banding algorithm can readily be implemented in computer code that will probably achieve a tighter banding than the user could arrange, even after some attention is given to the order in which equations should be listed and loops formed. One approach to minimizing the band width is described by Jeppson and Davis (1976). This approach is implemented in SOLDQBAN.FOR, which is on the CD. Also on the CD is SOLDQEQ1 that does not use the band width of the Jacobian but instead uses the standard linear algebra solver SOLVEQ, as do SOLQEQS and SOLHEQS, as it solves the ΔQ-equations.

Example Problem 4.15

In the sketch is a network with 10 pipes and 6 nodes which contains three pumps and one turbine. Prepare input data files for SOLQEQS, SOLHEQS and SOLDQEQS so these programs can be used to analyze this network. Use the pairs of (Q, h_p) data in the table to define the pump curves. Then replace the pump curve for pump 1 with the new pump data listed later in the solution, and resolve the problem with all three programs.



Pump 1		Pump 2		Pump 3		Turbine	
Q m³/s	**H** m	**Q** m³/s	**H** m	**Q** m³/s	**H** m	**Q** m³/s	**H** m
0.40	20.0	0.12	16.0	0.06	8.0	0.09	- 8.0
0.42	18.0	0.15	15.0	0.08	7.5	0.10	- 7.5
0.44	15.0	0.18	13.6	0.10	6.8	0.11	- 6.8

Since SI units are used, options must be changed from the default values. The input data file for each of these three programs are listed next, using two columns for each set:

Input Data For SOLQEQS

```
10 7 2 4 2 'U' 1 'D' 2
1 0 1 1000 0.45 0.0001
2 1 2   800 0.35 0.0001
3 2 3 2000 0.25 0.0001
4 1 6 2000 0.25 0.0001
5 3 6   800 0.20 0.0001
6 1 4   900 0.25 0.0001
7 4 5 2000 0.20 0.0001
8 5 6   900 0.20 0.0001
9 6 7   600 0.20 0.0001
10 7 0 800 0.20 0.0001
1 0.05 200 4 1 -2 -4 -6
2 0.05 228 2 2 -3
3 0.10 220 2 3 -5
4 0.06 180 2 6 -7
5 0.04 170 2 7 -8
6 0.07 160 4 4 5 8 -9
7 0.04 160 2 9 -10
1 245
10 200
1 0.40 20 0.42 18 0.44 15
2 0.12 16 0.15 15 0.18 13.6
4 0.06 8 0.08 7.5 0.1 6.8
5 0.09 -8 0.10 -7.5 0.11 -6.8
4 1 4 9 10
4 2 3 5 -4
4 4 -8 -7 -6
```

Input Data For SOLHEQS

```
10 7 2 4
&OPTIONS IU=1,IQ=3,ID=2/
1 0 1 1000 0.45 0.0001
2 1 2  800 0.35 0.0001
3 2 3 2000 0.25 0.0001
4 1 6 2000 0.25 0.0001
5 3 6  800 0.20 0.0001
6 1 4  900 0.25 0.0001
7 4 5 2000 0.20 0.0001
8 5 6  900 0.20 0.0001
9 6 7  600 0.20 0.0001
10 7 0 800 0.20 0.0001
1 0.05 200 4 1 -2 -4 -6
2 0.05 228 2 2 -3
3 0.10 220 2 3 -5
4 0.06 180 2 6 -7
5 0.04 170 2 7 -8
6 0.07 160 4 4 5 8 -9
7 0.04 160 2 9 -10
1 245
10 200
1 0.40 20 0.42 18 0.44 15
2 0.12 16 0.15 15 0.18 13.6
4 0.06  8 0.08 7.5 0.10 6.8
5 0.09 -8 0.10 -7.5 0.11 -6.8
```

Input Data For SOLDQEQS

```
10 7 2 4
&OPTIONS IU=1,IQ=3,ID=2/
1 0 1 1000 0.45 0.0001  0.44
2 1 2 0800 0.35 0.0001  0.20
3 2 3 2000 0.25 0.0001  0.15
4 1 6 2000 0.25 0.0001  0.12
5 3 6 0800 0.20 0.0001  0.05
6 1 4 0900 0.25 0.0001  0.07
7 4 5 2000 0.20 0.0001  0.01
8 5 6 0900 0.20 0.0001 -0.03
9 6 7 0600 0.20 0.0001  0.07
10 7 0 800 0.20 0.0001  0.03
1 0.05 200
2 0.05 228
3 0.10 220
4 0.06 180
5 0.04 170
6 0.07 160
7 0.04 160
1 245
10 200
1 0.4 20 0.42 18 0.44 15
2 0.12 16 0.15 15 0.18 13.6
4 0.06 8.0 0.08 7.5 0.10 6.8
5 0.09 -8.0 0.10 -7.5 0.11 -6.8
4 1 4 9 10
4 2 3 5 -4
4 4 -8 -7 -6
```

The solution tables from SOLQEQS and SOLDQEQS are identical, as shown below. SOLHEQS failed to converge. The failure was caused by the relative inaccuracy of the initial values of the heads that were provided to the Newton method by the automated estimator in the code; the values were too crude in relation to the sensitivity of the code to the way that the three pairs of points for pump 1 define its operating characteristics. If

PIPE DATA

PIPE NO.	NODES FROM	TO	L m	DIA. m	e $\times 10^4$ m	Q m^3/s	VEL. m/s	HEAD LOSS m	HLOSS/ 1000
1	0	1	1000	0.450	1.0	0.436	2.74	12.61	12.61
2	1	2	800	0.350	1.0	0.163	1.70	5.39	6.73
3	2	3	2000	0.250	1.0	0.113	2.31	36.76	18.38
4	1	6	2000	0.250	1.0	0.118	2.40	39.64	19.82
5	3	6	800	0.200	1.0	0.013	0.42	0.74	0.92
6	1	4	900	0.250	1.0	0.105	2.14	14.29	15.88
7	4	5	2000	0.200	1.0	0.045	1.43	19.20	9.60
8	5	6	900	0.200	1.0	0.005	0.16	0.14	0.15
9	6	7	600	0.200	1.0	0.066	2.10	11.92	19.87
10	7	0	800	0.200	1.0	0.026	0.82	2.56	3.20

Pump 1 in pipe 1: Head = 15.71 m, Q = 0.436 m^3/s
Pump 2 in pipe 2: Head = 14.44 m, Q = 0.163 m^3/s
Pump 3 in pipe 4: Head = 6.02 m, Q = 0.118 m^3/s
Pump 4 in pipe 5: Head = - 5.17 m, Q = 0.013 m^3/s

NODE DATA

NODE	D E M	A N D	ELEV.	HEAD	PRESSURE	HGL ELEV.
	Estimate	m^3/s	m	m	kPa	M
1	0.1	0.050	200.0	48.10	471.7	248.10
2	0.1	0.050	228.0	29.15	285.8	257.15
3	0.1	0.100	220.0	0.39	3.8	220.39
4	0.1	0.060	180.0	53.81	527.7	233.81
5	0.0	0.040	170.0	44.61	437.5	214.61
6	0.1	0.070	160.0	48.46	475.2	208.46
7	0.0	0.040	160.0	42.56	417.3	202.56

this pump curve is plotted, we see immediately how the curve turns steeply downward outside each end of the given data. Using pump curves of this nature should be avoided. To obtain a solution from SOLHEQS, either the points that define the pump curve must be adjusted, or the code must be modified so the user can supply the initial estimates of the heads for the Newton method. If the pump curve for pump 1 is modified so the three discharge-head data pairs are (0.40, 16.0), (0.43, 15.8), and (0.46, 15.5), then SOLHEQS can solve the modified problem.

* * *

4.5 CONCLUDING REMARKS

This chapter concentrated on the analysis of pipeline networks. The first area of emphasis was on the development of the three kinds of systems of equations to describe mathematically the flow in a pipe network, first for simpler networks and then for networks which contain pumps or turbines and loss-producing devices such as a pressure reducing valve or a back pressure valve. The Newton method for the solution of these equation systems was introduced and later included in computer programs to solve the equation systems. Later sections of the chapter developed solution routines and implemented them for each of the three types of equation systems.

There are features that a production network program would usually include that these programs do not have. For example, rather than requiring the user to provide a set of estimated initial discharges Q_{oi} that satisfy all of the junction continuity equations, the program should develop these values. One way to create these values is to reduce the network to a branched system by deleting some pipes with smaller diameters and then using the methods in this chapter to obtain a solution for the branched network.

Another burden that would not be placed on the user of a production program is the need to supply the pipe numbers that define the loops around which the energy equations are written and the corrective loop discharges circulate. An algorithm for this task must satisfy two criteria: (a) the pipes that define any loop should be minimum in number, and also (b) these loops must lead to the creation of energy equations that are independent so that none of the equations are a combination of any group of the other equations. The first criterion can be achieved by using a "minimum path algorithm," and the second criterion requires each new loop to contain at least one pipe that does not exist in any of the previous loops.

Production programs will also take full advantage of the sparsity of the Jacobian matrix in computing network solutions in an efficient manner.

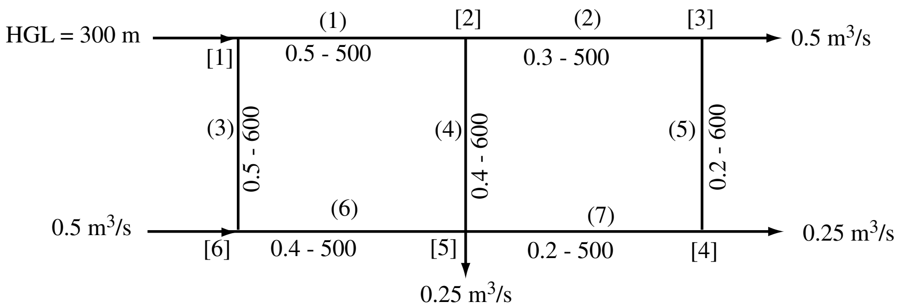
Network solvers can also allow the user to obtain time-dependent solutions. Such solutions, which do not account for the forces that are required to accelerate or decelerate the fluid columns, have become known as "extended time simulations." To develop an extended time simulation, additional information of several kinds is needed, such as demand functions which describe how one or several external nodal demands QJ vary with time, rules based on pressures at nodes or on water surface elevations in tanks or reservoirs can determine how many pumps should operate in series or parallel, and storage-elevation-capacity curves can be used to describe the behavior of tanks, and so on.

The use of programs for network analysis can also allow designers to obtain answers for the many questions that naturally occur during the design process. For example, what head and capacity should a pump produce to maintain a prescribed pressure and/or discharge at the far end of the network? What is the discharge from a junction if the pressure is known from a measurement there? How much head should a PRV dissipate so the pressure does not exceed a set value? How much flow can be obtained from a fire hydrant if its discharge characteristics are known? What are the flows from sprinkler heads if their sizes are known? How does a contaminant spread through a pipe network if it is accidentally introduced at a point?

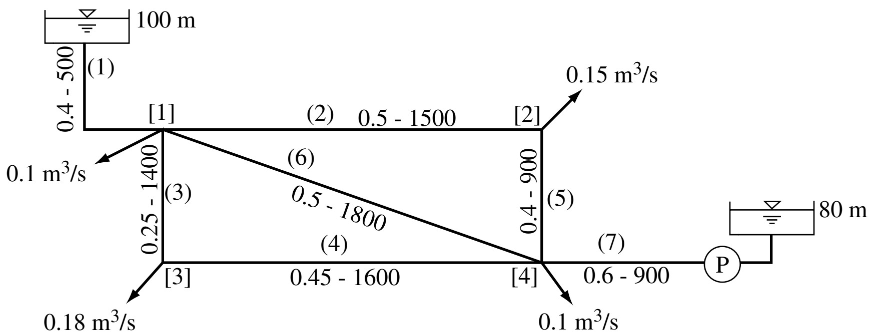
Chapter 5 will explore the design of these pipe networks, and Chapter 6 will examine further several topics, including extended-time simulations.

4.6 PROBLEMS

4.1 For the two pipe networks shown below, write the system of Q-equations. In writing these equations, use K and n with subscripts that correspond to the pipe number.



Diameters in mm
Lengths in m
All e's for both networks = 0.00002 m

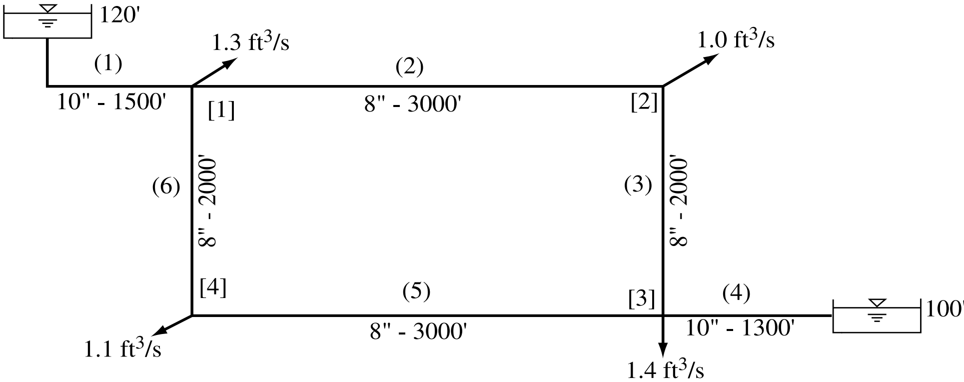


Pump

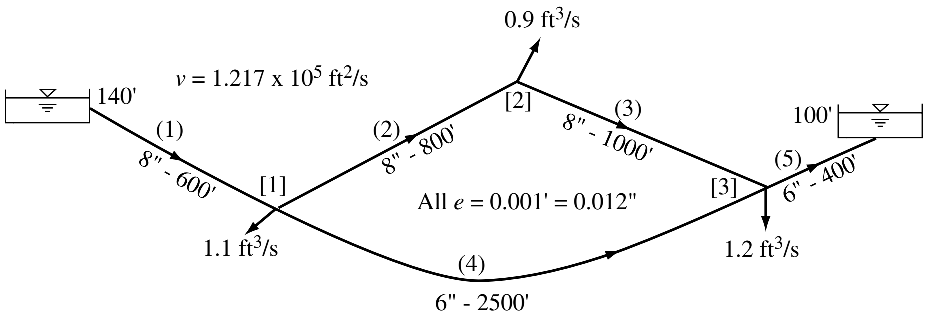
Q m³/s	h_p m
0.2	30
0.4	27
0.7	21

4.2 Write the system of Q-equations for the network shown. It is not necessary to substitute the values of K and n from the table into the equations; instead use K_i and n_i where i is the pipe number. If the discharge in pipe 1 is $Q_1 = 3.1$ ft^3/s, then what is the friction factor f for this pipe?

Pipe	K	n
1	1.841	1.928
2	11.47	1.871
3	7.47	1.839
4	1.615	1.914
5	11.08	1.828
6	7.69	1.884

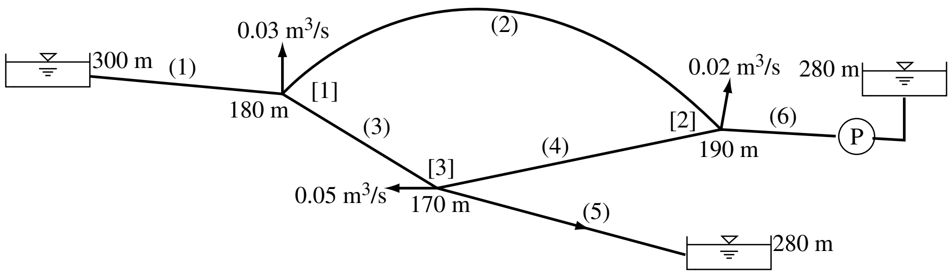


4.3 A 5-pipe, 3-node network appears below. On this diagram the first number along each line is the pipe diameter in inches, and the second number is the pipe length in feet. All pipes have an equivalent sand roughness $e = 0.001$ ft $= 0.012$ inches. Do the following: (a) compute the values of K and n in $h_f = KQ^n$ for pipe 1, based on the Darcy-Weisbach equation, and (b) write the system of Q-equations for this network. (Use subscripts on K, n and Q corresponding to the pipe number.)



Pipe	K	n
1		
2	3.53	1.961
3	4.44	1.929
4	48.6	1.934
5	6.40	1.817

4.4 In the sketch the network consists of 6 pipes and 3 nodes. A source pump and one reservoir supply the network, and the lower reservoir receives water. Do the following tasks: (a) write the system of Q-equations; (b) write the system of H-equations; (c) write the system of ΔQ-equations; (d) if the discharge in pipe 5 is $Q_5 = 0.026$ m^3/s into the reservoir, what is the elevation of the HGL at node 3; (e) if the discharge in pipe 6 is $Q_6 = 0.112$ m^3/s, what are the HGL and pressure at node 2?

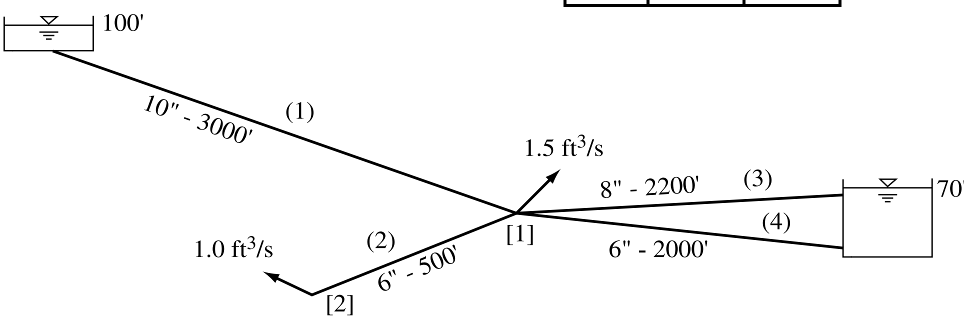


Pipe	Dia. m	Length m	K	n
1	0.30	1000	543	1.886
2	0.20	2500	13700	1.946
3	0.20	1000	3270	1.839
4	0.30	1500	1077	1.965
5	0.15	1000	27400	1.974
6	0.35	800	260	1.968

Pt.	Q m^3/s	h_p m
1	0.05	35
2	0.10	31
3	0.15	24

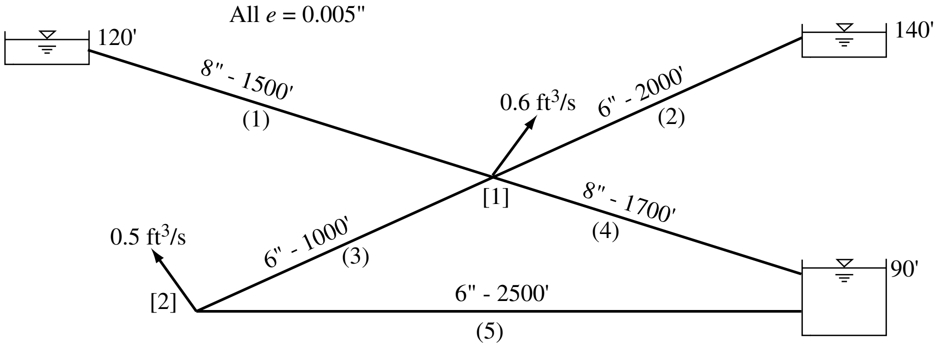
4.5 For the network below: (a) write the Q-equations; (b) write the H-equations; and (c) write the ΔQ-equations. (Use the symbols K and n with correct subscripts for the pipes in the equations.)

Pipe	K	n
1	3.59	1.922
2	7.97	1.917
3	7.94	1.821
4	28.80	1.809



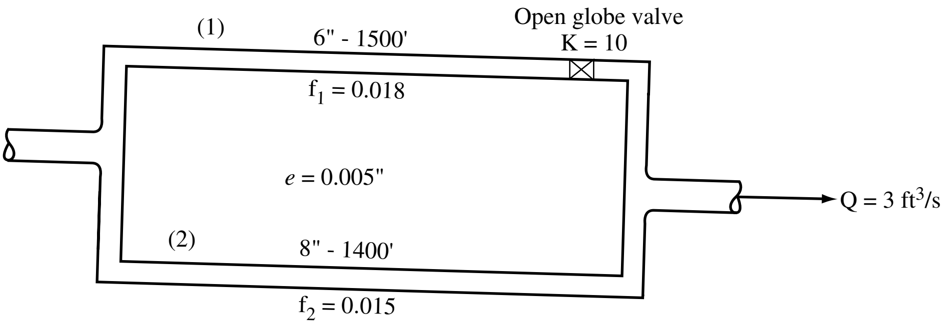
4.6 For the network shown, write (a) the Q-equations, (b) the H-equations, and (c) the ΔQ-equations. The K and n values for the pipes in this network are given in the table which follows. (Your equations should contain only numerical values and unknowns.)

Pipe	D in.	L ft.	K	n
1	8	1500	5.72	1.930
2	6	2000	33.00	1.931
3	6	1000	16.30	1.889
4	8	1700	6.53	1.913
5	6	2500	40.70	1.890

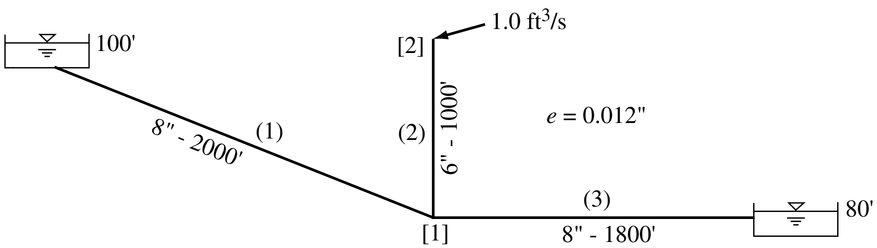


4.7 Prepare the input data for, and obtain the solution from, NETWK for the network described in Problem 4.6.

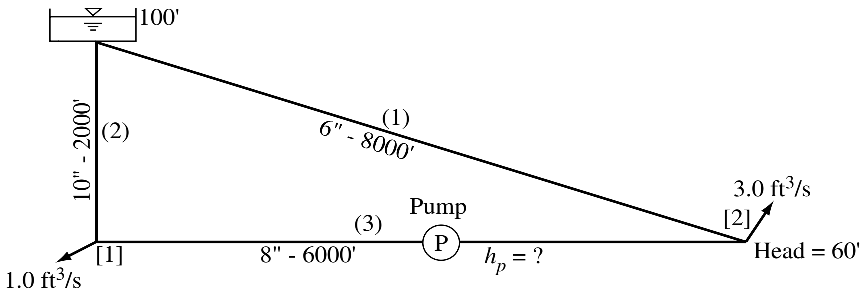
4.8 A pipe branches into a 6-in diameter, 1500-ft long pipe and a 8-in diameter, 1400-ft long pipe and then rejoins, so the two pipes are in parallel. Pipe 1 contains an open globe valve with a local loss coefficient $K = 10$. If the total discharge is $Q = 3$ ft^3/s, determine the discharges Q_1 and Q_2 in the individual pipes. For simplicity, we shall assume $f_1 = 0.018$ and $f_2 = 0.015$.



4.9 This sketch of a small water system shows two reservoirs, with a pipe connected to the center node with an inflow of 1.0 ft^3/s at the other end. Set up the three equation systems that could be used to solve this problem, and then obtain a solution by using one of them.

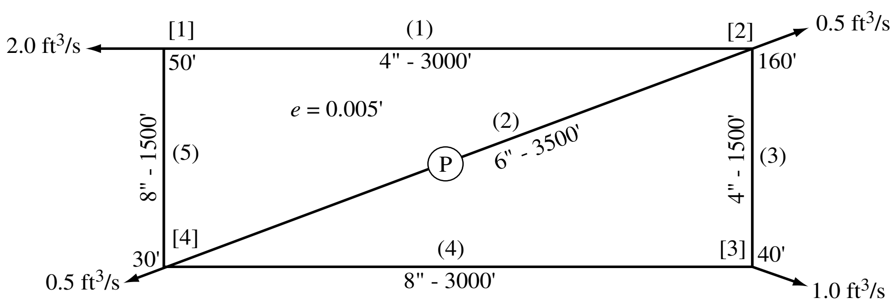


4.10 In the diagram three pipes that form a triangle are supplied by a reservoir at one vertex of the triangle, and demands of 1.0 and 3.0 ft^3/s are found at the other two vertices. A booster pump exists in pipe 3. What head should the pump in pipe 3 produce so the pressure at node 2 causes an HGL of 60 ft there. The ground elevation is everywhere 0 ft. You may solve this problem by using any, or all, of the equation systems that are available. Assume $e = 0.012$ in and $\nu = 1.41 \times 10^{-5}$ ft^2/s.



4.11 In the network of Problem 4.10 the diameters of pipes 2 and 3 have been enlarged to 12 inches. In place of the pump a pressure reduction valve is now needed in pipe 3 to create a pressure head at node 2 of 60 ft. Determine the required setting, i.e. HGL, of the pressure reduction valve. You can use any of the equation systems.

4.12 A network is shown in the diagram. Write the system of ΔQ-equations for this network and complete one Newton iteration toward a solution of the problem. Verify this result by using an application software package such as MathCAD or TK-Solver.



Pump Characteristics

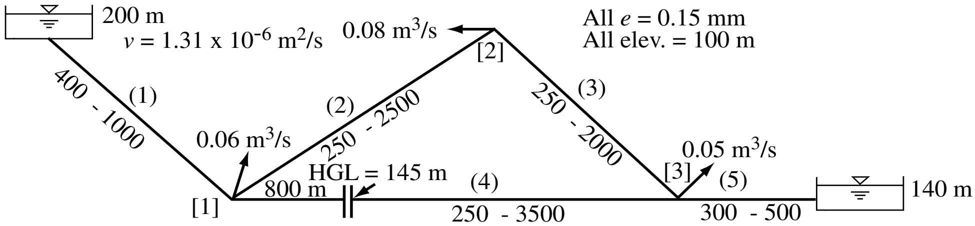
Q ft^3/s	h_p ft
0.3	40
0.5	35
0.8	28

Pipe	K	n
1	375.0	1.860
2	57.4	1.902
3	190.0	1.898
4	11.5	1.880
5	5.72	1.930

4.13 Write the system of H-equations for the two networks in Problem 4.1.

4.14 Write the system of ΔQ-equations for the two networks in Problem 4.1.

4.15 The following network contains a pressure reducing valve (PRV) that is set so it will produce a HGL of 145 m on its downstream side. This valve is 800 m downstream from node 1. Do the following: (a) write the system of Q-equations; (b) write the system of H-equations; (c) write the system of ΔQ-equations; (d) using the Newton method, solve the system of ΔQ-equations; (e) and what is the HGL on the upstream side of the PRV?



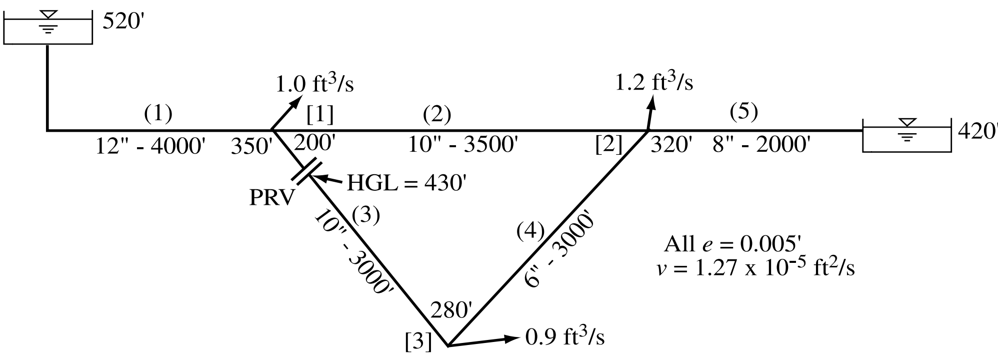
Pipe	1	2	3	4	5
K	196	3520	2380	4130	192
n	1.819	1.955	1.895	1.892	1.834

4.16 The reservoir water surface elevation at the beginning of pipe 1 in Problem 4.15 is lowered by 50 m so it is WS_1 = 150 m, and a pump with the characteristics given below is installed in pipe 1. Write the three equation systems and solve one of them, also finding the HGL elevation on the upstream side of the PRV.

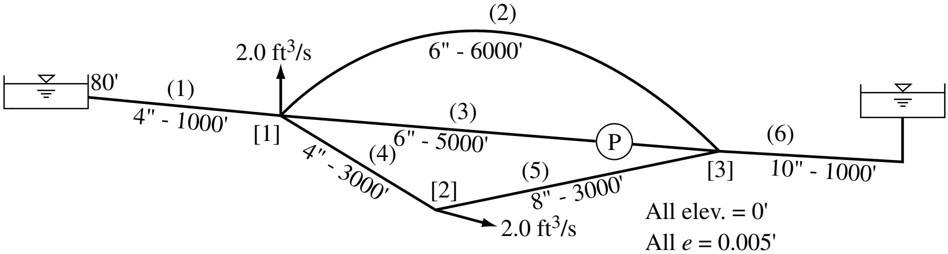
Q m³/s	h_p m
0.18	55
0.22	51
0.26	44

4.17 For the network shown below write (a) the Q-equations, (b) the H-equations, and (c) the ΔQ-equations. Pipe 3 contains a pressure reduction value 200 ft downstream from node 1 that is set to maintain an HGL = 430 ft on its discharge side. Use the notation K_i and n_i in these equations.

Pipe	K	n
1	1.93	1.935
2	4.44	1.940
3	3.50	1.840
4	47.90	1.866
5	7.67	1.917



4.18 For the network below: (a) write the Q-equations; (b) write the H-equations; (c) write the ΔQ-equations; and (d) solve the system of ΔQ-equations. The water surface elevation of the right reservoir is 300 ft, and the following three (Q, h_p) pairs define the pump characteristic curve: (1.0, 26), (1.5, 24), (2.2, 20).



4.19 Solution tables from NETWK follow, with four values omitted. Fill in the missing values. What head drop occurs across the PRV? What horsepower does this loss represent?

PIPE DATA

PIPE NO.	NODES FROM	TO	L	DIA.	e x10^3	Q	VEL.	HEAD LOSS	HLOSS/1000
			ft.	in	in	ft^3/s	ft/s	ft.	
1	0	1	4000	12.0	5.0	4.72	6.01		9.69
2	1	2	3500	10.0	5.0	3.26	5.97	41.87	11.96
3	1	3	3000	10.0	5.0	0.46	0.85	0.91	0.30
4	2	3	3000	6.0	5.0	0.44	2.22	10.21	3.40
5	2	0	2000	8.0	5.0	1.62	4.64	19.36	9.68

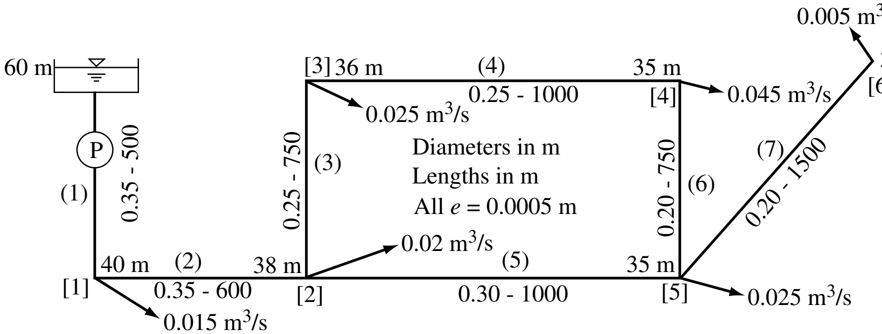
AVE. VEL. = 3.94 ft/s, AVE. HL/1000 = 7.01, MAX. VEL. = 6.01 ft/s, MIN. VEL. = 0.85 ft/s

NODE DATA

NODE	DEMAND ft^3/s	DEMAND gal/min	ELEV. ft.	HEAD ft.	PRESSURE lb/in^2	HGL ELEV. ft.
1	1.0	449	350	131.2	56.87	481.2
2	1.2	539	320	119.4	51.72	439.4
3	0.9	404	280			

AVE. HEAD = 133.3 ft, AVE. HGL = 450.0 ft
MAX. HEAD = 149.2 ft, MIN. HEAD = 119.4 ft

4.20 For the network shown: (a) write the Q-equations; (b) write the H-equations; (c) write the ΔQ-equations; and (d) solve the ΔQ-equation system.



Q m^3/s	h_p m
0.120	40
0.140	38
0.165	35

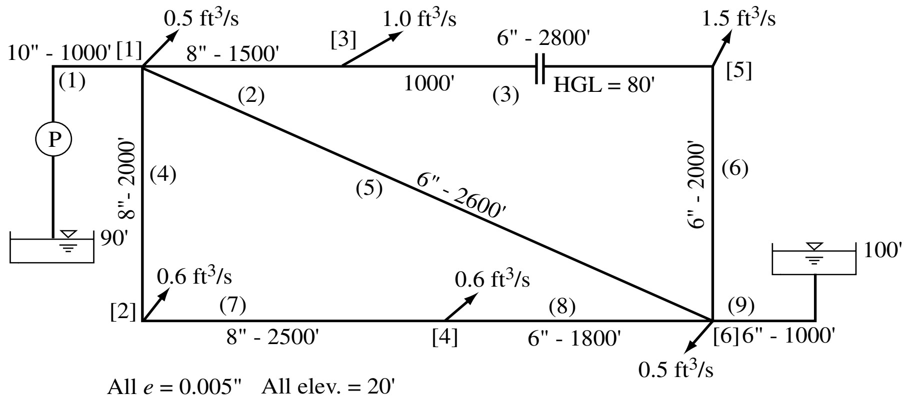
4.21 For the two networks in Problem 4.1, solve the Q-equation system using the Newton method.

4.22 For the two networks in Problem 4.1, solve the H-equation system using the Newton method.

4.23 For the two networks in Problem 4.1, solve the ΔQ-equation system using the Newton method.

4.24 Determine the pressures in lb/in^2 at the six nodes of Problem 4.1a.

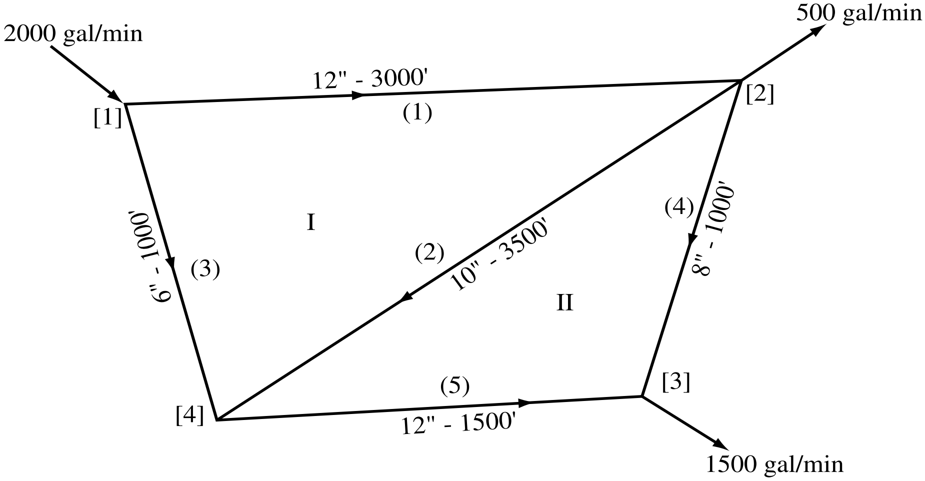
4.25 For the network below, write the ΔQ-equation system and solve them, and verify your solution by obtaining a computer solution by using NETWK.



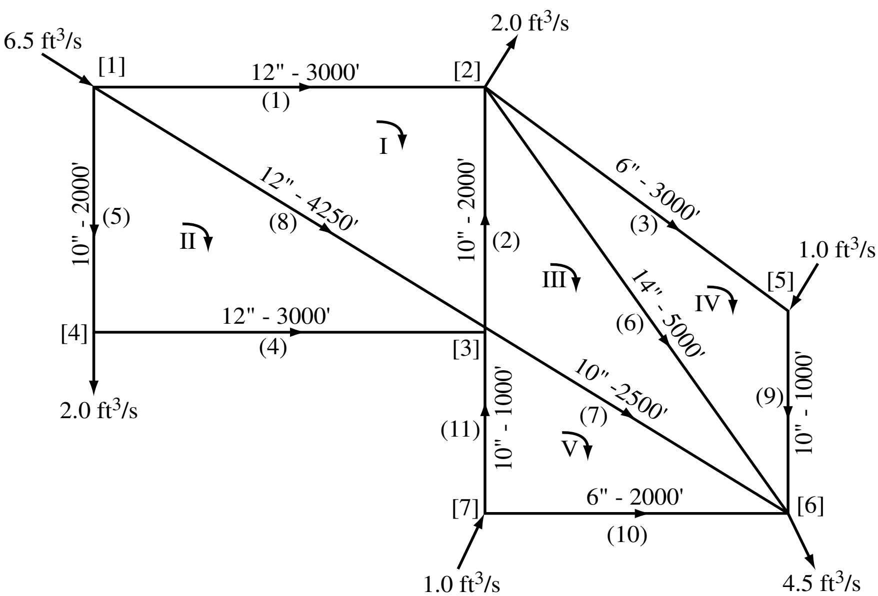
Q ft^3/s	h_p ft
1.0	50
2.0	48
3.0	45

4.26 Determine the discharge and head loss in each pipe of the networks shown on the following pages by first determining a set of values for the coefficients K and n; then setting up and solving the equations without using a computer, except perhaps to solve each linear system of equations that is formed as a part of the Newton method.

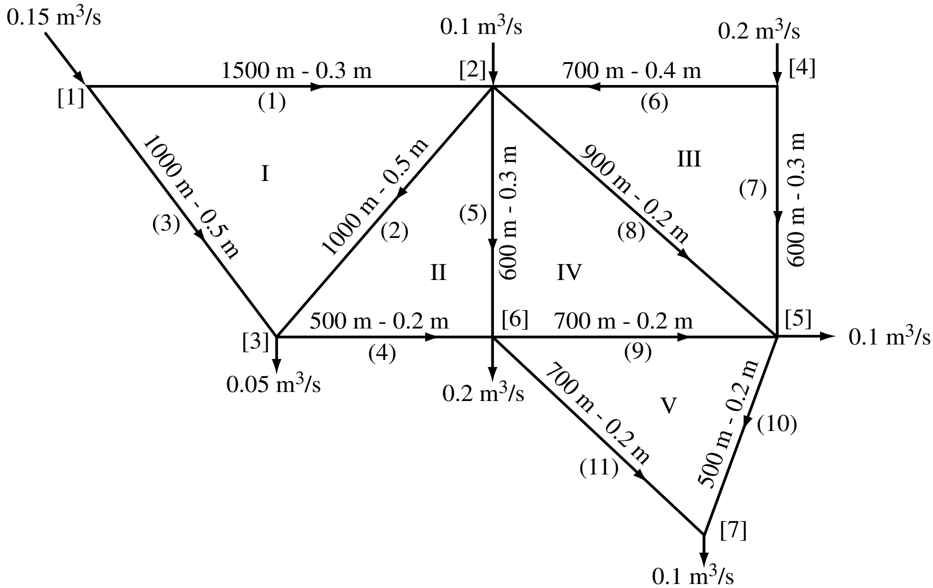
(a) Analyze this network with the Hazen-Williams equation and $C_{HW} = 120$ for all pipes.



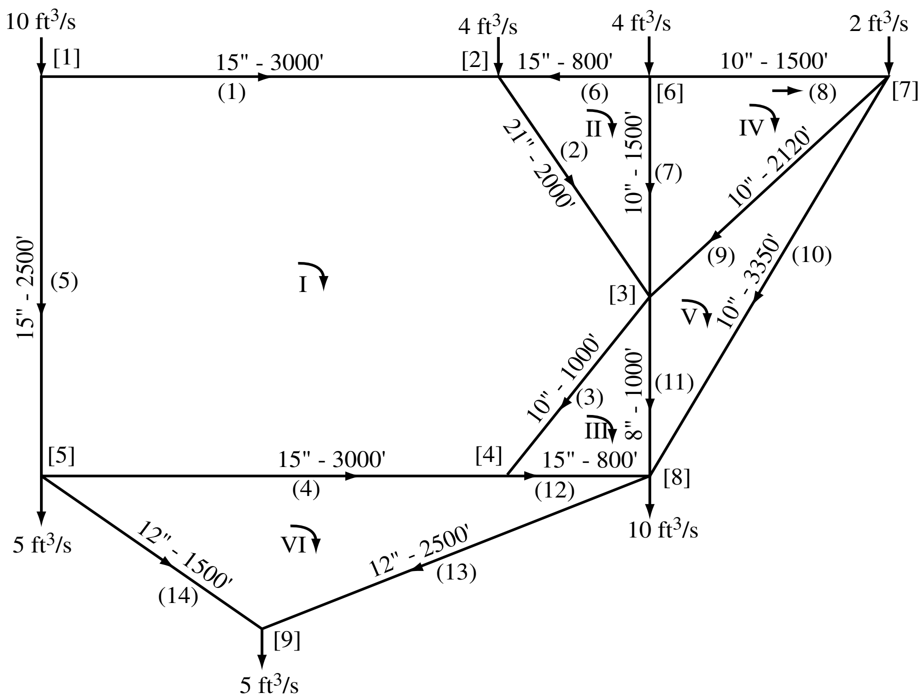
(b) Analyze this network by using the Hazen-Williams equation; for pipes 1 through 5 use $C_{HW} = 120$, and for pipes 6 through 11 use $C_{HW} = 100$.



(c) Use the Hazen-Williams equation to analyze this network; all pipes are cast iron.



(d) Analyze this network with the Darcy-Weisbach equation, assuming $e = 0.001$ ft for the 8-in and 10-in pipes and $e = 0.0005$ ft for all other pipes.

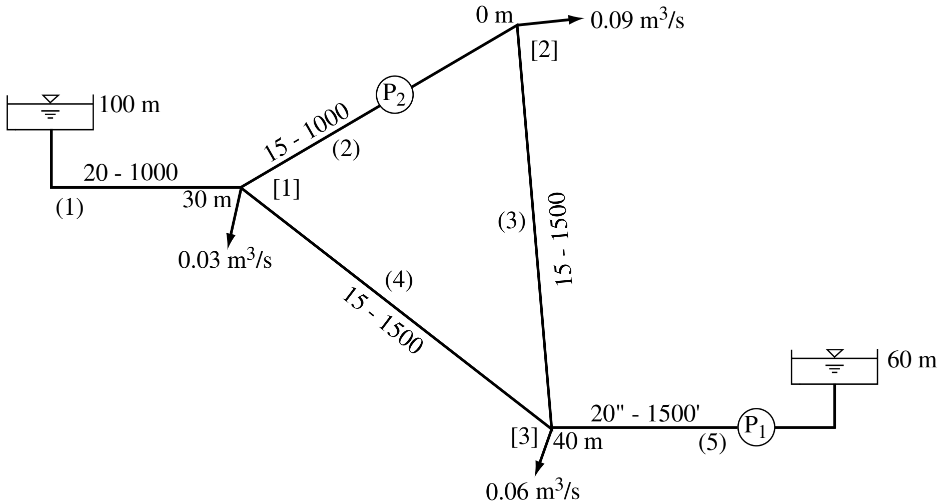


(e) Analyze the network of part (a) by using the Darcy-Weisbach equation with a roughness of $e = 0.0005$ ft for all pipes.

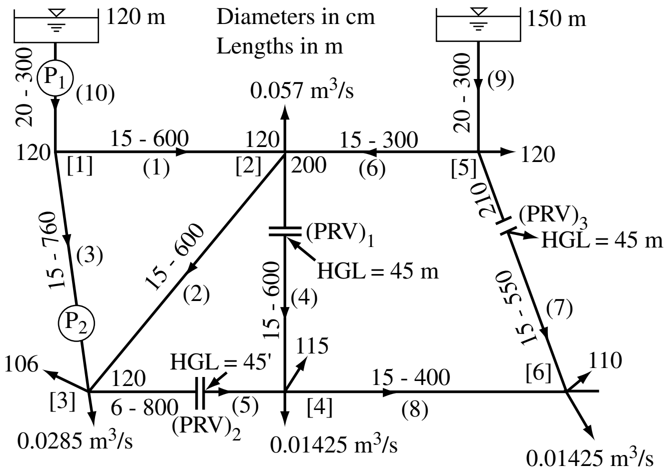
(f) Analyze the network of part (b) by using the Darcy-Weisbach equation; for pipes 1 through 5 use $e = 0.005$ ft, and for pipes 6 through 11 use $e = 0.006$ ft.

(g) Use the Hazen-Williams equation to analyze this network; all pipes are made of cast iron. The diameters are given in centimeters; lengths and elevations are in meters. Pump performance data are listed in the table.

Pump 1		Pump 2	
Q m^3/s	h_p m	Q m^3/s	h_p m
0.10	50	0.05	10
0.15	48	0.10	8
0.20	44	0.15	5



(h) Use the Hazen-Williams equation to analyze this network that is diagrammed atop the next page; all pipes are made of cast iron. The diameters are given in centimeters; lengths and elevations are in meters. Pump performance data are listed in the table.



Pump 1		Pump 2	
Q m^3/s	h_p m	Q m^3/s	h_p m
0.0425	36.6	0.0283	12.2
0.0708	30.5	0.0425	10.7
0.0991	22.9	0.0566	8.5

4.27 Only the Colebrook-White equation is used in subroutine COMPK_N that determines the values of K and n in the exponential formula for programs SOLQEQS, SOLQEQS and SOLDQEQS. Modify this subroutine so it will allow laminar flow in the pipe. Also modify the subroutine so the discharge can become zero; e.g., this might commonly occur for initial discharges Q_{oi} for use with the ΔQ-equations.

4.28 Assume that the flow in all pipes will always be turbulent; however, a user might select initial values for Q_{oi} that are zero when solving the ΔQ-equations. Then the subroutine COMPK_N would fail, as it is now written in SOLQEQS, SOLHEQS and SOLDQEQS. Modify COMPK_N so it can accept a value of zero for the discharge.

4.29 Modify SOLQEQS so an option allows the user to supply starting values for Q that will be used in the Newton method rather than generating these values internally.

4.30 Modify SOLHEQS so it has an option that allows the user to supply initial values for H that will be used in the Newton method rather than generating these values internally. The additional input could be supplied from another read statement, or these heads could be listed after the nodal elevations in the node data.

4.31 Modify SOLDQEQS so it will allow PRV's to exist in the network. This change will require two sets of loop data to be read as input data (unless you wish to obtain these loops internally), one around which the ΔQ's circulate, and one around which the energy equations are written. Since the two sets of loops will not be identical, the Jacobian will not be symmetric.

4.32 Modify SOLHEQS and/or SOLDQEQS so it calls a symmetric matrix solver such as SYMMAT.

4.33 SOLQEQS, SOLHEQS and SOLDQEQS can all analyze networks that contain local losses if the user will provide the actual length of the pipe and the additional length of pipe that would cause a frictional pipe loss that is equivalent to what the local loss device would cause. Modify one, or all, of these programs so each equivalent pipe length is computed internally within the program and then added to the actual length before the problem is solved.

4.34 Rather than compute an equivalent length of pipe for a local loss, as in Problem 4.33, modify SOLQEQS so that local losses, where they occur, are treated by adding a head loss term of the form $h_L = KQ^2/(2gA^2)$ to the energy loop equation.

4.35 Repeat Problem 4.34, but modify SOLDQEQS.

4.36 Use SOLQEQS, SOLHEQS, and SOLDQEQS to analyze the network depicted in Problem 4.20.

4.37 Use SOLQEQS to analyze the network in Problem 4.25. This network contains a PRV in pipe 3 that is located 1000 ft downstream from the beginning of this pipe.

4.38 SOLQEQS, SOLHEQS and SOLDQEQS all represent pump performance by fitting three (Q, h_p) pairs of pump characteristic curve data with a second-order polynomial. Modify one or all of these programs so they accept the normal capacity (discharge at best efficiency) and head at this discharge as input, and then the relation between h_p and Q is obtained from the power equation $P = \gamma Q h_p$ under the assumption that the power P remains constant.

4.39 Modify the program that was developed in Problem 4.38 so the efficiency of the pump is a linear function of the difference of the discharge from its normal capacity.

4.40 Place a PRV in pipe 2 of the network in Example Problem 4.5 with a pressure setting of HGL = 445 ft. Obtain a solution for this network using SOLQEQS. Verify this solution using NETWK.

4.41 SOLQEQS contains a code segment that cross checks the connectivity of the network by looking at the two node numbers at the ends of a pipe and at the pipe numbers that join at a junction. It also checks that upstream node numbers are negative and that downstream node numbers are positive. But the algorithm currently can not determine whether an extra pipe might be connected to a node. Modify the code so a check can identify any extra pipe(s) that might be specified in the data that lists the pipes that are connected to nodes.

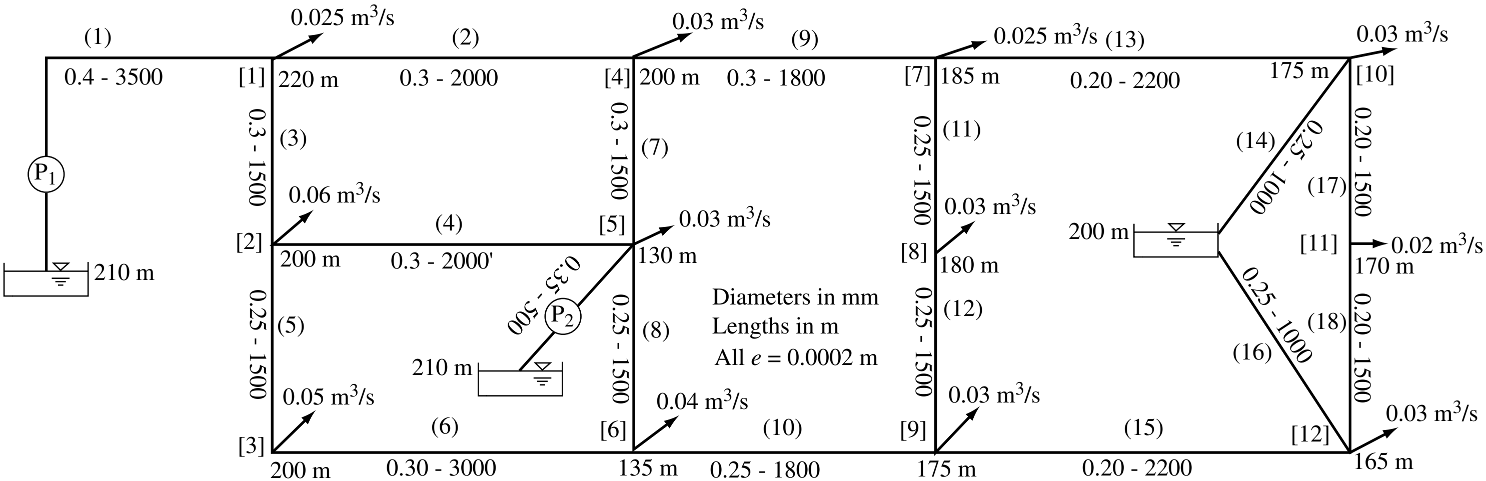
4.42 Modify SOLDQEQS so PRV's can exist in the network. Now two separate kinds of loops will exist, those around which the corrective loop discharges circulate and those around which the energy equations are written. Therefore two sets of loop data must be included in the input data file.

4.43 Use the resulting computer program from Problem 4.42 to obtain a solution to the network in Example Problem 4.5 with a PRV in pipe 2 having a pressure setting that causes the downstream head to be HGL = 445 ft. Verify this solution by (1) using NETWK and by (2) changing subroutine FUNCT in program EQUSOL1.

4.44 The network diagram below lists average demands on it. The storage tank that is connected to the network by pipes 14 and 16 has a 20-m diameter; at 6 a.m. its water surface elevation should be 200 m. The demands at all nodes change according to the peaking factors in the table. The pump characteristics represent two pumps in parallel at each location. Obtain a series of solutions for the times at which the peaking factors are given. For each solution of this series determine the new water level in the tank and the electrical energy consumed by each pump during the latest time increment. Suggest when one pump at each station should be shut off. What might be done to improve the design and thereby the operation of the system?

Time	6 a.m.	9 a.m.	12 Noon	4 p.m.	7 p.m.	10 p.m.	12 Mid.	3 a.m.
PF	1.0	1.8	1.3	1.3	1.7	1.5	0.6	0.2

Pump 1		**Pump 2**	
Q m^3/s	h_p m	Q m^3/s	h_p m
0.15	50	0.20	30
0.25	47	0.25	28
0.35	42	0.30	25



(1)
0.4 - 3500
P_1
210 m
0.025 m^3/s
(2)
[1] 220 m
0.3 - 2000
0.3 - 1500
(3)
0.06 m^3/s
[2] 200 m
(4)
0.3 - 2000'
0.25 - 1500
(5)
0.05 m^3/s
[3] 200 m
(6)
0.30 - 3000
0.03 m^3/s
[4] 200 m
0.3 - 1500
(7)
[5] 130 m
0.03 m^3/s
0.35 - 500
P_2
210 m
0.25 - 1500
(8)
[6] 135 m
0.04 m^3/s
(9)
0.3 - 1800
(10)
0.25 - 1800
Diameters in mm
Lengths in m
All e = 0.0002 m
0.025 m^3/s
[7] 185 m
0.25 - 1500
(11)
0.03 m^3/s
[8] 180 m
0.25 - 1500
(12)
0.03 m^3/s
[9] 175 m
(13)
0.20 - 2200
175 m
(14)
0.25 - 1000
200 m
0.25 - 1000
(16)
(15)
0.20 - 2200
0.03 m^3/s
[10]
0.20 - 1500
(17)
[11]
0.02 m^3/s
170 m
0.20 - 1500
(18)
[12]
0.03 m^3/s
165 m

CHAPTER 5

DESIGN OF PIPE NETWORKS

5.1 INTRODUCTION

When dealing with problems associated with pipelines (or pipe networks) for which all diameters, lengths, roughness coefficients, and demands are known, then the nodal HGL elevation, or *H*'s, and pipe discharges are the unknown quantities to be found. Problems of this nature are classified as analysis problems since a known piping system is being ana-lyzed for a given demand pattern. Chapter 4 dealt with the analysis of networks. In an analysis problem for a network, the demands at all nodes of the network are specified, and the elevation of the HGL is known at one or more positions (where reservoirs exist), and the solution seeks to find the discharges (and head losses) in all pipes, and the HGL elevation, head, and pressure at each node in the network.

The focus of this chapter is on the design of pipe networks, which most frequently means that the pipe diameters are unknown and are to be determined. A brief introduction to design problems was presented in Chapter 4, where the equations for mass and energy conservation were used in determining any desired variables associated with the problem. This chapter will greatly expand upon these principles, but we start with a single pipe.

5.1.1. SOLVING FOR PIPE DIAMETERS

A typical design problem consists of sizing, i.e., determining the size of, as many pipes as the equations allow to meet specified pressures and discharges throughout the network. For such design problems the pressures at all nodes, the heads at all nodes, or the HGL elevations are typically specified. (Knowing any one of these allows the others to be com-puted if the nodal elevations are known.) In addition to finding pipe diameters, one might want to determine the heads that pumps must produce to satisfy the specified pressures.

Consider a single pipe that conveys water from a reservoir with a known water surface elevation H_1 to another reservoir with a known water surface elevation H_2, as shown in Fig. 5.1, as the simplest possible design problem. For this case there is one unknown diameter D, a known length L, and a known roughness e. The problem is to determine the smallest pipe diameter that will convey the known discharge Q between the two reservoirs.

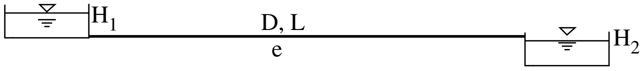


Figure 5.1 A simple two-reservoir design problem.

5.1.2. SOLUTION BASED ON THE DARCY-WEISBACH EQUATION

The Darcy-Weisbach equation will be used here to describe the head loss in a pipe as a function of the discharge in that pipe. The next section will base solution procedures on the Hazen-Williams equation. We recall the Darcy-Weisbach equation

$$h_f = f\frac{L}{D}\frac{V^2}{2g} = f\frac{L}{D}\frac{Q^2}{2gA^2} \tag{5.1}$$

in which h_f is the head loss due to friction in units of energy per unit weight, i.e., a length, the friction factor f is in general a function of the Reynolds number and the relative roughness e/D of the pipe, and the cross-sectional area of the pipe is $A = \pi D^2/4$. Since nearly all water flows are in the transitional zone of the Moody diagram, the behavior of the friction factor can be defined by the implicit Colebrook-White equation in the form

$$\frac{1}{\sqrt{f}} = 1.14 - 2Log_{10}\left[\frac{e}{D} + \frac{9.35}{Re\sqrt{f}}\right] = 1.14 - 2Log_{10}\left[\frac{e}{D} + \frac{7.3434728\,\nu D}{Q\sqrt{f}}\right] \tag{5.2}$$

in which $Re = VD/\nu = 4Q/(\pi\nu D) = 1.27324\ Q/(\nu D)$ is the Reynolds number. Since Eq. 5.2 merges into the equation that describes the wholly rough zone on the Moody diagram well, and it also merges into the equation that describes hydraulically smooth flow, it will be used whenever the flow is turbulent. If the flow is laminar with Re below 2100, then Eq. 5.2 must be replaced by

$$f = 64/Re = 64\nu/(VD) = 81.487\nu D/Q \tag{5.3}$$

The basic problem that seeks to determine a diameter now requires that Eqs. 5.1 and 5.2 (or possibly Eq. 5.3) be solved simultaneously for the two unknowns D and f. Several methods will be applied to obtain a simultaneous solution of these equations. These methods will be implemented in the computer programs DIAPIP, DIAPIPA, DIAPIP2, and DIAPIP3. The reader will benefit most by printing a copy of these programs now and consulting the listings as the methods are described.

The first method uses the Newton method to solve simultaneously the Darcy-Weisbach and Colebrook-White equations for D and f. This approach is similar to that used in program DW_CW in Chapter 4, with the difference that D is chosen to be the second unknown in place of some other variable of the problem. In solving Eqs. 5.1 and 5.2 simultaneously by the Newton method, we first rewrite the original equations in the generic form $F(D, f) = 0$. One way of rewriting these equations is as follows:

$$F_1(D, f) = \frac{1}{\sqrt{f}} - 1.14 + 2Log_{10}\left[\frac{e}{D} + \frac{7.3434728\,\nu D}{Q\sqrt{f}}\right] = 0 \tag{5.2a}$$

$$F_2(D, f) = h_f - f\frac{L}{D}\frac{Q^2}{2gA^2} = 0 \tag{5.1a}$$

The Jacobian matrix for this system of equations is a 2x2 square matrix J:

$$J = \begin{bmatrix} \dfrac{\partial F_1}{\partial D} & \dfrac{\partial F_1}{\partial f} \\ \dfrac{\partial F_2}{\partial D} & \dfrac{\partial F_2}{\partial f} \end{bmatrix} \tag{5.4}$$

Program DIAPIP implements the Newton method to determine simultaneously the friction factor f and diameter D. Prompts in the program ask the user for the data that are required to define the problem. The acceleration of gravity is required so that problems in either of the ES or SI unit systems can be solved. If the solution is to be written to a disk file and also displayed on the monitor, then the Output unit number (the second input item) should not be 6. Microsoft's Fortran version 5 and higher versions prompt the user for the disk file if writing to a logic unit other than 6 and this unit is not already open. The next input statement requests values for the desired discharge Q, the roughness e, the pipe length L, and the frictional head loss h_f. For our problem the difference in the water surface elevations H_1 and H_2 is this frictional head loss. Since $1/\sqrt{f}$ occurs on both sides of Eq. 5.2a, let it be the unknown in place of f. In the program this variable is SF.

The Jacobian is defined by the expanded 2x3 array DJ. The first two columns in this array contain the Jacobian derivatives, and the third column contains the equation vector F. The derivatives are determined with respect to SF, rather than f, because this is slightly simpler. The two unknowns are SF and D, which are initialized to 8 and 0.5 ft, respectively, for the Newton method. The two equations are denoted by F1 and F2; after they are evaluated for the first time in each Newton iteration, they are stored in the third column of matrix DJ. Then the two statements that define the equations are evaluated twice more by the IF and GO TO statements. The last two times repeat the first computations with incremented values of SF and D. The program variable NCT counts the number of iterations. The number of Newton iterations should always be limited to avoid the possibility of an infinite loop in these computations. With two unknowns the solution by Gaussian elimination requires only one element D21 to be eliminated. Thereafter, the solution vector z is obtained by back substitution. Thus the approach is much like that in Chapter 4 to solve simultaneously for the discharge Q and the friction factor f (or SF). The major difference is the change in unknowns to D and f (or SF); when the unknowns are treated properly, the Newton method works in the same way.

If we want to find the diameter that will convey 2.0 ft^3/s when the difference is 40 ft in a 3000 ft long pipe of roughness 0.002 inches, the computer program DIAPIP will produce the solution $f = 0.01668$, D = 7.941 in, listed below as case 1. Although in practice these results would be rounded, we present them in this way to aid the checking of the computer output. To verify that the program works properly, the reader should use DIAPIP to solve the four problems in Table 5.1; these steps will also augment the reader's understanding of the program logic. We assume either $\nu = 1.41 \times 10^{-5}$ ft^2/s or $\nu = 1.31 \times 10^{-6}$ m^2/s.

Table 5.1 Test Problems

No.	**1**	**2**	**3**	**4**
L	3000 ft	1000 m	1000 m	10,000 ft
h_f	40 ft	8 m	80 m	15 ft
e	0.002 in	0.0001 m	0.0001 m	0.0004 ft
f	0.0168	0.01598	0.01679	0.01559
D	7.941 in (0.662 ft)	0.3664 m	0.3335 m	23.052 in (1.921 ft)

The attractive convergence behavior of the Colebrook-White equation, Eq. 5.2, using Gauss-Seidel iteration is the basis for an alternative to the simultaneous solution of Eqs. 5.1 and 5.2 by the Newton method. By starting with some reasonable value for f, Eq. 5.2 must only be solved a few times by always using the newly computed value of f to recompute f. When Gauss-Seidel iteration is used to solve Eq. 5.2, then Eq. 5.1 can be solved via the Newton method with f treated as if it were known in each Newton iteration. In this process the Newton method is therefore used to solve only one equation for the one unknown, D. Since D does affect the value of f, the Gauss-Seidel iteration must be repeated within each new Newton iteration, however. Therefore this alternative solution process consists of applying the Newton method to solve Eq. 5.1 for D, and within this iteration Gauss-Seidel iteration is used to resolve Eq. 5.2 for f. The Newton iteration is achieved via the equation

$$D^{(m+1)} = D^{(m)} - \frac{F(D^{(m)})}{dF(D^{(m)})/dD} \tag{5.5}$$

in which $F(D)$, under the assumption that f is known, is Eq. 5.1 written as follows:

$$F(D) = h_f - f(L/D)Q^2/(2gA^2) = 0 \tag{5.6}$$

This method is implemented by program DIAPIPA.

The approach in DIAPIPA can be used in a slightly modified manner in solving for D and f with an HP48 or equivalent handheld calculator. Retrieve both the Colebrook-White and Darcy-Weisbach equations from memory. Using an estimate for D, solve the Colebrook-White equation with the SOLVR function. Next solve the Darcy-Weisbach equation using SOLVR, and repeat this process until small changes in D occur between consecutive iterations.

A third alternative is to replace the Newton solution of the Darcy-Weisbach equation with a direct solution of this equation. Since the area $A = \pi D^2/4$, this equation can be written as

$$D = \left[\frac{fLQ^2}{2g(\pi/4)^2 h_f}\right]^{0.2} = \left[\frac{0.8105695 fLQ^2}{gh_f}\right]^{0.2} \tag{5.1b}$$

Because f depends upon D, Eq. 5.1b must be solved iteratively, with the Colebrook-White equation being solved either by the Gauss-Seidel method or the Newton method as soon as a new D is available. The program DIAPIP2 implements this solution method, applying the Gauss-Seidel method to the Colebrook-White equation. In previous programs a conversion factor CONV allowed D and e to be given in inches when using ES units, but program DIAPIP2 requires consistent units for all variables. One could use this same approach with an HP48 calculator. However, now one does not use SOLVR in obtaining the solution to the Darcy-Weisbach equation.

Yet another possible approach is to eliminate the friction factor by solving for it in the Darcy-Weisbach equation and substituting the result into the Colebrook-White equation; then the resulting equation for D is solved by using the Newton method. The Darcy-Weisbach equation, with f on the left of the equal sign, is

$$f = h_f D(2g)A^2/(LQ^2) = h_f D(2g)/(LV^2) = 1.2337 h_f g D^5/(LQ^2) \tag{5.1c}$$

or

$$\frac{1}{\sqrt{f}} = \frac{Q\sqrt{L}}{A(2gh_f D)^{1/2}} = \left(\frac{L}{2gh_f}\right)^{1/2}\left(\frac{4Q}{\pi D^{2.5}}\right) = 0.90031632 Q\left[L/(gh_f)\right]^{1/2}/D^{2.5} \tag{5.7}$$

The equation to be solved for D is obtained by replacing $1/\sqrt{f}$ in this last equation with the expression on the right wherever it appears in the Colebrook-White equation. In implementing the solution in a computer program it is better to use two lines of code, one for the above expression for $SF = 1/\sqrt{f}$ and the other for the Colebrook-White equation.

The program DIAPIP3 uses this method to determine the diameter, with the derivative of the equation with respect to diameter being obtained numerically. After the diameter has been found, Eq. 5.1c is used to determine f.

5.1.3. SOLUTION BASED ON THE HAZEN-WILLIAMS EQUATION

The empirical Hazen-Williams equation is widely used in practice to define the discharge-head loss relation for water flows in full pipes. The Hazen-Williams equation is

$$Q = KC_{HW}AR_h^{0.63}S^{0.54} \tag{5.8}$$

in which $K = 1.318$ for ES units and $K = 0.849$ for SI units, C_{HW} is the Hazen-Williams roughness coefficient which ranges from 150 for smooth-walled pipes to as low as 80 for old, corroded cast iron pipes (see Table 2, Chapter 2), R_h is the hydraulic radius, and S is the slope of the HGL or energy line so that $S = h_f/L$. Another convenient form of the Hazen-Williams equation is

$$h_f = \frac{K_1 L}{C_{HW}^{1.852} D^{4.87}} Q^{1.852} \tag{5.9}$$

in which $K_1 = 4.727$ with ES units, and $K_1 = 10.7$ with SI units. If the Hazen-Williams equation is solved directly for the pipe diameter D, it then appears as

$$D = \left[\frac{QK_1^{0.54}}{C_{HW}S^{0.54}}\right]^{0.380228} = K_2\left[\frac{Q}{C_{HW}S^{0.54}}\right]^{0.38} \tag{5.10}$$

in which $K_2 = 1.376$ for ES units and $K_2 = 1.626$ for SI units. As Eq. 5.10 indicates, use of the Hazen-Williams equation allows the pipe diameter to be found directly if the discharge Q, head loss h_f, length L, and roughness coefficient C_{HW} are known. This obvious computational advantage, simplicity, is the main reason for its popularity. Program DIAPIPH obtains a solution for D from the Hazen-Williams equation.

When computers (and programmable pocket calculators) are used, the ease of computation will be of minor importance in relation to the validity of the formula over a large range of flow conditions. The Hazen-Williams equation agrees closely with results produced by the Darcy-Weisbach equation for water flowing in relatively smooth-walled pipes with Reynolds Numbers in the range of 10^5 to 10^6 (the typical range for pipe design). However, it does not produce results that agree well with the Darcy-Weisbach equation over a range of flow conditions in rough-walled pipes. In fact, the Manning equation is a better empirical equation for the representation of flow in rough-walled pipes, especially if the pipe does not flow full.

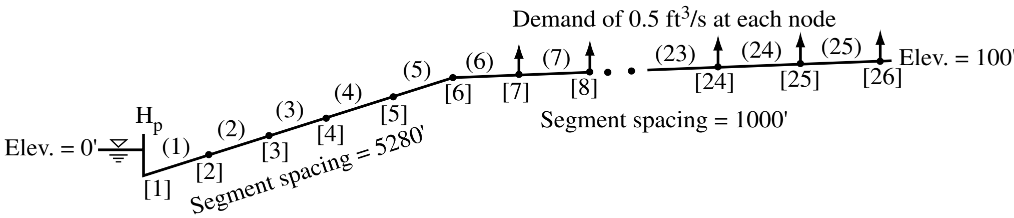
5.1.4. BRANCHED PIPE NETWORKS

In a branched pipe system it is easy to determine the discharge that must be carried by each pipe if all external demands are specified. If the pressures, heads, or HGL elevations are also known, then it is possible to use the methods described above to find the diameter of every pipe in the system, simply by repeating the computation for a single pipe. This can be done because the head loss and discharge for each pipe can be determined from simple preliminary computations. Thus no additional computational methods are needed to compute results for a branched system. Even though a more detailed look at the variables in pipe systems is presented later, it may be instructive to look at an example now.

Example Problem 5.1

As a consulting engineer you have been asked by an irrigation district to prepare a preliminary study of a pipeline using PVC pipe (Assume e = 0.000084 in.) that will bring irrigation water from a river that is 5 miles from the first farm. There are 20 farms. The turnout for each is to receive 0.5 ft^3/s, and these turnouts are spaced at 1000 ft intervals along the pipeline. The water level in the river is 100 ft below the elevation of the irrigated land, which is essentially flat. The water at the last turnout is to be delivered at a pressure of 40 lb/in^2. The pipeline will be laid on a constant grade between these two elevations, and a pump will be required at the river to provide sufficient head.

You decide to base computations on a 1-mile increment for the first 5 miles, and on a 1000 ft increment thereafter, with each turnout at a junction between pipe segments. A sketch of this pipe system is shown below. To determine the pipe size that will result in the least (or near least) cost, you decide to obtain a series of design solutions in which the slope of the HGL will vary. The sum of the pipe cost and the energy cost for pumping will be plotted as a function of the slope of the HGL, and the minimum cost on this graph will identify the best design for the piping system.



The following tables present the solution to this problem with the slope of the HGL specified to be 1.2424×10^{-3}. The discharges in column 9 are obtained first. Thereafter the diameters are computed by using any of the methods described in this section. The last column lists the incremental head losses (because this is commonly given), but since the slope of the HGL has been specified here, they are directly related to the pipe lengths. You should verify some of these results. If the Hazen-Williams equation is used in place of the Darcy-Weisbach equation, then a solution such as that given below can easily be completed by using a spread sheet. If the spread sheet has the ability to solve an implicit equation, then the Darcy-Weisbach equation could also be used. The design solution is followed by an analysis, in which the nearest standard pipe sizes have replaced the computed values. The correctness of some of these head losses for the standard pipe sizes should be verified. The cost analysis assumes the life expectancy of 45 years and energy costs of \$0.09/kWh. A knowledge of engineering economic analysis will allow the pumping cost for this system to be verified. Pumping is assumed to occur 365 days per year and has a combined motor-pump efficiency of 70 percent.

DESIGN PIPE DIAMETERS

PIPE NO.	NODES FROM	TO	DIA. in	AREA ft^2	NOM. DIA. in	L ft	e x10^5 in	Q ft^3/s	VEL. ft/s	HEAD LOSS ft.
1	1	2	23.22	2.940	24.0	5280	8.4	10.0	3.40	6.56
2	2	3	23.22	2.940	24.0	5280	8.4	10.0	3.40	6.56
3	3	4	23.22	2.940	24.0	5280	8.4	10.0	3.40	6.56
4	4	5	23.22	2.940	24.0	5280	8.4	10.0	3.40	6.56
5	5	6	23.22	2.940	24.0	5280	8.4	10.0	3.40	6.56
6	6	7	23.22	2.940	24.0	1000	8.4	10.0	3.40	1.24
7	7	8	22.77	2.828	24.0	1000	8.4	9.5	3.36	1.24
8	8	9	22.31	2.715	24.0	1000	8.4	9.0	3.31	1.24
9	9	10	21.84	2.601	20.0	1000	8.4	8.5	3.27	1.24
10	10	11	21.34	2.484	20.0	1000	8.4	8.0	3.22	1.24
11	11	12	20.83	2.366	20.0	1000	8.4	7.5	3.17	1.24
12	12	13	20.29	2.246	20.0	1000	8.4	7.0	3.12	1.24
13	13	14	19.74	2.124	20.0	1000	8.4	6.5	3.06	1.24
14	14	15	19.15	2.000	20.0	1000	8.4	6.0	3.00	1.24
15	15	16	18.53	1.873	18.0	1000	8.4	5.5	2.94	1.24
16	16	17	17.88	1.743	18.0	1000	8.4	5.0	2.87	1.24
17	17	18	17.18	1.610	18.0	1000	8.4	4.5	2.79	1.24
18	18	19	16.44	1.474	15.0	1000	8.4	4.0	2.71	1.24
19	19	20	15.63	1.333	15.0	1000	8.4	3.5	2.63	1.24
20	20	21	14.75	1.187	15.0	1000	8.4	3.0	2.53	1.24
21	21	22	13.78	1.035	15.0	1000	8.4	2.5	2.42	1.24
22	22	23	12.67	0.875	12.0	1000	8.4	2.0	2.28	1.24
23	23	24	11.37	0.706	12.0	1000	8.4	1.5	2.13	1.24
24	24	25	9.77	0.521	10.0	1000	8.4	1.0	1.92	1.24
25	25	26	7.54	0.310	8.0	1000	8.4	0.5	1.61	1.24

NODE DATA

NODE	DEMAND ft^3/s	ELEV. ft.	HEAD ft.	PRESSURE lb/in^2	HGL ELEV. ft.
1	-10.0	100.	150.00	65.00	250.00
2	0.0	100.	143.44	62.16	243.44
3	0.0	100.	136.88	59.31	236.88
4	0.0	100.	130.32	56.47	230.32
5	0.0	100.	123.76	53.63	223.76
6	0.0	100.	117.20	50.79	217.20
7	0.5	100.	115.96	50.25	215.96
8	0.5	100.	114.72	49.71	214.72
9	0.5	100.	113.47	49.17	213.47
10	0.5	100.	112.23	48.63	212.23
11	0.5	100.	110.99	48.09	210.99
12	0.5	100.	109.75	47.56	209.75
13	0.5	100.	108.50	47.02	208.50
14	0.5	100.	107.26	46.48	207.26
15	0.5	100.	106.02	45.94	206.02
16	0.5	100.	104.78	45.40	204.78
17	0.5	100.	103.53	44.86	203.53
18	0.5	100.	102.29	44.33	202.29
19	0.5	100.	101.05	43.79	201.05
20	0.5	100.	99.81	43.25	199.81
21	0.5	100.	98.56	42.71	198.56
22	0.5	100.	97.32	42.17	197.32
23	0.5	100.	96.08	41.63	196.08
24	0.5	100.	94.84	41.10	194.84
25	0.5	100.	93.59	40.56	193.59
26	0.5	100.	92.35	40.02	192.35

An analysis based on the nearest standard pipe diameter yields the following results:

STANDARD PIPE DIAMETER SOLUTION

PIPE NO.	NODES FROM	NODES TO	L ft.	DIA. in	e x10^5 in	Q ft^3/s	VEL. ft/s	HEAD LOSS ft.	HLOSS /1000
1	1	2	5280.	24.0	8.4	10.0	3.18	5.59	1.06
2	2	3	5280.	24.0	8.4	10.0	3.18	5.59	1.06
3	3	4	5280.	24.0	8.4	10.0	3.18	5.59	1.06
4	4	5	5280.	24.0	8.4	10.0	3.18	5.59	1.06
5	5	6	5280.	24.0	8.4	10.0	3.18	5.59	1.06
6	6	7	1000.	24.0	8.4	10.0	3.18	1.06	1.06
7	7	8	1000.	24.0	8.4	9.5	3.02	0.96	0.96
8	8	9	1000.	24.0	8.4	9.0	2.86	0.87	0.87
9	9	10	1000.	20.0	8.4	8.5	3.90	1.90	1.90
10	10	11	1000.	20.0	8.4	8.0	3.67	1.70	1.70
11	11	12	1000.	20.0	8.4	7.5	3.44	1.51	1.51
12	12	13	1000.	20.0	8.4	7.0	3.21	1.33	1.33
13	13	14	1000.	20.0	8.4	6.5	2.98	1.17	1.17
14	14	15	1000.	20.0	8.4	6.0	2.75	1.01	1.01
15	15	16	1000.	18.0	8.4	5.5	3.11	1.43	1.43
16	16	17	1000.	18.0	8.4	5.0	2.83	1.20	1.20
17	17	18	1000.	18.0	8.4	4.5	2.55	0.99	0.99
18	18	19	1000.	15.0	8.4	4.0	3.26	1.93	1.93
19	19	20	1000.	15.0	8.4	3.5	2.85	1.52	1.52
20	20	21	1000.	15.0	8.4	3.0	2.44	1.15	1.15
21	21	22	1000.	15.0	8.4	2.5	2.04	0.83	0.83
22	22	23	1000.	12.0	8.4	2.0	2.55	0.61	1.61
23	23	24	1000.	12.0	8.4	1.5	1.91	0.96	0.96
24	24	25	1000.	10.0	8.4	1.0	1.83	1.11	1.11
25	25	26	1000.	8.0	8.4	0.5	1.43	0.94	0.94

AVE. VEL. = 2.87 ft/s, AVE. HL/1000 = 1.22, MAX. VEL. = 3.90 ft/s, MIN. VEL. = 1.43 ft/s

In one more table we can summarize the information that describes this solution fully by listing various data associated with each node.

NODE DATA

NODE	DEMAND ft^3/s	gal/min	ELEV. ft.	HEAD ft.	PRESSURE lb/in^2	HGL ELEV. ft.
1	- 10.00	- 4490.0	100.0	150.00	65.00	250.00
2	0.00	0.0	100.0	144.41	62.58	244.41
3	0.00	0.0	100.0	138.82	60.16	238.82
4	0.00	0.0	100.0	133.23	57.73	233.23
5	0.00	0.0	100.0	127.64	55.31	227.64
6	0.00	0.0	100.0	122.05	52.89	222.05
7	0.50	224.4	100.0	120.99	52.43	220.99
8	0.50	224.4	100.0	120.03	52.01	220.03
9	0.50	224.4	100.0	119.15	51.63	219.15
10	0.50	224.4	100.0	117.25	50.81	217.25
11	0.50	224.4	100.0	115.55	50.07	215.55
12	0.50	224.4	100.0	114.04	49.42	214.04
13	0.50	224.4	100.0	112.71	48.84	212.71
14	0.50	224.4	100.0	111.54	48.34	211.54
15	0.50	224.4	100.0	110.54	47.90	210.54
16	0.50	224.4	100.0	109.11	47.28	209.11
17	0.50	224.4	100.0	107.90	46.76	207.90
18	0.50	224.4	100.0	106.91	46.33	206.91
19	0.50	224.4	100.0	104.98	45.49	204.98
20	0.50	224.4	100.0	103.46	44.83	203.46
21	0.50	224.4	100.0	102.32	44.34	202.32
22	0.50	224.4	100.0	101.49	43.98	201.49
23	0.50	224.4	100.0	99.88	43.28	199.88
24	0.50	224.4	100.0	98.92	42.86	198.92
25	0.50	224.4	100.0	97.81	42.38	197.81
26	0.50	224.4	100.0	96.87	41.98	196.87

AVE. HEAD = 114.9 ft., AVE. HGL = 214.91 ft.,
MAX. HEAD = 150.0 ft., MIN. HEAD = 96.87 ft.

COSTS ASSOCIATED WITH THIS NETWORK

ITEM	TYPE	PRESENT WORTH	ANNUAL COST
1	ELEC. POWER	$ 101,898,590	$ 10,277,391
2	PIPE	2,969,690	299,520
	TOTAL	$ 104,868,280	$ 10,576,910

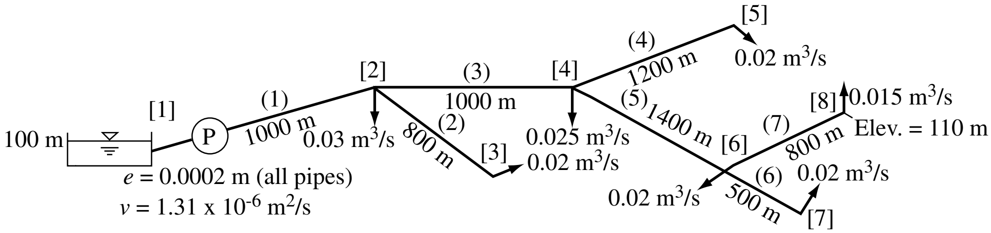
The solution was obtained by applying the NETWK program with this input data file:

```
EXAMPLE PROBLEM 5.1,  PIPE BRANCHED NETWORK
/*
$SPECIF IHGL=-2,NOMSOL=1,DESIGN=1,ICOST=1 $END
250. -10 100 0 .000084
1 6  .001242424 5280./
DEMAND
.5
6 26 .00124242 1000./
END
RUN
1 250.
END
```

* * *

Example Problem 5.2

This branched network is to be designed (i.e., pipe sizes determined) for the stated demands so the slope of the EL-HGL is 1/500 and a pressure of 50 kPa exists at node [8], the downstream node. What will be the cost per 30-day period for pumping if electricity costs \$0.09/kWh and the combined efficiency of the motor and pump is 75 percent?



To determine the solution, first the discharge in each pipe is calculated by starting at the downstream nodes and working upstream, applying continuity at each node, and then the diameters are found by using any of the programs DIAPIP*. The results are given below in the table. The head that the pump must supply can be determined by starting at node [8] and computing successively the elevations of the HGL at the nodes that are farther upstream; finally the supply water surface elevation is subtracted to obtain the net rise that is needed in the HGL, or $h_p = 123.5 - 100 = 23.5$ m. The cost per month is the cost per kWh multiplied by the number of hours in 30 days and the power rate in kW; thus

$$\text{Cost} = 0.09(30 \times 24)(0.095 \times 9.81 \times 23.5)/0.75 = \$1892 \text{ per month.}$$

Pipe	**Q** m^3/s	**h_f** m	**D** m
1	0.095	2.0	0.370
2	0.020	1.6	0.206
3	0.075	2.0	0.339
4	0.020	2.4	0.206
5	0.055	2.8	0.301
6	0.020	1.0	0.206
7	0.015	1.6	0.184

Node	**HGL** m
1	123.5
2	121.5
3	119.9
4	119.9
5	117.5
6	116.7
7	115.7
8	115.1

* * *

5.2 LARGE BRANCHED SYSTEMS OF PIPES

Section 5.1 has shown how to determine the diameters of pipes in branched systems. First the discharges in all pipes are determined from the nodal external demands; second, once the discharge in each pipe is known, one of the methods described in Section 5.1 is applied repeatedly until all of the diameters have been computed. The discharges in all pipes of a branched system are obtained by satisfying the junction continuity equations. If we assume that the node which supplies the system is numbered (and that its demand is negative), then in general for a branched system there will be one more node or junction than there are pipes. Therefore the number of pipe flow equations will be NJ - 1, and a junction continuity equation will not be written for one of the nodes. The node that is omitted is seemingly arbitrary, but typically the omitted junction continuity equation is associated with either the last or first node. Let's examine how this approach can be implemented effectively in computer codes.

Three somewhat disparate methods can be used to obtain the discharges in a systematic manner that can be implemented in computer code. The three methods focus on either (1) the network layout, (2) the coefficient matrix produced by the junction continuity equations, or (3) the use of standard linear algebra. The reader can prepare best for the next three sections by obtaining now a listing of programs SOLBRAN, SOLBRAN2, and SOLBRAN3 from the CD.

5.2.1. NETWORK LAYOUT

The implementation of this method is based on the layout or topological connectivity of the network; it notes that pipes that have a dead end, i.e., that have at most one connection or nodal demand at one of their ends, must convey a discharge that is equal to the demand at that node. After the discharge in such a dead end pipe is determined, the demand at the other end of this pipe is modified to be the sum of the original nodal demand there and the discharge in the pipe, and then the dead end pipe is removed from the network of pipes. This reduced network will contain other dead end pipes, and the process is continued until the discharge is established for all pipes in the network. This process can be defined by the following steps:

1. Examine the network to find all nodes that have only one pipe connected to them, and assign the discharge in each such pipe to be the demand at this node.
2. Modify the demand at the node at the other end of each such pipe to reflect the original demand and the discharge in the pipe, and remove the pipe from the definition of the network.
3. Repeat steps 1 and 2 until the discharge is determined for all pipes in the branched network.

The 10-pipe network shown in Fig. 5.2 will be used to illustrate this method. In step 1 we note that pipes 1, 5, 9, and 10 are dead end pipes, i.e., pipes connected to nodes that have only one pipe connected to them, and the discharges in these pipes equal the

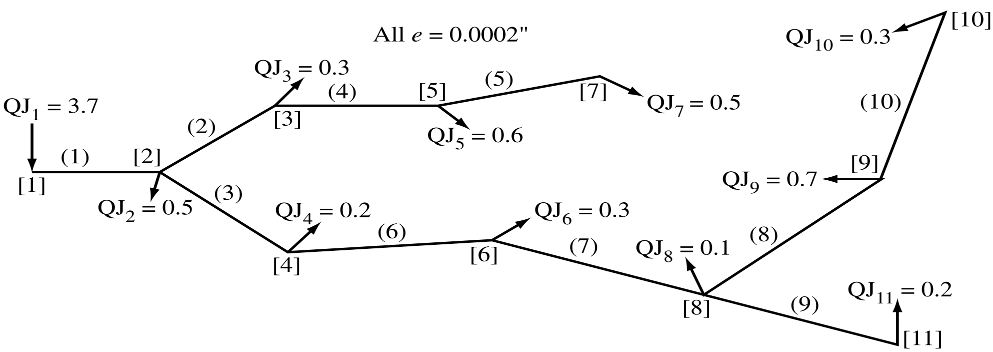


Figure 5.2 A 10-pipe network.

demands at these nodes: $Q_1 = QJ_1 = 3.7$ ft^3/s, $Q_5 = QJ_7 = 0.5$ ft^3/s, $Q_{10} = QJ_{10} = 0.3$ ft^3/s and $Q_9 = QJ_{11} = 0.2$ ft^3/s. Upon obtaining these discharges, step 2 is to reduce the branched system of pipes, by removing these pipes, to that shown in Fig. 5.3, in which the new demands account for the discharges in the pipes that have been removed:

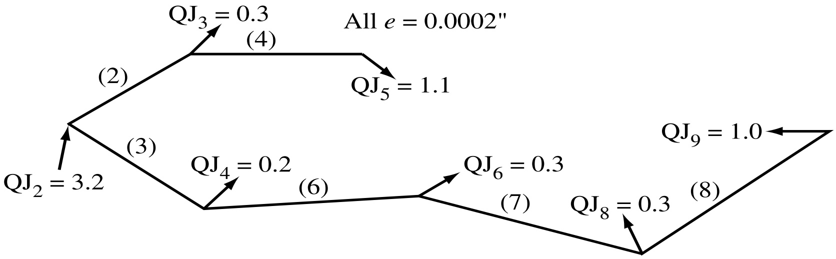


Figure 5.3 The reduced network.

For step 3 the process is repeated. After two additional applications (as shown below) there are only two pipes left, both of which are dead end pipes. The resulting discharges are Q_1 = 3.7 ft^3/s, Q_2 = 1.4 ft^3/s, Q_3 = 1.8 ft^3/s, Q_4 = 1.1 ft^3/s, Q_5 = 0.5 ft^3/s, Q_6 = 1.6 ft^3/s, Q_7 = 1.3 ft^3/s, Q_8 = 0.9 ft^3/s, Q_9 = 0.3 ft^3/s, and Q_{10} = 0.2

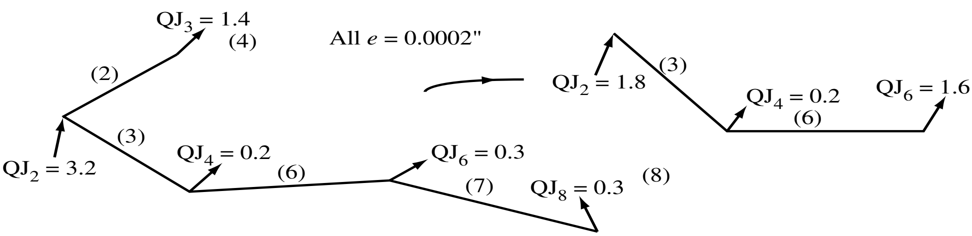


Figure 5.4 The final arrangement of the pipes.

Let's examine how this process can be implemented effectively in computer code. The details of the process will vary slightly, depending on the description of the network and one's sign convention. The description we will use for this purpose consists of a table with one line for each node. Each line contains the demand at the node, followed by a list of the pipes that join at this node. An extraction or outflow will be a positive demand, so if an external flow enters the network at a node it will be a negative demand. Pipes that receive flow from a node will be given positive numbers, whereas a pipe having flow into a node will be given a negative number. Using this nomenclature, the description of the branched network example is given by the two lists in Table 5.2. These lists are prepared in the order in which the nodes are numbered, and the entries under the second heading are the numbers of the pipes that join at this node. Thus dead end pipes are identified immediately by the single number on one row in this list.

Table 5.2

Demand QJ_i ft^3/s	Pipes at Node
- 3.7	1
0.5	- 1 2 3
0.3	- 2 4
0.2	- 3 6
0.6	- 4 5
0.3	- 6 7
0.5	- 5
0.1	- 7 8 9
0.7	- 8 10
0.3	- 10
0.2	- 9

The process for determining the discharge in each pipe can consist of these steps:

1. Scan the list "Pipes at Node." If only one pipe number appears in a row, assign the demand at this node to the discharge in this pipe. To account properly for the direction of flow, the discharge in this pipe k can be assigned as $Q_k = - QJ_i|k|/k$. The absolute value of the pipe number, divided by its number, will give the proper sign to the discharge.
2. Mark this node for deletion, as it is not needed during the next pass through the list.
3. Scan the list of nodes and note all other appearances of this same pipe number. Modify the demand at any node that has this pipe joining it by the discharge of this pipe, i.e. modify demand QJ_j by $(QJ_j)_{new} = (QJ_j)_{old} + Q_k|k|/k$, and remove this pipe from the list "Pipes at Node."
4. Delete all nodes that have been marked for deletion.
5. Repeat steps 1 through 4 until all nodes have been deleted from the list.

The program SOLBRAN executes the procedure that has just been described. After the discharges in the pipes are determined, then the diameters can be computed by the procedures described earlier. In this program these diameters are determined by solving the Darcy-Weisbach and Colebrook-White equations simultaneously; thus the previous program is now a subroutine that finds the diameter D (program variable DIA) given the discharge (program variable Q) and pipe roughness e. Then this subroutine finds D and f simultaneously.

The input to this program consists of the following:

1. The first line, which comes from the keyboard, gives the number of pipes NP (and in the C program the file names of the input and output units INPUT and IOUT);
2. The acceleration of gravity (32.2 for ES units or 9.81 for SI units) G, the kinematic viscosity of the fluid VISC, and the slope $S = h_f/L$ of the HGL line;
3. A list of pipe lengths;
4. A list of pipe roughnesses e in inches when using ES units and in meters when using SI units (by ending this list with / the missing e's will be equated to the last one supplied);
5. The list of demands and pipes at node as described above. Each line of item 5 must terminate with a / with the Fortran program. The program is dimensioned to allow up to four pipes to join at any node, but this can be changed by assigning PARAMETER N4 a different value.

The input file for this problem is presented in Fig. 5.5.

```
Input to FORTRAN program
32.2  1.41E-5  0.001
1000  1100  1200  1300  1400  1500  1600  1700  1800  1900
0.0002/
-3.7  1/
0.5  -1  2  3/
0.3  -2  4/
0.2  -3  6/
0.6  -4  5/
0.3  -6  7/
0.5  -5/
0.1  -7  8  9/
0.7  -8  10/
0.3  -10/
0.2  -9/
```

Figure 5.5 Input file for program SOLBRAN.

Table 5.3 Solution for a 10-pipe, 11-node Branched System

Pipe	Length ft.	e in $x10^4$	Dia. in.	Area ft^2	Discharge ft^3/s	Velocity ft/s	Head Loss ft.
1	1000.0	2.0	16.7139	1.52	3.7	2.43	1.00
2	1100.0	2.0	11.6044	0.73	1.4	1.91	1.10
3	1200.0	2.0	12.7509	0.89	1.8	2.03	1.20
4	1300.0	2.0	10.6023	0.61	1.1	1.79	1.30
5	1400.0	2.0	7.8967	0.34	0.5	1.47	1.40
6	1500.0	2.0	12.2000	0.81	1.6	1.97	1.50
7	1600.0	2.0	11.2867	0.69	1.3	1.87	1.60
8	1700.0	2.0	10.2308	0.57	1.0	1.75	1.70
9	1800.0	2.0	5.6149	0.17	0.2	1.16	1.80
10	1900.0	2.0	6.5282	0.23	0.3	1.29	1.90

5.2.2. COEFFICIENT MATRIX

This method writes the junction continuity equations in matrix form as [C]{Q} = {QJ}. The elements in the coefficient matrix [C] consist of three possible values, 0, 1, or - 1. The vector of unknowns {Q} contains the discharges in the pipes, and the known vector {QJ} lists the demands at the nodes. This method uses a very efficient method, rather than standard methods such as Gaussian or Gauss-Jordan elimination, to solve the linear algebra problem. The approach to the linear algebra problem can be very similar to the process employed in our first method, but the focus is on the coefficient matrix rather than the layout of the network. The steps can be identified as follows:

1. Examine the coefficient matrix for rows that contain only one element that is not zero, and solve this equation. (The solution of this equation will force the discharge in the pipe identified by the column in this coefficient matrix to be the demand at the downstream end of this pipe, i.e., equal to QJ in this row.) Then mark this equation as solved; i.e., remove this row from the existing linear equation system.
2. Find all other rows in the coefficient matrix that are not zero in this column; for each of these modify the known vector {QJ} in this row by multiplying the coefficient (1 or - 1) by the discharge determined in step 1, and subtract this amount from the existing value of QJ in this row. In effect this step removes this column from the coefficient matrix so that it has been reduced in size by one row and one column.
3. Repeat steps 1 and 2 until all rows and columns of the linear algebra problem have been removed.

The implementation of this method should not form the coefficient matrix as a N-row by N-column matrix, with N being the number of junctions NJ minus 1. Instead, identify which columns of the coefficient matrix contain the nonzero elements (the 1's or - 1's) for each of its rows, to save the storage needed for a two-dimensional array. Listing the pipe numbers that join at a node, as was done in implementing the first method, provides this identification, i.e. the node number identifies the row of the matrix, and the pipes joining at this node provide the column numbers that contain the non-zero elements. In program SOLBRAN this pipe information was read into the two-dimensional integer array JN(NJ,4) (the second subscript is the number of pipes that can join at any junction). Thus step 1 will identify those rows, i.e. the first subscript of JN, that have only one pipe and use only one position in the second subscript of JN. For these rows the *Q*'s will be determined, and the row will be marked and eliminated. For step 2 all of the rows not yet marked as eliminated will be searched for the same pipe number, and whenever this number is found it will be removed, and the number of elements used in the second subscript will be reduced by one. Thus the actual solution process becomes very similar to the first method. The program SOLBRAN2 shows one way to implement the second method. The subroutine DIAPIP is unchanged from the listing in SOLBRAN.

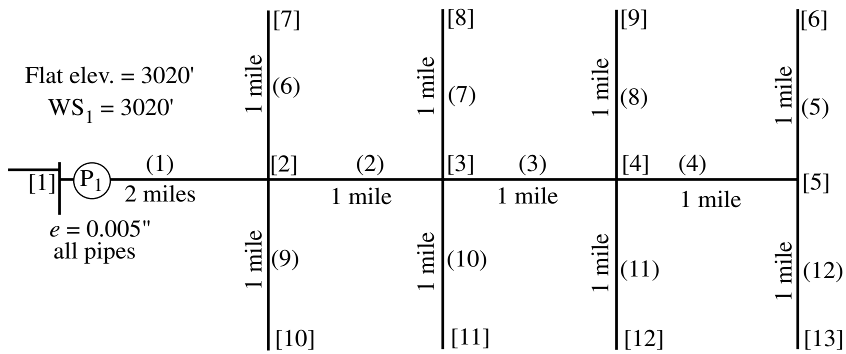
5.2.3. STANDARD LINEAR ALGEBRA

In this method the junction continuity equations are written as a coefficient matrix that multiplies the vector of unknown discharges (of length NP pipes) in the system. This product equals the known vector which consists of the demands at NP = NJ - 1 nodes of the network. This method requires the coefficient matrix to be a square matrix with NP rows and columns. The coefficient matrix elements will have the values 0, 1, or - 1. The row numbers correspond to the junction numbers for which the NP junction continuity equations are written, and the column numbers correspond to the pipe numbers. Upon properly defining the coefficient matrix and the known vector, a standard linear algebra subroutine (function) is called to solve the linear system of equations. One implementation of such a solution is given below in program SOLBRAN3. In this program the junction continuity equation is not written at the last junction of the network. Since the linear algebra solver SOLVEQ (see Appendix A) returns the solution in the same array that originally contained the known vector, the demands are now placed in the array Q at the outset, and the array QJ has been removed. In studying this listing you should strive to understand how the coefficient matrix is stored as 0's, 1's, or - 1's in the two-dimensional array C.

This method can be implemented easily by using spread sheets and general-purpose mathematics application software such as MathCAD, MATLAB, or TK-Solver. While the use of such software will result in computationally inefficient solutions, as is the case with SOLBRAN3, especially for large branched networks, the near-zero cost associated with such computations and the large PC RAMS makes it a viable approach. The CD contains a TK-Solver model and a brief description of it as files SOLBRAN3.TK2 and SOLBRAN3.DOC. A variation of the C program SOLBRAN.C is also on the CD under the name SOLBRAN4.C. This C program calls special pointer functions to allocate arrays beginning with 1, rather than 0, as is standard in C. (See Appendix A and the file SOLVEQC.DOC on the CD for more information.)

Example Problem 5.3

Water from a reservoir with a water surface elevation of 3020 ft passes through a pump to a pipeline that supplies twelve center-pivot irrigation sprinklers, each receiving a discharge of 1.5 ft^3/s at elevation 3020 ft and having a 1-mile spacing, as shown in the diagram. A pressure of 60 lb/in^2 or more is needed at each pivot location. Design the system to minimize costs. The capital cost of the pump is $100,000. Electrical energy costs $0.0935/kWh (actually $0.11/kWh, accounting for the 85% pump efficiency).



The cost per unit length for different pipe sizes is as follows (The NETWK program uses these default values.):

Diameter, in	10	12	15	18	20	24	30	36
Cost, cents/ft	10.67	16.67	24.00	43.33	56.67	80.00	100.00	120.00

The life expectancy of all components is 50 years, and the interest rate for acquiring capital for the project is 11 percent.

The cost of a system with pipes that are too small will be excessive, owing to the large energy cost of pumping the water. On the other hand the capital recovery cost for the pipes will be excessive if they are too large. The minimum total cost will be somewhere between these two extremes and will be determined by solving this branched system for several slopes of the HGL along the main line from node 1 though node 6 so that the pressure at node 6 is 60 lb/in^2. Likewise the pressures at nodes 7 through 13 will be specified as 60 lb/in^2. Thus a number of tentative designs will be required, and for each of these the costs will be determined. Since standard pipe sizes will be used, the nearest standard pipe size will be used in computing these costs.

The solution procedure will consist of the following steps:

1. Select a slope for the HGL along the main branch.
2. With a pressure of 60 lb/in^2 at node 6, or $HGL_6 = 3020 + 60(144)/62.4 = 3158.46$ ft, and the slope chosen in step 1, find the HGL slopes of pipes 6 through 12.
3. Compute all of the pipe diameters based on these HGL slopes.
4. Select standard pipe sizes that are nearest to the computed diameters.
5. Analyze the system that is composed of these standard pipe sizes, and compute the head and power that the pump must supply; then compute the electrical energy cost.
6. Determine the cost of the pipes, and convert this cost to an equivalent uniform annual cost by applying the capital recovery factor.
7. Repeat steps 1 through 6 until the least total cost is found.

SOLBRAN can not be used to seek this solution in a single run because the slope of the HGL is not the same for all pipes. The code would require modification to allow different slopes for different pipes. In its present form it could use the following input data to size pipes 1 through 6, but separate runs would be needed for the pipe pairs 6 and 9, 7 and 10, and 8 and 11 owing to the different HGL slopes. It is an instructive exercise to use the following input with SOLBRAN to compute the diameters of the pipes; those results can then be compared with those from NETWK.

Input to SOLBRAN

```
32.2  1.41E-5  0.001
10560  5280  5280  5280  5280/
0.005/
-18.  1/
4.5  -1 2/
4.5  -2 3/
4.5  -3 4/
3.0  -4 5/
1.5  -5/
```

The program NETWK will accomplish steps 1 through 6 with the input file below. In this input file the option IHGL = - 2 allows the main branch to be described by 2 lines of input, and the regular input is added to describe the lateral pipes. This input file has a HGL slope of 0.001 (and this slope results in the least cost). To obtain a solution for a different slope, this value (0.001) is changed; additional required changes are the HGL elevation at the beginning node (3190.14) and, on the line after the RUN command, a beginning HGL elevation for the analysis that is requested with the option NOMSOL=1. To pursue this solution process further, you should now obtain a solution from NETWK. The input file is on the CD under the name EXP5_3.IN. In obtaining the solution you should note that NETWK first computes a design solution in which the pipe diameters are

```
Example Problem  5.3
/*
$SPECIF IHGL=-2,NOMSOL=1,DESIGN=1,ICOST=1 $END
3190.14 -18. 3020. 1.5 .005
1 6 0.001  10560. 5280./
END
PIPES
6 2 7 5280. 0. .005
7 3 8/
8 4 9/
9 2 10/
10 3 11/
11 4 12/
12 5 13
NODES
7 1.5 3020. 3158.46
8 1.5
9 1.5
10 1.5
11 1.5
12 1.5
13 1.5
RUN
1 3190.14
PUMPS
UNIT=0.11
CAPI=100000
END
```

determined. Then the nearest standard pipe sizes are used to "analyze" the network. The final cost is based on this analysis and should agree with the data in this table:

COSTS ASSOCIATED WITH THIS NETWORK

ITEM	TYPE	PRESENT WORTH	ANNUAL COST
1	PIPE	$ 2,749,243	$ 277,286
2	ELEC. ENERGY	2,575,937	259,857
	TOTAL	$ 5,325,180	$ 537,143

The least cost is $537,143 per year with the energy costing $259,857 per year and the amortized cost of the pipes being $277,256 per year. Pipes 1 and 2 should be 30 inches in diameter, pipe 3 should be 24 inches in diameter, pipe 12 should be 12 inches in diameter, and the other pipes should be 10 inches in diameter.

*　　　　*　　　　*

5.3 LOOPED NETWORK DESIGN SOLUTION CRITERIA

This section will discuss the means for establishing equations to determine diameters and other desired quantities associated with looped pipe network problems. As background it is appropriate to review some of the fundamental relations that apply to the analysis of pipe networks, whether looped or branched. If the number of pipes that exist in a network is denoted by NP, the number of nodes (or junctions) is denoted by NJ, and the number of independent loops is denoted by NL, then this basic relation must be satisfied:

$$NP = NJ + NL \quad \text{if the network has two or more supply sources}$$

or

$$NP = (NJ - 1) + NL \quad \text{if the network has fewer than two supply sources.}$$

Actually a network can never be devoid of supply sources, but often problems are shown without a supply source. Instead the supply source is simply a node that has a negative demand or a flow into the system. If a network has only one supply source, it can always be shown as a network with no reservoir, or source pump, by obtaining the sum of the other demands and then indicating that this discharge amount enters at a particular point. For this relation to apply we tacitly assume that supply sources are not numbered as nodes.

The two kinds of *basic* equations are (1) junction continuity equations (NJ or NJ - 1 in number) that simply give mathematical expression to the fact that the mass rate of flow (or volumetric discharge for an incompressible fluid) from a junction must equal the mass rate of flow (or discharge) to a junction, and (2) equations that describe the relation between head loss and discharge in a pipe, e.g., the Darcy-Weisbach or Hazen-Williams equations. Of course other equations could be written and may be needed, but these are not considered to be basic equations. For example, in using the Darcy-Weisbach equation a friction factor f is introduced for each pipe, but alternative equations such as the Colebrook-White equation could express this relation. In a similar way pipe cross-sectional areas could be introduced as variables, and for each such area an additional equation becomes available. These secondary equations will not be included in the subsequent discussion.

One might wonder whether the equations around the loops constitute additional independent equations? The answer is no; they are not independent if all of the pipe head loss equations are written. The connectivity of the network, in conjunction with the pipe head loss equations, can be used to obtain the loop equations around both pseudo and real loops. To demonstrate this situation, consider the 16-pipe, 9-node network in Fig. 5.6.

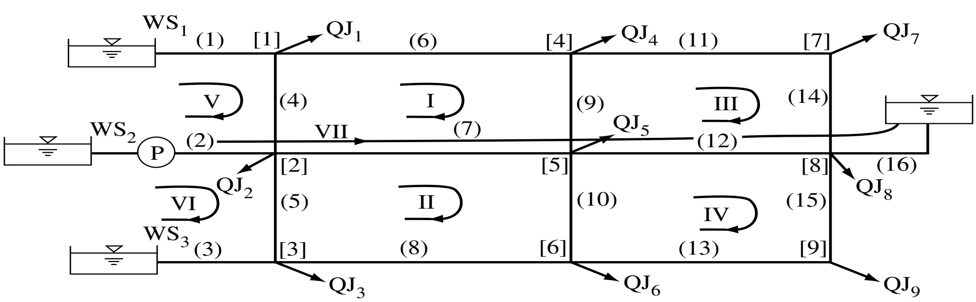


Figure 5.6 A 16-pipe, 9-node network.

The number of independent loop equations that can be written is NP - NJ = 16 - 9 = 7. These 7 loops are clear; four of them are real loops and three are pseudo loops connecting the four supply sources in some manner. However, the total number of basic equations consists of NJ = 9 junction continuity equations, and NP = 16 pipe head loss equations, for a total of 25. Thus 25 variables might be regarded as unknown, and if the other variables of the problem were all known, a solution for them could be sought.

To verify that the loop equations do not constitute additional independent equations, consider the four pipes (6, 9, 7, and 4) in loop I, using the exponential formula to express the head loss in each pipe:

$$H_1 - H_4 = K_6 Q_6^{n_6} \tag{5.11}$$

$$H_4 - H_5 = K_9 Q_9^{n_9} \tag{5.12}$$

$$H_2 - H_5 = K_7 Q_7^{n_7} \tag{5.13}$$

$$H_2 - H_1 = K_4 Q_4^{n_4} \tag{5.14}$$

Adding Eqs. 5.11 and 5.14 gives

$$H_2 - H_4 = K_4 Q_4^{n_4} + K_6 Q_6^{n_6} \tag{5.15}$$

Subtracting Eq. 5.12 from 5.13 gives

$$H_2 - H_4 = K_7 Q_7^{n_7} - K_9 Q_9^{n_9} \tag{5.16}$$

Now the subtraction of Eq. 5.16 from Eq. 5.15 produces

$$K_4 Q_4^{n_4} + K_6 Q_6^{n_6} + K_9 Q_9^{n_9} - K_7 Q_7^{n_7} = 0 \tag{5.17}$$

which is the loop equation for loop I. In a similar way writing the pipe head loss equations for pipes 1, 4, and 2 leads to

$$WS_1 - H_1 = K_1 Q_1^{n_1} \tag{5.18}$$

$$H_2 - H_1 = K_4 Q_4^{n_4} \tag{5.19}$$

$$WS_2 + h_p - H_2 = K_2 Q_2^{n_2} \tag{5.20}$$

Subtracting Eq. 5.19 from 5.18 results in

$$WS_1 - H_2 = K_1 Q_1^{n_1} - K_4 Q_4^{n_4} \tag{5.21}$$

Finally subtract Eq. 5.20 from 5.21 to eliminate H_2 and obtain

$$WS_1 - WS_2 - h_p + K_2 Q_2^{n_2} + K_4 Q_4^{n_4} - K_1 Q_1^{n_1} = 0 \tag{5.22}$$

which is the loop equation for pseudo loop V.

If a pipe head loss equation were written for every pipe in the network and the H's were then eliminated from these equations, an independent set of loop equations would be obtain-ed. Thus we see that loop equations are not independent of the pipe head loss equations and cannot also be used if the head loss equations are used. It is the way in which pipes are connected in a network that allows the loop equations to replace the pipe head loss equa-tions. This realization was the basis for the development of the Q-

equations in Chapter 4 to analyze a network. If one desires, it is always possible to omit pipe head loss equations and use loop equations in their place. Doing this, however, generally results in more arithmetic in obtaining the solution.

For the present we regard a design problem as one in which pipe diameters are to be determined. The definition of a design problem could be given a broader meaning, but at this time we are not concerned with the sizing of other components of a pipe system. Design problems can be further divided into two categories: (1) those in which we seek to determine as many diameters as there are nodes in the network (branched networks are a special case here); and (2) those in which we seek only certain individual pipe diameters to meet specified pressures. The latter category of problems will be treated in a later section. In the first category it is not possible to solve for more pipe diameters than there are nodes because the number of unknowns would then exceed the number of available equations. If the maximum possible number of pipe diameters is to be found (category 1), then it is assumed that the HGL elevations, or the heads H (pressure heads, or pressures), are specified at all nodes of the network. The number of basic equations is then NP + NJ (or NP + NJ - 1 if no supply sources are specified), but some of these must be used to determine other variables, usually the individual pipe discharges. Thus a basic difference between the first type of design problem and an analysis problem is that the H's at the nodes are known (specified) rather than unknown, and pipe diameters are to be found in place of the H's. The discharges are unknown variables in both the first type of design problem and the analysis problem. Thus diameters replace H's in the list of unknowns. The number of diameters in the list of unknowns must equal the number of H's which are specified. Looking again at the most recent network as an example, if the H's at all 9 nodes are given, one can in principle determine 9 pipe diameters. In this case the 25 independent equations would be used to determine 16 discharges plus 9 diameters.

To gain further insight into how this interchange of unknowns for knowns occurs, and what works and what won't work, consider the three-pipe looped system in Fig. 5.7, for

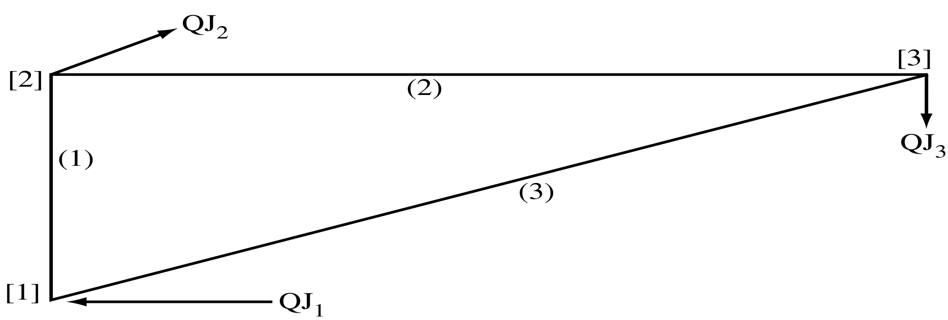
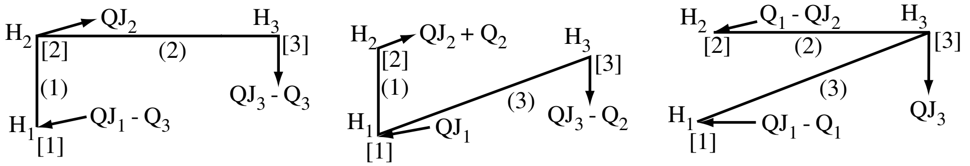


Figure 5.7 The three-pipe looped system.

which there exist two independent junction continuity equations and three head loss equations. If this were an analysis problem, all pipe diameters (and their lengths and roughnesses) would be given, and the five unknowns to be found would be Q_1, Q_2, Q_3, H_2, and H_3 (assuming H_1 is known). For the design problem H_2 and H_3 are given, along with H_1, and two diameters can then be found. The unknowns in the design problem would be Q_1, Q_2, Q_3, and two diameters. There are three possible combinations of two diameters: D_1 and D_2, D_1 and D_3, and D_2 and D_3. In the first combination D_3 must be given, in the second D_2, and in the third D_1. Specifying a diameter plus the head at both ends of a pipe establishes from the head loss equation the discharge in that pipe. These three combinations of diameters create the three problem cases shown in Fig. 5.8.



Case 1. D_3 known (Q_3 fixed)   Case 2. D_2 known (Q_2 fixed)   Case 3. D_1 known (Q_1 fixed)

Figure 5.8 The three cases.

One approach to the solution of these three cases is to write the 5 basic equations (plus the secondary equations), specify the knowns and solve for the unknowns. In other words the independent equations are simultaneously solved for as many unknowns as there are equations. This approach is illustrated by the "Rule Sheet" from TK-Solver, shown below with the three variable sheets for these three cases. The diameter that is regarded as known is listed in the "Input" column, and the diameters that are to be found are listed in the "Output" column.

However, from these cases one may be able to see a computationally more efficient means of solving the problem. First, the discharges in the pipes with given diameters can be computed by solving a head loss equation. Next, by removing these pipes and

RULE SHEET

```
S Rule-----------------------------------------------------------------------
 Q2+Q3=QJ3
 Q1-Q2=QJ2
   H1-H2=f1*(L1/D1)*Q1^2/(G2*(pi()/4.*D1^2)^2)
   H2-H3=f2*(L2/D2)*Q2^2/(G2*(pi()/4.*D2^2)^2)
   H1-H3=f3*(L3/D3)*Q3^2/(G2*(pi()/4.*D3^2)^2)
     1/sqrt(f1)=1.14-2*log(e/D1+7.34347283*v*D1/(Q1*sqrt(f1)))
     1/sqrt(f2)=1.14-2*log(e/D2+7.34347283*v*D2/(Q2*sqrt(f2)))
     1/sqrt(f3)=1.14-2*log(e/D3+7.34347283*v*D3/(Q3*sqrt(f3)))
```

Case 1

VARIABLE SHEET

St Input----	Name---	Output---
	D1	.1209918
	D2	.1244984
.125	D3	
	Q1	.0459944
	Q2	.0159944
	Q3	.0290056
150	L1	
400	L2	
550	L3	
.045	QJ3	
.03	QJ2	
100	H1	
85	H2	
80	H3	
.0000	e	
1		
1.3E-6	ν	
19.62	G2	
	f2	.0176878
	f1	.0148337
	f3	.0159636

Case 2

VARIABLE SHEET

St Input----	Name---	Output---
	D1	.1211631
.125	D2	
	D3	.1247196
	Q1	.0461667
	Q2	.0161667
	Q3	.0288333
150	L1	
400	L2	
550	L3	
.045	QJ3	
.03	QJ2	
100	H1	
85	H2	
80	H3	
.0000	e	
1		
1.3E-6	ν	
19.62	G2	
	f2	.0176642
	f1	.0148277
	f3	.0159747

Case 3

VARIABLE SHEET

St Input----	Name---	Output---
.12	D1	
	D2	.1215503
	D3	.1265911
	Q1	.0450043
	Q2	.0150043
	Q3	.0299957
150	L1	
400	L2	
550	L3	
.045	QJ3	
.03	QJ2	
100	H1	
85	H2	
80	H3	
.0000	e	
1		
1.3E-6	ν	
19.62	G2	
	f2	.0178295
	f1	.0148689
	f3	.0159017

Figure 5.9 The TK-Solver variable and rule sheets for the three cases.

modifying the demands on the reduced network, the discharges in the remaining pipes can be determined so they satisfy the junction continuity equations. Finally, for the remaining two pipes whose discharges are now known, the head loss equations can be solved for the diameters. Thus for all three cases the problem can be reduced to the solution of three separate equations, in proper order, each with only one unknown. (If the Darcy-Weisbach equation is selected for use, then actually pairs of equations must be solved, because the Colebrook-White equation for f must also be employed.)

For Case 1 this procedure would consist of the following steps if the heads are specified as $H_1 = 100$ m, $H_2 = 85$ m and $H_3 = 80$ m, each $e = 0.00001$ m, $L_1 = 150$ m, $L_2 = 400$ m, and $L_3 = 550$ m:

(a) Find Q_3 from $H_1 - H_3 = f_3(L_3/D_3)Q_3^2/(2gA_3^2)$ and the Colebrook-White equation using $D_3 = 0.15$ m, $L_3 = 550$ m, $e_3 = 0.00001$ m, $QJ_2 = 0.03$ m^3/s, and $QJ_3 = 0.045$ m^3/s; the solution is $Q_3 = 0.029$ m^3/s, $f_3 = 0.016$.

(b) From continuity (i.e. inspection) $Q_1 = 0.0460$ m^3/s, $Q_2 = 0.0160$ m^3/s.

(c) Seek D_1 from $H_1 - H_2 = f_1(L_1/D_1)Q_1^2/(2gA_1^2)$ and the Colebrook-White equation; the result is $D_1 = 0.1210$ m.

(d) Finally, find D_2 from $H_2 - H_3 = f_2(L_2/D_2)Q_2^2/(2gA_2^2)$ and the Colebrook-White equation; the result is $D_2 = 0.1245$ m.

When our discussion indicated that the number of pipe diameters that can be sought is equal to the number of junction continuity equations, one might infer that a simultaneous solution of continuity equations would provide all of the unknown diameters. This is not the case. In fact a simultaneous solution of the junction continuity equations provides the discharges in the pipes with unknown diameters (since this case uses only one continuity equation at a time), and the head loss equations (e.g., the Darcy-Weisbach equation) are used to find the unknown diameters and also to determine the discharges in pipes whose diameters are specified. In other words, all of the equations were used.

Before moving on to additional and more complex networks, we must note that it is quite possible to create combinations of specifications that lead to impossible situations. In this three-pipe network, for example, if the diameter of pipe 3 in case 1 were specified to be too large so that the discharge it conveys in response to the head loss $H_1 - H_3$ exceeds the demand QJ_3, an impossible problem is defined in which the flow in pipe 2 must be from node 3 to node 2, but this is not possible because H_2 is greater than H_3. A specified diameter which is too small can also create impossible conditions: if in case 3 $D_1 = 0.1$ m, then the discharge in pipe 1 must be (with $H_1 = 100$ m and $H_2 = 85$ m) $Q_1 = 0.028$ m^3/s, which is less than the demand QJ_2, and so the flow in pipe 2 must be from node 3 to node 2, but this is not possible because $H_2 = 85$ m is larger than $H_3 = 80$ m. The prescribed diameters and the heads at the pipe ends must be within certain limits so the flow pattern is consistent with what is required by continuity at both ends of these pipes and with the head distribution in nearby pipes.

Consider next the design of a simple network consisting of only two pipes with reservoirs at both ends, as shown in Fig. 5.10. If this network is viewed as a design

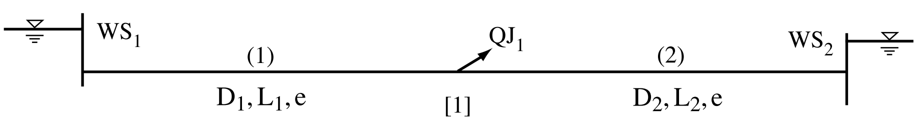


Figure 5.10 A simple two-pipe, two-reservoir network.

problem in which the head is specified at the one node, then only one pipe diameter can be found. There are three basic equations available, one junction continuity equation and two head loss equations, since NJ + NP = 1 + 2 = 3. Below are results from TK-Solver for a set of known values for two cases; in the first case D_1 is unknown, and in the second case D_2 is unknown. This simple network with a pseudo loop (because there are two supply sources) shows that the same principles govern how many diameters can be found for a looped network with two or more supply sources and for a looped network with one supply source. Clearly the number of diameters that can be regarded as unknown equals the number of junction continuity equations. This same principle applies to branched networks.

RULE SHEET

```
S Rule-------------------------------------------------------------------------
  Q1-Q2=QJ1
    WS1-H1=f1*(L1/D1)*Q1^2/(G2*(pi()/4.*D1^2)^2)
    H1-WS2=f2*(L2/D2)*Q2^2/(G2*(pi()/4.*D2^2)^2)
      1/sqrt(f1)=1.14-2*log(e/D1+7.34347283*v*D1/(Q1*sqrt(f1)))
      1/sqrt(f2)=1.14-2*log(e/D2+7.34347283*v*D2/(Q2*sqrt(f2)))
```

Case 1

VARIABLE SHEET

St Input-----	Name----	Output------
	D1	.67179122
.5	D2	
	Q1	2.0806609
	Q2	.58066088
1200	L1	
1000	L2	
1.5	QJ1	
85	H1	
.0001	e	
1.217E-5	ν	
64.4	G2	
	f1	.01569440
	f2	.01840944
100	WS1	
80	WS2	

Case 2

VARIABLE SHEET

St Input-----	Name----	Output------
.8	D1	
	D2	.76556319
	Q1	3.299349
	Q2	1.799349
1200	L1	
1000	L2	
1.5	QJ1	
85	H1	
.0001	e	
1.217E-5	ν	
64.4	G2	
	f1	.01494752
	f2	.01613281
100	WS1	
80	WS2	

Figure 5.11 The rule and variable sheets for the network of Fig. 5.10.

Another view of this two-pipe network problem might be as in Fig. 5.12; now a desired pressure at the downstream end is sought. This specified pressure could equally well be interpreted as a reservoir with a specified water surface elevation, as in the previous example. If so, then the demand at node 2, QJ_2, is unknown since it is the discharge into the downstream reservoir. If the diameters of both pipes are specified, then this is an analysis problem.

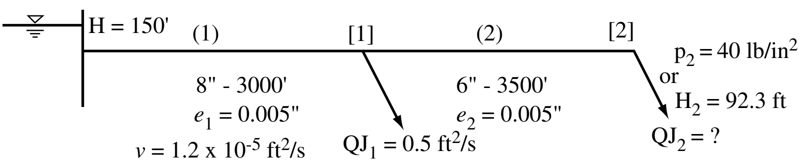


Figure 5.12 An alternative view of the network in Fig. 5.10.

The unknowns are Q_1, Q_2, and QJ_2, and the three equations that are to be solved to determine these unknown values are

Node 1 continuity	$Q_1 - Q_2 - QJ_1 = 0$	(5.23)
Node 2 continuity	$Q_2 - QJ_2 = 0$	(5.24)
Pseudo loop equation	$h_{f1} + h_{f2} = 150 - 92.3 = 57.7$	(5.25)

For this problem the pressure specification in place of the demand at node 2 allows this unknown demand to be computed. The solution requires the first continuity equation and the loop equation to be solved simultaneously (if the Darcy-Weisbach equation is used, then one Colebrook-White equation must be added for each f; so we actually require the simultaneous solution of four equations for the four unknowns Q_1, Q_2, f_1, and f_2), followed by noting from the second continuity equation that $Q_2 = QJ_2$. The results are $Q_2 = QJ_2 = 0.815$ ft^3/s, $Q_1 = 1.315$ ft^3/s, $f_1 = 0.01935$, and $f_2 = 0.02056$. We encourage you to verify this solution.

An alternative would be to pose the question: What pipe diameter D_2 would be needed if the demand QJ_2 were to be 0.6 ft^3/s and the pressure at node 2 were to be $p_2 = 40$ lb/in^2 (HGL = 92.3 ft)? This is now a design problem; in our three equations QJ_2 is known, and the unknowns are Q_1, Q_2, and D_2. A logical sequence in solving this problem would first note that $Q_2 = 0.6$ ft^3/s (the specified demand); next find $Q_1 = 1.1$ ft^3/s from the first continuity equation, and with Q_1 known compute $h_{f1} = 13.77$ ft, leading to $h_{f2} = f_2(L_2/D_2)(Q_2/A_2)^2/(2g) = 31.7f_2/D_2^5 = 43.9$ ft from the loop equation, which when solved with the Colebrook-White equation would produce $f_2 = 0.02157$ and $D_2 = 5.194$ in. If the pressure is also specified at node 2, then both pipe diameters can be found. Then the problem is converted into a branched system with demands known at all three nodes, and the heads are also known at these nodes.

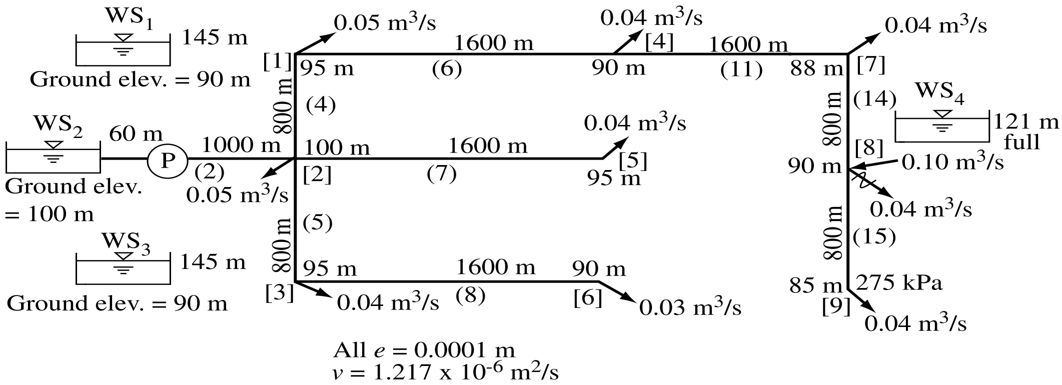
This example illustrates a principle that can be applied to our second looped-network category: each alternate specification allows us to regard another variable as a member of the set of unknowns. In this case if pressures are specified, then diameters can be left unspecified, and the resulting equations can be used to determine these diameters. However, if the pipe roughness coefficients are unknown, then we must specify the diameters. In brief, any variable in a pipe network may be left unspecified while another is specified in its place, so long as the number of independent equations equals the number of unspecified variables, or unknowns, for which a solution is sought.

Flow through a single pipe illustrates this principle. For the Darcy-Weisbach approach six variables appear in the problem: L, D, e, Q, f, and h_f. Two independent equations are available, the Darcy-Weisbach equation $h_f = f(L/D)Q^2/(2gA^2)$ and the Colebrook-White equation $1/\sqrt{f} = 1.14 - 2\,\mathrm{Log}\{e/D + 9.35/(Re\sqrt{f})\}$, that allow two unknowns to be found. Any pair of variables may be selected as unknown, so long as the other four variables are given values. Other equations may appear in this process, such as $A = \pi D^2/4$, $Re = VD/\nu$, and $V = Q/A$; these equations define secondary quantities. More fundamentally, however, these additional variables may be added to the list of variables, and the equations may be added to the list of equations. Then V may be counted as an unknown, for example. For each additional pipe in a network one can add variables to the list of unknowns, and at the same time equations are added to that list. Thus for two pipes the list of six variables becomes 12, and the number of unknowns that can be found increases to four, etc. Almost any combination of variables may be chosen as unknowns.

In summary: (1) We use two basic fluid mechanics principles in the design of pipe systems, the continuity principle (conservation of mass) and the energy principle. (2) The continuity principle assures that the discharge into each junction (or node) in the network equals the discharge from that junction. Mathematically, ΣQ_i - $QJ_j = 0$, in which the subscript on Q denotes the pipe numbers that join at that junction, and QJ is the demand at this junction. (QJ is positive from, and negative to, the junction. The reverse convention applies for pipe discharges; Q_i is positive if to the junction, and negative if from the junction.) (3) The energy principle accounts for the head loss that occurs in a pipe, $H_i - H_j = h_{fk}$, in which subscript k denotes the pipe number and subscripts i and j denote the upstream and downstream node numbers. If every pipe head loss equation is used, then the network connectivity guarantees that the head losses around loops sum to zero and through pseudo loops equals the difference in water surface supply elevations. (4) These two principles provide all of the basic equations that are available. (5) The number of unknowns and independent equations must match for a unique solution to exist. (6) Any variable may be selected as an unknown. Once the unknowns have been chosen, then the remaining variables must be specified. (7) It is possible, however, to assign values to known variables in such a way that physically impossible situations are created.

No one set procedure exists for the design of looped networks. Professional judgment is required to balance the concern for redundancy (i.e., the ability to satisfy large emergency demands, or to allow components to be pulled out of service) with the desire to minimize costs. Since the equations will allow only NJ pipe diameters to be determined, one workable procedure would first select NL = NP - NJ pipes, for which we specify the diameters. The selection of these pipes should be such that, if they were to be removed, the remaining network would be a branched network. Normally there are several pipe combinations that could be selected to reduce a network to a branched system, and this branched system should be considered to be the main transmission lines. The specification of diameters for the pipes (NL in number) that have been selected is also based on judgment; if these pipes are secondary, they might be given diameters that are the minimum size that is allowed for this network. Second, with the heads known at the ends of these pipes, compute their discharges, and then modify the demands at the two pipe ends to include these discharges in defining the branched system. Third, solve the branched network. The diameters that are found for this branched system are normally then replaced by the nearest standard pipe sizes, but they may be rounded up to the next larger standard pipe size. Fourth, conduct analyses that cover a variety of conditions that the proposed network is expected to encounter, and study these results. If deficiencies are noted, adjust the pipe diameters (or other network components) so these deficiencies no longer exist.

To illustrate this procedure, assume that the 16-pipe network in Fig. 5.6 is the subject of a design study. The supply sources denoted by WS_1 and WS_3 are imported from another water supplier with a head of 50 m, but this water is costly and will be used only when demands are large. The other source, WS_2, is from a groundwater well with an aquifer water surface elevation that is 40 m below the ground surface. Lastly, WS_4 is a storage tank with a 45-m diameter, a bottom elevation at 118 m and a maximum depth of 3 m. The average demands are given in the accompanying table, and the demands (in m^3/s) for the hour of greatest demand, on which the design is to be based, are twice these values.



Node	1	2	3	4	5	6	7	8	9
Demand m^3/s	0.025	0.025	0.020	0.020	0.020	0.015	0.020	0.020	0.020

Figure 5.13 Another view of the network of Fig. 5.6.

First NP - NJ = 16 - 9 = 7 pipes must be given diameters, say 150 mm, the smallest size allowed in this system. Pipes 1, 3, 9, 10, 12, 13, and 16 are selected, based on judgment. Pipes 1 and 3 are chosen because they supply the expensive water and will be shut off during this design process. Using the maximum capacity of the pumping station, which is 0.22 m^3/s with all pumps on, the demands are summed, and it is determined that the storage tank must supply 0.14 m^3/s. Therefore, the demand at node 8 is changed from 0.04 m^3/s to an inflow of 0.10 m^3/s, and pipe 16 is removed. Elimination of these pipes results in the branched system in the figure. Of course, depending upon the choice for the main transmission system, there are several alternatives that could be explored. For this reduced system, node 9 is the farthest downstream, and its pressure should be set to the minimum allowable pressure, say 275 kPa, which corresponds to $H_9 = 113$ m for the elevation of the HGL. The pipe discharges in this branched network can now be determined directly, as given below in the first table. Based on energy-line slopes, which are also in this table, and on economic considerations, the heads at the nodes can be computed; they are listed in the second table. By solving the Darcy-Weisbach and Colebrook-White equations simultaneously for the 9 single pipes, the diameters can then be computed. These computed diameters, also listed in the first of Tables 5.4, should be replaced by sizes chosen from a set of standard sizes (such as the following: 150, 205, 255, 305, 355, and 405 mm). As a final step, analyses of the full network should be completed for several different demand levels, storage tank levels, fire flows, etc.

Table 5.4

Pipe	**Q** m^3/s	S_f	h_f m	**Dia.** mm
2	0.23	0.005	5.0	428.6
4	0.07	0.006	4.8	263.1
5	0.07	0.005	4.0	272.7
6	0.02	0.010	16.0	148.1
7	0.04	0.002	4.0	253.0
8	0.03	0.005	8.0	197.9
11	0.02	0.0025	4.0	194.8
14	0.06	0.005	4.0	257.5
15	0.04	0.005	4.0	220.6

Node	**H** m
1	125.0
2	129.8
3	125.8
4	109.0
5	125.8
6	117.8
7	113.0
8	117.0
9	113.0

The newly-found diameters in Table 5.4 ignore any influence of the pipes that were removed. This approach assumes that the discharges in the other (ignored) pipes assist the network in performing adequately under the variety of conditions that will occur, but the branched system can by design supply the required demands without the other pipes. In a sense it also assumes that the discharges carried by the pipes that are ignored is small in comparison to the discharges in the pipes that are retained throughout the computations.

Next let us explore the design process further by attempting another design without ignoring the flows in pipes 1, 3, 9, 10, 12, and 13 that were removed to form the branched network. Assume these all have a diameter of 150 mm and that pipes 1 and 3 (that bring water from the more costly water supply) are open. With the additional flow from these two pipes let us also assume that no flow enters or leaves the storage tank through pipe 16, and that the head losses in the pipes are as specified in the third part of Tables 5.5. Under these assumptions the flow in pipes 11 and 14 now reverse direction from what they were previously. Consequently the heads at the nodes are as given in the second part of Tables 5.5, and with these heads the discharges in the other pipes can be computed, as listed in the first of Tables 5.5:

Tables 5.5

Pipe	h_f m	L m	Dia. mm	Q m^3/s
1	16.0	800	150	0.030
3	15.8	800	150	0.029
9	4.8	800	150	0.016
10	8.0	800	150	0.021
12	12.8	1600	150	0.018
13	8.8	1600	150	0.015

Node	H m
1	129.0
2	133.8
3	129.8
4	125.0
5	129.8
6	121.8
7	121.0
8	117.0
9	113.0

Pipe	h_f m	Q m^3/s	Dia. Mm
2	5.0	0.311	480.8
4	4.8	0.131	334.1
5	4.0	0.034	207.5
6	4.0	0.111	372.4
7	4.0	0.095	351.1
8	8.0	0.025	161.2
11	4.0	0.087	339.5
14	4.0	0.047	234.5
15	4.0	0.025	184.7

If the assumption was, as before, that the storage tank was supplying $0.14\ m^3/s$ along with the supply through pipes 1 and 3, and the HGL-elevations at the nodes was as before, then the results in Tables 5.6 would be obtained:

Tables 5.6

Pipe	h_f m	L m	Dia. mm	Q m^3/s
1	20.0	800	150	0.033
3	19.2	800	150	0.033
9	8.8	800	150	0.022
10	8.0	800	150	0.021
12	8.8	1600	150	0.015
13	4.0	1600	150	0.011
16	4.0	1200	384	0.140

Node	H m
1	125.0
2	129.8
3	125.8
4	109.0
5	125.8
6	117.8
7	123.0
8	117.0
9	113.0

Pipe	h_f m	Q m^3/s	Dia. Mm
2	5.0	0.114	480.8
4	4.8	- 0.011	*
5	4.0	0.027	190.2
6	8.0	- 0.028	*
7	4.0	0.098	355.2
8	8.0	0.020	169.8
11	4.0	0.046	266.8
14	4.0	0.086	294.9
15	4.0	0.029	195.4

* The heads do not allow a negative Q.

The newer set of assumptions has led to an impossible situation in which the junction con-tinuity equations require flows in pipes 4 and 6 in the opposite direction from what the heads at their ends require. The specified flow from the storage tank was too large to be compatible with the heads and pipe diameters that were specified. In the earlier case the absence of flow from the storage tank avoided the impossible situation that was created in the last set of specifications. However, we see clearly that various combinations of specified variables can lead to situations in which the direction of flow is inconsistent with the heads at some nodes.

One even simpler example of an inappropriately specified diameter consists of two pipes which meet at junction [2]; the HGL at this junction is smaller than the HGL at the other ends of these pipes, as shown in Fig. 5.14. The discharge for each pipe must be toward the common junction. If the diameter of either pipe is specified so that the resulting discharge in that pipe exceeds the demand QJ_2, then an impossible situation has been created, since the direction of the discharge in the other pipe must oppose the direction of flow implied by the HGL for that line.

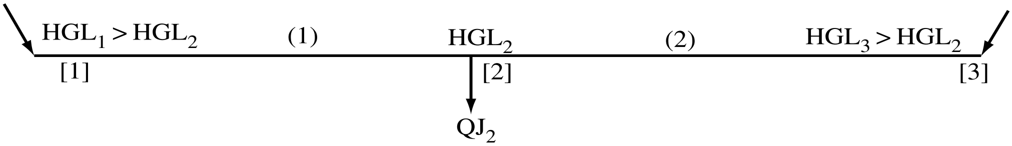
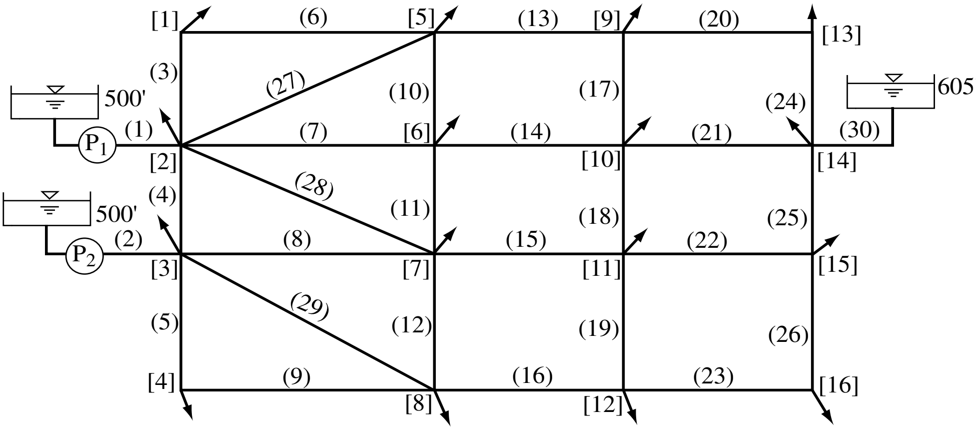


Figure 5.14 A problem with an inappropriate diameter.

For example, let HGL_1 = 100 ft., HGL_2 = 88 ft., HGL_3 = 90 ft., L_1 = 2000 ft., L_2 = 2500 ft. and QJ_2 = 1.0 ft^3/s. If D_1 = 8.0 in, then Q_1 = 1.15 ft^3/s, and we must have Q_2 = - 0.15 ft^3/s to satisfy continuity at the junction, which is inconsistent with the set of specified heads. If Q_1 = 1.0 ft^3/s, then D_1 = 7.60 in; hence D_1 must be less than 7.6 in for a solution to be possible.

These analyses illustrate the important fact that the outcome of a design depends directly upon the assumptions that go into that design. While cost has not yet been considered in these designs, the usual objective is to minimize the total cost of meeting a set of specified demands. We will include cost considerations later in the chapter.

Now let's examine a larger network of 30 pipes and 16 nodes, as shown in Fig. 5.15. For this network with 3 supply sources there are 16 junction continuity equations, and it is therefore possible to determine 16 pipe diameters if the heads are given at all 16 nodes. The input data to obtain a "design" solution by using NETWK is in the file FIG5_15.IN on the CD. The reader can list this file and use it as input to NETWK to obtain a solution. This input lists the pipe lengths and the nodal demands. If the option DESIGN=1 is given in the $SPECIF list, then (1) NETWK interprets a 0 for a diameter as one that is to be determined, and (2) the elevation of the HGL at each node must be listed after the elevation of the node under the NODES command. Thus for this network with DESIGN=1 we must assign 16 pipes a zero diameter in the input data. The example input data set has assigned diameters of 18 in and 15 in, respectively, to the two pipes from the source pumps, and pipes 10, 11, 12, 17, 18, 19, 24, 25, 26, 27, 28, and 29 have been given diameters of 6 inches. This problem is quite large for a hand solution, but the approach to a solution, if done by hand, could follow precisely the approach that was applied to the networks that have just been examined. First, the discharges in the pipes with specified diameters would be computed so that each pipe head loss matches the difference in head between its end nodes. These pipes would then be removed from the network, and the demands at their ends would be adjusted for their discharges. Next, from the junction continuity equations the discharges in the remaining pipes would be determined, and finally, with these discharges known, the diameters of the remaining pipes would be found.



Pump 1

Q, ft^3/s	8	15	22
h_p, ft	157	152	144

Pump 2

Q, ft^3/s	6	12	18
h_p, ft	152	147	139

Figure 5.15 A 30-pipe, 16-node network.

In the solution from NETWK we find after the design solution has computed the pipe diameters that NETWK then selects the closest standard diameter from its default list of standard diameters and performs an analysis of this system since the option NOMSOL=1 is included in the $SPECIF list. From the analysis the column giving HGL elevations will change from the initially specified values because the standard pipe sizes will not produce the same frictional head losses.

The program NETWEQST in the NETWK program package is intended specifically for design problems. It allows the user to specify the unknown variables and allows any of the variables in the network to be regarded as an unknown. The mechanics of this solution will be explained later in this chapter. By selecting all of the pipe discharges and a number of pipe diameters that is equal to the number of nodes in the network, this type of design problem can be solved. The input data file for NETWEQST for this problem follows.

```
Large Design Example
/*
$SPECIF IUNENT=4 $END
PIPES
1 0 2 500 18 .004 15
2 0 3 500 15 .004 11
3 2 1 800 12 .004 5
4 2 3 800 6  .004 1
5 3 4 800 12 .004 6
6 1 5 1800 6 .004 5
7 2 6 1800 12 .004 6
8 3 7 1800 12 .004 6
9 4 8 1800 12 .004 3
10 6 5 800 6  .004 .5
11 6 7 800 6  .004 .5
12 7 8 800 6  .004 .5
13 5 9 1600 12 .004 5
14 6 10 1600 12 .004 5
15 7 11 1600 12 .004 5
16 8 12 1600 12 .004 5
17 10 9 800 6 .004 1
18 10 11 800 6 .004 .2
19 11 12 800 6 .004 .2
20 9 13 1600 12 .004 4
21 10 14 1600 8 .004 1
22 11 15 1600 8 .004 2
23 12 16 1600 8 .004 1
24 14 13  800 6 .004 1
25 14 15  800 6 .004 1
26 15 16  800 6 .004 1
27 2 5 2500 6 .004 1
28 2 7 2500 6 .004 1
29 3 8 2500 6 .004 1
30 0 14 1000 10 .004 2.5
RESER
30 605
PUMPS
1 8 157 15 152 22 144 500
2 6 152 12 147 18 139 500
NODES
1 1.2 500 630
2 1.2 490 645
3 .8 485 640
4 1.6 480 632
5 1.4 495 618
6 1.2 494 620
7 1. 490 616
8 .8 483 613
9 2. 493 605
10 2 492 608
11 3.6 488 605
12 2.8 484 603
13 4. 480 595
14 2 478 600
15 1.8 475 594
16 2 470 586
RUN
```

Figure 5.16 Input for NETWEQST.

The option IUNENT=4 tells NETWEQST that the HGL-elevations at the nodes are listed as the last item after the NODES command, and the last item on each line following the PIPES command is an initial estimate of the discharge in that pipe, to be used to start the Newton solution method. The manual for NETWEQST is on the CD as file NETWEQST.DOC. To solve this problem with NETWEQST, the responses listed in Fig. 5.17 should be provided in response to the **bold** prompts from NETWEQST.

Pipes = 30, Nodes = 16, Sources = 3
46 unknowns must be given. Give no. of each:
1. HGLs at nodes 0
2. Nodal demands 0
3. Pipe discharges 30
4. Pipe diameters 16
Give 16 pipe diameter numbers 3-9 13-17 20-23 30

Figure 5.17 Prompts and responses for the 30-pipe example.

In obtaining solutions to these design problems, one must have considerable understand-ing of either the system performance or the sizes of the specified diameters; otherwise the corresponding set of specified heads can lead to an impossible situation. We have already seen how such a situation can occur for the 16-pipe network. So we cannot select arbitrari-ly all of the pipes that will be assigned a prescribed diameter. Since the pipes having specified diameters carry a fixed discharge, the network problem becomes in essence one with these pipes removed. The reduced network must still be able to satisfy all specified demands at the nodes. There are different combinations of circumstances that may make this impossible to do. First, if in creating the reduced network the original network has become divided into two or more separate networks, then each separate network must have at least one supply source. Second, in the reduced network the specified heads must allow the flow to move in the direction that is dictated by the demands. Furthermore, we know it will not be possible to prescribe a diameter for every supply pipe in the network, because the resulting set of computed discharges (that are fixed by the prescription of diameters and of heads at the end nodes) will generally not sum to the total demand in the network.

In the 30-pipe problem the diameters of the pipes connected to the source pumps were both given, but the reservoir pipe diameter was not given. If D_{30} is given, then either D_1 or D_2 must not be given. The heads may remain unchanged if D_{30} is given and D_1 is not given (but still with D_2 = 18 in). However, if D_2 is not given when D_{30} is given as 6 in, then D_1 must be given a diameter that is larger than 18 in, because with D_1 = 18 in the solution of the continuity equations produces a negative flow in pipe 4, but this is not possible for the heads that are given at the ends of that pipe.

Assigning diameters to pipes that connect to the source pumps fixes the discharge that these pumps can supply; therefore the discharge through the pipe from the reservoir must equal the difference between the sum of the demands on the network and the amount of the discharge from the two source pumps. With these restrictions it is difficult to create even one loop in the reduced network. Therefore, we must verify that a branched network is obtained when the pipes having specified diameters are removed from the network, and if the removal of these pipes separates the original network into two or more smaller networks, then each of these new networks must have a supply source. Nor can we specify the diameter of a dead end pipe because, with a specified diameter and a pair of given heads at the ends of the line, the computed discharge usually will not match the specified demand at the end of the line.

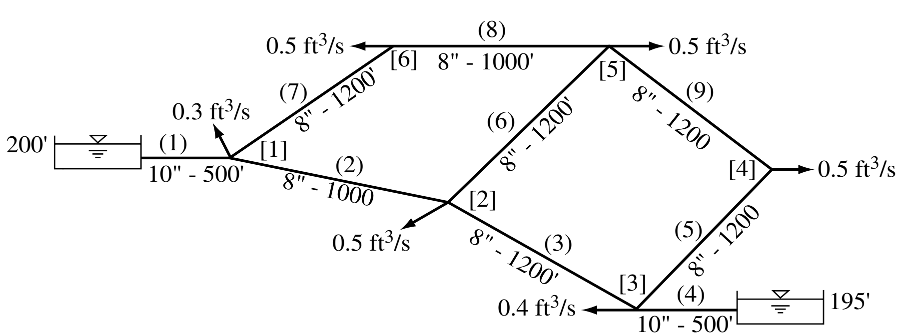
Less obvious impossible situations can develop. For example, if the diameter of one or more pipes that contain source pumps is specified to be too large so that the inflow to the network from this/these source/s exceeds the sum of the network demands, and if furthermore the reservoir water surface elevation were specified to be above H_{14}, then no way exists for the surplus inflow from the source pumps to leave the system. Since the specification of the pipe diameter and the head at the ends of this pipe fixes its discharge, this discharge value may not match the external demand at the end of the pipe. For example, if in the 30-pipe network the diameter of pipe 1 were set at 24 in rather than 18 in, such an impossible situation would be the result. On the other hand, if D_{30} is given and D_2 is not, then $D_1 = 18$ in would cause an impossible situation but $D_1 = 24$ in would not. When more than two pipes meet at a junction, the possibilities of creating impossible situations are more numerous and more complex. NETWK does detect the existence of impossible situations and then prints a brief message related to the problem. When this occurs (and it will occur frequently if one is not sufficiently careful and/or experienced in the specification of diameters and heads), it is important first to examine carefully the possible causes of this situation; then the values of the specified diameters and/or heads can be adjusted, or an alternate set of pipes can be selected for the specification of diameters. In making these adjustments, it may be helpful to sketch the reduced network after the pipes with specified diameters have been removed, and then to keep in mind the process that will be followed in obtaining this design.

Example Problem 5.4

Designs are to be obtained for the looped water distribution system below, given the heads at the nodes listed in the table. Since three independent loops (two real loops and one pseudo loop) exist here, the sizes of three pipes must be specified, and the diameters of the remaining six pipes are to be found. All specified pipe diameters are to be 6 inches. Determine whether the assignment of 6-inch diameters to the following combinations of pipes will allow a solution of the remainder of the branched system; if an impossible situation has been created, determine why this is the case. Otherwise solve the branched network.

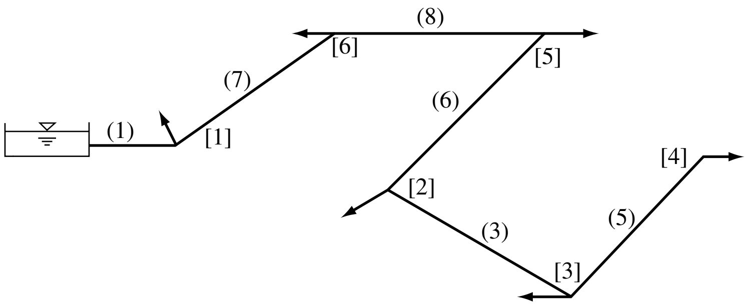
Node	Elev. ft.	H ft.
1	100	197
2	98	194
3	100	194.5
4	95	190
5	95	191
6	90	188

Case	Spec. Pipe
1	2, 4, 9
2	4, 6, 9
3	4, 8, 9
4	3, 8, 9
5	2, 8, 9
6	2, 6, 9

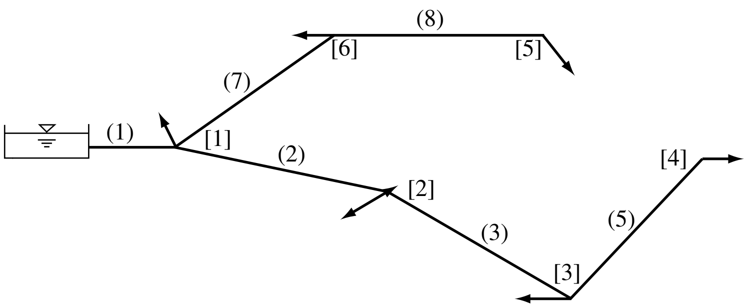


The first steps are to determine the discharges in the pipes with specified diameters and then reduce the network to the branched system. Sketches for these reduced branched systems will be presented.

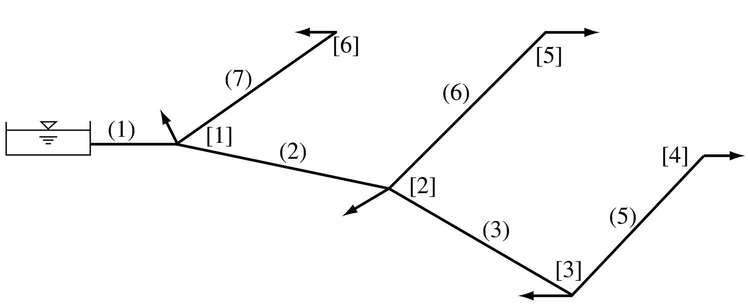
Case 1:
This case is not valid since $H_3 > H_2$, so the flow cannot pass through pipe 3 to meet downstream demands.



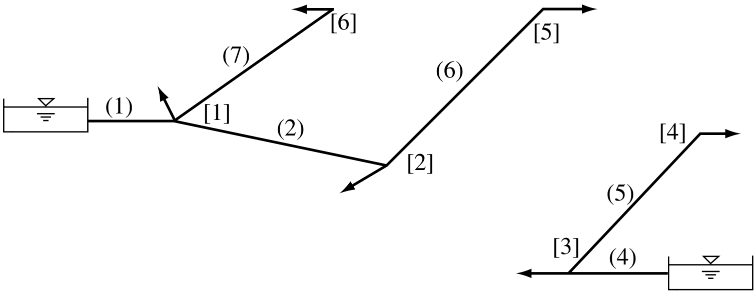
Case 2:
This case is not valid for the same reason as Case 1.



Case 3:
This case is not valid since $H_3 > H_2$; thus the flow cannot satisfy the demand at nodes 3 and 4.



Case 4:
This configuration is valid. We first compute the discharges in the 6-inch pipes.



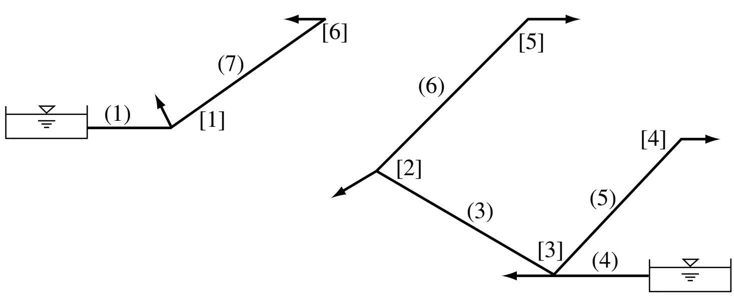
Pipe	Q ft^3/s	f	Changed QJ's ft^3/s
3	0.145	0.024	$QJ_3 = 0.545$, $QJ_2 = 0.355$
8	0.435	0.020	$QJ_5 = 0.935$, $QJ_6 = 0.065$
9	0.214	0.023	$QJ_5 = 1.149$, $QJ_4 = 0.286$

The solution for the reduced branched system provides the following discharges and pipe diameters:

Pipe	Q ft^3/s	Dia. in.
1	1.869	9.000
2	1.504	9.556
4	0.831	9.592
5	0.286	4.902
6	1.149	8.965
7	0.065	2.446

Case 5:

This case is valid. A solution can be obtained with NETWEQST. (Alternatives are to use NETWK or apply HYDEQS to obtain individual discharges and/or diameters.) The table of input data, the list of prompts and responses, and two tables of results follow:




```
Input Data
Example Problem 5.4
/* 1 .3 100 197
$SPECIF IUNENT=4 $END
PIPES
1 0 1 500 10 .001 2.4
2 1 2 1000 6 .001 1
3 3 2 1200 6 .001 .2
4 0 3 500  6 .001 1
5 3 4 1200 6 .001 .5
6 2 5 1200 6 .001 .5
7 1 6 1200 1 .001 1
8 5 6 1000 6 .001 .2
9 5 4 1200 6 .001 .2
NODES

2 .5 98 194
3 .4 100 194.5
4 .5 95 190
5 .5 95 191
6 .5 90 188
RESER
1 200
4 195
RUN
```

Pipes= 9, Nodes=6
15 Unknowns must be given. Give no. of each:
1. HGLs at nodes 0
2. Nodal demands 0
3. Pipe discharges 9
4. Pipe diameters 6
Give 6 pipe diameter numbers 1 3-7

PIPE DATA

PIPE NO.	NODES FROM	TO	L ft.	DIA. in	e x10³ in	Q ft³/s	HEAD LOSS ft.
1	0	1	500	6.539	1.0	0.800	3.00
2	1	2	1000	6.000	1.0	0.435	3.00
3	3	2	1200	13.247	1.0	1.214	0.50
4	0	3	500	13.079	1.0	1.900	0.50
5	3	4	1200	4.902	1.0	0.286	4.50
6	2	5	1200	8.965	1.0	1.149	3.00
7	1	6	1200	2.445	1.0	0.065	9.27
8	5	6	1000	6.000	1.0	0.435	3.00
9	5	4	1200	6.000	1.0	0.214	1.00

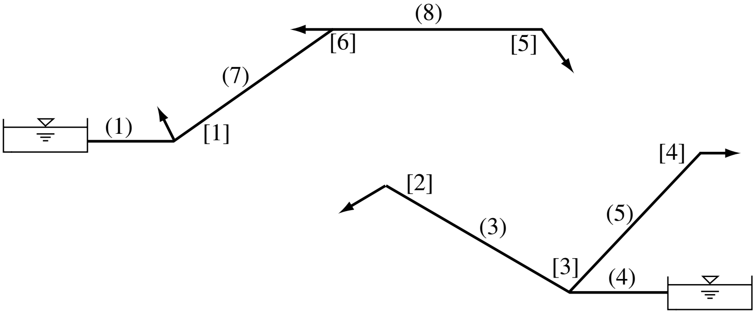
NODE DATA

NODE	DEMAND ft³/s	ELEV. ft.	HEAD ft.	PRESSURE lb/in²	HGL ELEV. ft.
1	0.300	100	97.0	42.0	197.0
2	0.500	98	96.0	41.6	194.0
3	0.400	100	94.5	41.0	194.5
4	0.500	95	95.0	41.2	190.0
5	0.500	95	96.0	41.6	191.0
6	0.500	90	98.0	42.5	188.0

Case 6:

The case is valid. Using the same input as in case 5, the solution can be found with the following responses to the prompts from NETWEQST (Solution not given):

Pipes = 9, Nodes = 6
15 unknowns must be given. Give no. of each:
1. HGLs at nodes 0
2. Nodal demands 0
3. Pipe discharges 9
4. Pipe diameters 6
Give 6 pipe diameter numbers 1 2-5 7 8



* * *

5.4 DESIGNING SPECIAL COMPONENTS

Section 5.3 defined two types of design problems: (1) problems in which as many pipe diameters are sought as there are nodes in the network; and (2) problems in which an individual pipe diameter is sought so that some specified condition (e.g., a pressure) occurs at a prescribed node. That section examined the first problem category. This section considers the second problem type. Previously the solution of such problems involved a trading of known and unknown variables. For example, if a nodal pressure was specified, then a nodal demand (or pipe diameter or length, etc.) was placed in the list of unknowns. Now, however, it will not be necessary to swap a variable from the known to the unknown list. Instead a new unknown will be introduced into the network problem, and a new equation will be added to the list of equations, thus satisfying the requirement that the number of independent equations and the number of unknowns must match.

How is it possible to obtain another independent equation, one might ask. As was stated before, the basic network relations are

$$NP = NJ + NL \quad \text{if the network has two or more supply sources}$$

or

$$NP = (NJ - 1) + NL \quad \text{if the network has fewer than two supply sources.}$$

These relations apply to both branched and looped networks and cannot be changed. In a branched network $NP = NJ - 1$ and therefore $NL = 0$. The key in introducing an additional unknown is also to create another independent equation. This additional equation will enforce another condition, usually a nodal HGL (or pressure) that is required. This unknown will be called a differential head device, and in mathematical equations it will be given the symbol ΔH; it will represent a variety of devices such as a booster pump, a pressure-reducing station, a valve, a pipe with an unknown diameter, or a wall roughness. A differential head device will be something that creates a (positive or negative) head difference, other than the frictional head loss of the original pipe, between the ends of a pipe. If the pipe diameter or roughness is to be unknown, then the new pipe will produce a head loss that is the sum of the frictional head loss of the original pipe and the computed differential head. The equation for this additional unknown, ΔH, that will be added to the equation system will be an energy equation that is written between the node where the pressure (or pressure head, or HGL elevation) is specified and another point of known head in the network. This other point of known head will usually be a supply source such as a reservoir or source pump. However, if two or more differential head devices are introduced, then the added equation might be an energy equation between two nodes with specified pressures. The additional equation is a special pseudo loop that will generally be independent of the other loops because it imposes an additional condition on the network that requires a pipe to have a different head loss than that which is caused by fluid friction alone. The phase "generally independent" is used because, as described later, inappropriate specifications can cause the added equation not to be independent. This loop is called special because it consists of a continuous path along pipes from an internal pipe, one end of which has a specified HGL, to a supply source, whereas the usual pseudo loop follows a sequence of pipes between two supply sources. Thus this special loop is similar to a pseudo loop connecting the downstream end of a pressure-reducing valve (PRV) or the upstream end of a back-pressure valve (BPV) to a supply source or to another PRV or BPV where the HGL is specified.

To illustrate the concept and implementation of a differential head device, the small network consisting of 7 pipes and 5 nodes in Fig. 5.18 will be examined. In this network

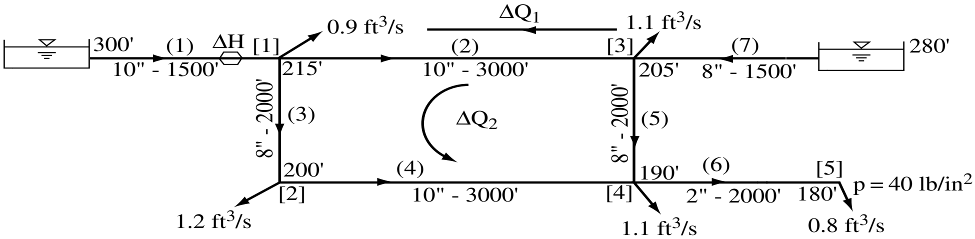


Figure 5.18 Small network with a differential head device.

the amount (magnitude and sign) of the differential head ΔH that is needed in pipe 1 is to be determined so that the pressure at node 5 is 40 lb/in^2. In addition to ΔH, the solution is to determine the discharges in all 7 pipes and the elevations of the HGL (and pressures) at the 4 internal nodes. (The pressure is specified at node 5, so this pressure can not be part of the solution.) In this problem ΔH in pipe 1 could be a booster pump if it is positive, or it could be a valve if it is negative. If it is negative, the differential head device may instead be a pressure-reducing valve; once ΔH is known, it is a simple computation to determine the pressure setting for a PRV which would produce this same additional head loss. Or if ΔH is positive but smaller in magnitude than the frictional head loss in the pipe containing it, then one could compute an "equivalent" pipe diameter that would produce the same head loss as the present frictional head loss and ΔH. Thus, depending upon what the solution produces for ΔH and what you want the differential head device to represent, it can be any of a variety of appurtenances.

If the ΔQ-system of equations is to model this network, the system may be written as

$$\begin{aligned} K_7\left(Q_{o7}+\Delta Q_1\right)^{n_7} - K_2\left(Q_{o2}-\Delta Q_1-\Delta Q_2\right)^{n_2} \\ - K_1\left(Q_{o1}-\Delta Q_1\right)^{n_1} + \Delta H + 20 = 0 \end{aligned} \tag{5.26}$$

$$\begin{aligned} K_3\left(Q_{o3}+\Delta Q_2\right)^{n_3} + K_4\left(Q_{o4}+\Delta Q_2\right)^{n_4} \\ - K_5\left(Q_{o5}-\Delta Q_2\right)^{n_5} - K_2\left(Q_{o2}-\Delta Q_1-\Delta Q_2\right)^{n_2} = 0 \end{aligned} \tag{5.27}$$

$$\begin{aligned} K_6 Q_{o6}^{n_6} + K_5\left(Q_{o5}-\Delta Q_2\right)^{n_5} \\ + K_7\left(Q_{o7}+\Delta Q_1\right)^{n_7} + \left(40(144)/62.4+180\right) - 280 = 0 \end{aligned} \tag{5.28}$$

in which the vector of initial flows that satisfy all junction continuity equations might be

$$\left\{\boldsymbol{Q}_o\right\} = \begin{Bmatrix} Q_{o1} \\ Q_{o2} \\ Q_{o3} \\ Q_{o4} \\ Q_{o5} \\ Q_{o6} \\ Q_{o7} \end{Bmatrix} = \begin{Bmatrix} 2.5500 \\ -1.3533 \\ 3.0033 \\ 1.8033 \\ 0.0967 \\ 0.8000 \\ 2.5500 \end{Bmatrix} \tag{5.29}$$

The three unknowns are ΔQ_1, ΔQ_2, and ΔH. The solution of these three equations (using Newton's method with the above vector for $\{\mathbf{Q_o}\}$) yields

$$\begin{Bmatrix} \Delta Q_1 \\ \Delta Q_2 \\ \Delta H \end{Bmatrix} = \begin{Bmatrix} -4.33 \\ -0.59 \\ 112.62 \end{Bmatrix} \tag{5.30}$$

To verify that the solution is correct, one must use the correct values for K and n for each pipe, which are listed in the following table:

Pipe No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**
K	1.70	2.55	7.34	3.70	7.46	31.8	5.51
n	1.957	1.931	1.932	1.873	1.865	1.905	1.918

Any verification should also employ ΔQ_1, ΔQ_2, and ΔH to compute the HGL elevations and the pressure at each node.

It is much easier to let the computer do the arithmetic. Below is the input data file for NETWK to solve this problem, followed by the output. The line of input data after DHEAD consists of the following items: (1) the pipe containing ΔH; (2) an estimate of this ΔH; (3) the pressure that is being specified (the minus indicates that the pressure is in lb/in^2, rather than being specified as a HGL); (4) the designation (pipe since NODESP = 0) for a supply source, to use in forming the energy equation loop; and (5) the pressure in lb/in^2 at node 5. This solution file contains an extra table for differential head devices; it reports an INCREMENTAL HEAD of 112.62 ft and also states NO EQUIVALENT DIA. POSSIBLE. Had the value for HEAD LOSS minus the INCREMENTAL HEAD been negative, then an EQUIVALENT DIAMETER would have been reported in the last column of this extra table, as well as e and the head loss in the equivalent pipe.

```
Problem for Differential Head Device
/*
$SPECIF NPRINT=-3,COEFRO=.004 $END
PIPE-
1 10. 1500. 1 .9  215.
2 10. 3000. 1 3 1.1 205.
3 8. 2000. 1 2 1.2 200.
4 10. 3000. 2 4 1.1 190.
5 8. 2000. 3 4
6. 2000. 4 5 .8 180.
7 8. 1500. 3
RESER
1 300
7 280
DHEAD
1 40 -5 7 40.
RUN
```

Figure 5.19 Input data for NETWK for the differential head device problem of Fig. 5.18.

Problem for Differential Head Device
ALL DEMAND FLOWS ARE MULTIPLIED BY 1.0000
FLOW FROM PUMPS AND RESERVOIRS EQUALS 5.100

PIPE NO.	ORIG. DIA.	Q	INCR. HEAD	HEAD LOSS	TOTAL HEAD	e	EQUIV. DIA.
	in	ft^3/s	ft.	ft.	ft.		in
1	10.0	6.88	112.62	74.2	NO EQUIV.	DIA. POSSIBLE	

PIPE DATA

PIPE NO.	NODES FROM	TO	L	DIA.	e x10^3	Q	VEL.	HEAD LOSS	HLOSS /1000
			ft.	in		ft^3/s	ft/s	ft.	
1	0	1	1500	10	4.0	6.88	12.62	74.2	49.5
2	1	3	3000	10	4.0	3.58	6.56	41.6	13.9
3	1	2	2000	8	4.0	2.41	6.89	40.1	20.0
4	2	4	3000	10	4.0	1.21	2.21	5.25	1.8
5	3	4	2000	8	4.0	0.69	1.99	3.77	1.9
6	4	5	2000	6	4.0	0.80	4.07	20.8	10.4
7	3	0	1500	8	4.0	1.78	5.11	16.9	11.3

NODE DATA

NODE	DEMAND ft^3/s	gal/min	ELEV. ft.	HEAD ft.	PRESSURE lb/in^2	HGL. ELEV ft.
1	0.9	404	215	123.4	53.5	338.4
2	1.2	539	200	98.4	42.6	298.4
3	1.1	494	205	91.8	39.8	296.9
4	1.1	494	190	103.1	44.7	293.1
5	0.8	359	180	92.3	40.0	272.3

Figure 5.20. Solution using the input data listed in Fig. 5.19.

This problem might also be altered, for example, to specify a pressure of 40 lb/in^2 at node 2. This could be done by placing a special differential head device in pipe 3. We encourage you to modify the input file in Fig. 5.19, compare it with file FIG19.IN, obtain a solution and then compare it with file FIG19.OUT on the CD.

The foregoing example might cause a person to believe that it is possible to specify a pressure anywhere within a network and place a differential head device in any pipe. However, this is not the case; a problem can be specified for which there is no solution. An example is the specification of pressures at nodes 4 and 5 in this network without a differential head device in pipe 6. The discharge in pipe 6 must be 0.8 ft^3/s to satisfy the specified downstream demand, and this discharge dictates the head loss in pipe 6. Therefore the specification of pressures at both ends of pipe 6 without placing a differential head device in this pipe will result in an insoluble problem. Similar situations can be created.

In using the DHEAD command with NETWK, one must be relatively familiar with the performance and nature of the network if the specification of impossible situations is to be avoided. Should an impossible situation be specified, then NETWK will be unable to complete a solution. In some instances the iterative solution process will simply fail to converge; this condition becomes apparent when the number of iterations exceed the allowable maximum and the residual, reported as SUM or SUM OF DIFFERENCE, is not becoming smaller. Or NETWK will indicate that a singular matrix exists; then an examination of the system of equations should allow one to discover why the singular matrix exists. However, often it is easier simply to examine the network and the system specifications until it becomes apparent how to change the input data to allow a solution.

Aside from dead-end pipes with pressures specified at both ends, let us look further at some other conditions that can lead to an improperly posed problem. If we specify a larger HGL downstream from a node with a smaller HGL, then we must place a differential head device in one of the pipes between these two nodes. Or in a network with all of its supply sources in one subregion or at one end of the system, the specification of the HGL elevations must allow it to decrease continually in the downstream direction through the network, unless differential head devices have been placed in some of the intermediate pipes. And differential head devices that produce negative incremental heads will be needed in some pipes if HGL elevations are specified to decrease more rapidly in the downstream flow direction than can be caused by pipe friction alone. Experience shows it is difficult to avoid the creation of an impossible situation if the differential head devices are all located near the supply sources. In general, if HGL elevations are to be specified at several nodes, then the pipes containing differential head devices should also be near these nodes.

A network, diagrammed in Fig. 5.21, will illustrate some less easily recognized specifications that will cause an impossible situation. To receive maximum benefit from this description, the reader is encouraged to prepare input data for this network and actually obtain solutions etc. as the next few pages are read. This network is a skeletonized system for a small city. The supply for the network comes from a single source outside of town. A storage tank has been installed near the old main part of town to supply some of the demand during periods of above-average usage, and to receive water from the pump when demand is low. The town has grown, expanding into some areas with slightly higher elevations, especially to the west of the storage tank.

The present pump is not adequate; to begin the study we decide to seek a solution that will tell us the pump head that will meet the demand shown in Fig. 5.21 when the storage tank neither receives nor supplies any water. To set up the problem for NETWK, the DHEAD command can be used to indicate that pipe 1 contains the differential head device and that the elevation of the HGL at node 3 should be 580 ft., which is exactlythe elevation of the water surface in the storage tank.

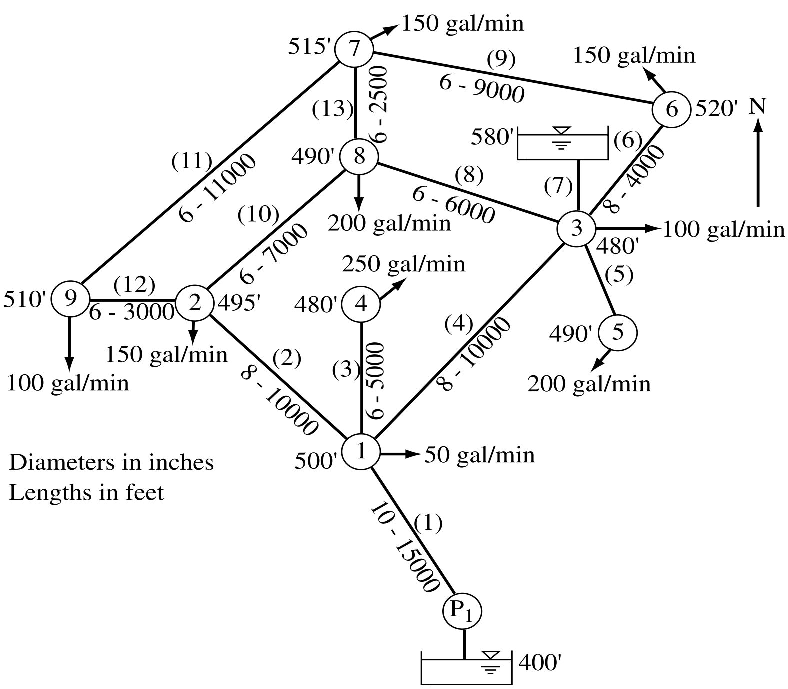


Figure 5.21 A skeletonized network for a small city.

We might remove the source pump, add up the demands and specify this sum as a negative demand at the node which replaces the source, i.e., 10. This problem might seem reasonable because the demands would be exactly satisfied by the flow from this new node, and the elevation of the HGL will be established throughout the system by retention of the connection between the reservoir and the network. However, we have created a problem for which there is no solution. The network has only one supply source. With a specified differential head device we must have a reservoir whose discharge is unknown in order to define the additional equation that is needed for a solution. We have created problem specifications that cause the flow from the reservoir to be known and equal to zero. The unknowns for this problem are the three corrective discharges ΔQ_1, ΔQ_2, and ΔQ_3, around the three loops of the system, plus the incremental head in pipe 1. Therefore four independent equations are required. Three of these are the energy equations for the three loops of the system. The fourth equation forces the head loss in pipe 7 to equal the dif-ference between the water surface elevation in the reservoir and the specified HGL at node 3; this equation is invalid because the flow in pipe 7 is not unknown. (We might run NETWK to attempt to solve this improperly-posed problem, using NPRINT=1 or larger, so the output could be studied.) From this problem we see that at least two supply sources must exist if a differential head device is to be used in a network.

The specification of the impossible might be avoided by treating the pump as a second supply source. It may be changed to a reservoir with a suitably chosen water surface elevation or the original pump could be retained. If the reservoir option is selected, the differential head reported by NETWK is the head that the pump should supply; if the existing pump is kept, the reported head will be the additional head needed by the new pump over that which is supplied by the existing pump. Figure 5.22 presents a suitable input data file for this problem. (The CD contains this file as FIG22.IN.)

```
SIZING A PUMP - RESERVOIR FLOW
SHOULD BE ZERO FOR DESIGN
/*
 $SPECIF NPRINT=10,NFLOW=1,NPGPM=1 $END
PIPES                                   NODES
 1 0 1 15000 10  .005                    1 50 500
 2 1 2 10000 8/                          2 150 495
 3 1 4 5000 6/                           3 100 480
 4 1 3 10000 8/                          4 250 480
 5 3 5 4000 6/                           5 200 490
 6 3 6 4000 8/                           6 150 520
 7 0 3 2000 8/                           7 150 515
 8 3 8 6000 8/                           8 200 490
 9 6 7 9000 6/                           9 100 510
 10 2 8 7000 6/                         RESER
 11 9 7 11000 6/                         7 580
 12 2 9 3000 6/                          1 400
 13 8 7 2500 6/                         DHEAD
                                         1 250 3 7 580
                                        RUN
```

Figure 5.22 NETWK input data file for the skeletonized network.

In this input data two reservoirs are given, the original storage tank with a water surface elevation of 580 ft. and the reservoir where the source pump really exists with a water surface elevation of 400 ft. The input line after the DHEAD command consists of (1) pipe 1 that contains the pump, (2) an estimate that this pump must supply about 250 ft of head, (3) the HGL is to be specified at node 3, (4) the source at the end of pipe 7 is to be a part of the additional equation containing the differential head, and (5) the specified HGL elevation.

To gain modeling experience with a differential head device, the following exercises are recommended:

(a) Extract from the CD the data listed in Fig. 5.22 and obtain a solution.

(b) Modify this data to designate supply sources as nodes.

(c) Modify the data from either (a) or (b) so that the original pump is now used in the problem specification. The original pump has the following pump characteristics:

Discharge, gal/min.	700	1200	1500
Head, ft.	370	350	280

(d) Add a second differential head device in pipe 9 that is to produce a HGL elevation of 605 ft. at node 9. In this analysis retain the requirement, as in (a), that the pump meet all of the demand.

Some study of the results from these four solutions will show the following: (a) The pump must develop a head of 391 ft to supply all of the flow. (b) An additional head of 70.2 ft above that produced by the present pump is needed, but if the diameter of pipe 1 is 11.64 in instead of the present 10 in, then the present pump would meet the requirements. (c) A booster pump that produces a head of 78.5 ft and a discharge of 3.01 ft^3/s is needed in pipe 9 to increase the HGL elevation at node 9 by 20 ft to 905 ft. (d) While the booster pump in pipe 9 does increase the pressure at several nodes where the pressure was low, it also decreased the pressure at node 6 to just under 16 lb/in^2. From these results the engineer must decide which improvements to propose for this water distribution system; proposals can then be examined further with appropriate analyses.

We turn now to a discussion of the twin questions of (1) how to select the pipes in which the pipe diameters will be specified, and (2) how to assign numerical values to these specified diameters. We begin by looking at the nine-pipe looped network in Fig. 5.23. Since this network has six nodes, diameters must be chosen for three pipes if all of the nodal demands and HGL elevations are given. The design demands are shown in the figure, and the target HGL elevations are listed in the table.

Node No.	**HGL Elev.** ft.
1	482
2	463
3	481
4	466
5	470
6	463

Pipe No.	**L** ft.
1	2000
2	2000
3	1200
4	1800
5	1200
6	2000
7	1300
8	2000
9	1300

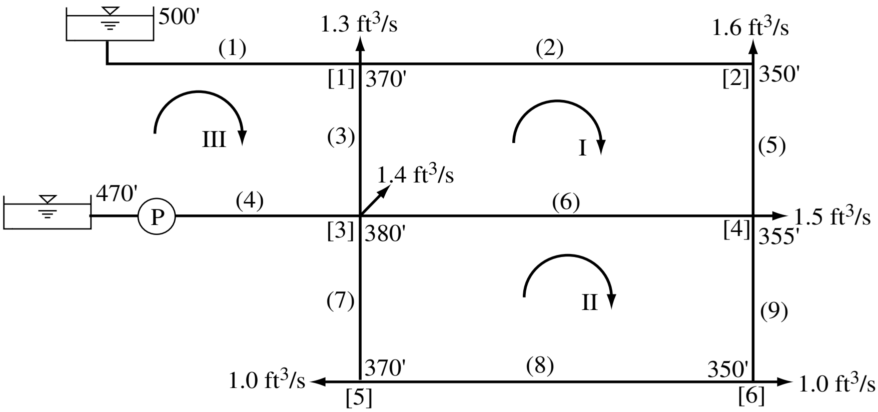


Figure 5.23 A nine-node pipe network.

In examining this network we note that the pipes to be deleted must prevent the existence of loops I, II, and III. The reduced network will be branched after three pipe diameters are specified. If pipes 5 and 9 are assigned diameters, then clearly loops I and II will not exist in the reduced network. But the effect of specifying the diameter of pipe 3 in order to eliminate the pseudo loop is not obvious; actually the network would be divided into two branched systems, but this step is permissible since each has a supply source. Thus pipes 3, 5, and 9 could be assigned diameters if the values for these diameters are suitably chosen. Many other combinations are also acceptable. If the diameter of pipe 3 is specified, then loops I and II can be broken by also specifying diameters for any of the following pairs of pipes: 6 and 9, 5 and 8, 5 and 7, or 5 and 6. And pipe 3 is not the only pipe whose removal would break pseudo loop III. Either pipe 1 or pipe 4 could replace the role of pipe 3. For these last two alternatives the reduced network would no longer be divided into two branched systems. Then the specification of an inflow into node 1 or node 3, respectively, would be required, because the assignment of a diameter to a pipe that connects a supply source to the network fixes the discharge from that source. Further thought will show for each of pipes 1, 3, and 4, that there exist five pairs of pipe numbers that could be chosen in order to arrive at a properly posed problem. For each triple of pipe numbers we can also choose reasonable pipe diameters. Table 5.7 lists the 15 combinations and reasonable sizes for these pipes. The reader will find it instructive to prepare input data and obtain, and study, the solutions from NETWK for several, if not all, of these combinations. File FIG5_23.IN contains the input for NETWK for the second combination which assigns diameters to pipes 3, 6, and 9.

Table 5.7 Possible pipe combinations whose diameters could be specified

Pipe Nos.	Sizes in	Pipe Nos.	Sizes in	Pipe Nos.	Sizes in
3, 5, 9	8, 6, 8	1, 5, 9	10, 6, 8	4, 5, 9	10, 6, 8
3, 6, 9	8, 10, 8	1, 6, 9	10, 10, 8	4, 6, 9	10, 10, 8
3, 5, 8	8, 6, 7	1, 5, 8	10, 6, 7	4, 5, 8	10, 6, 7
3, 5, 7	8, 6, 8	1, 5, 7	10, 6, 8	4, 5, 7	10, 6, 8
3, 5, 6	8, 6, 10	1, 5, 6	10, 6, 10	4, 5, 6	10, 6, 10

To investigate the appropriateness of values for diameters, suppose that pipe 6 had been assigned a diameter of 8 in instead of 10 in in the second combination. Since the head loss in pipe 6 is prescribed as 15 ft., the discharge must be $Q_6 = 1.503$ ft^3/s instead of a discharge of 2.784 ft^3/s that is obtained for the 10-in diameter. Since the diameter of pipe 5 is also specified, its flow is $Q_5 = 0.412$ ft^3/s from node 4. Since the energy line slopes from node 4 to node 6, the flow in pipe 9 must also be from node 4. The sum of the discharge in pipe 5 and the demand at node 4 is 1.912 ft^3/s; thus the flow into this node through pipe 6 must exceed 1.912 ft^3/s. However, since this is not the case, the specification $D_6 = 8$ in makes it impossible to find a consistent solution. To obtain a consistent solution the minimum diameter for pipe 6 can be computed by setting the discharge at 1.912 ft^3/s with a head loss of 15 ft in this pipe. This diameter is 8.765 in. If in this case the diameter of pipe 5 was a different prescribed value, it would cause us to compute a different minimum diameter for pipe 6. The specification of a diameter for a pipe must allow the junction continuity equations at both ends of it to be satisfied. In larger networks the satisfaction of this criterion at the ends of all pipes whose diameters are specified is often not an easy task.

5.5 DEVELOPING A SOLUTION FOR ANY VARIABLES

This section examines methods to determine any variable associated with a pipe network. The unknowns may be selected from the (1) hydraulic grade lines at nodes, (2) demands at nodes, (3) discharges in pipes, (4) pipe diameters, (5) pipe roughnesses, and (6) elevations of water supply surfaces. There are two restrictions: (1) the number of unknowns must equal the number of independent equations, i.e., NJ + NP, and (2) the knowns are such that an impossible flow situation is not created. Clearly the second restriction means that the specified variables must be appropriately configured and assigned values so that a solution for the unknowns will exist.

In Chapter 4 the computer program EQUSOL1 was introduced and discussed. The subroutine FUNCT must be rewritten for each individual network when this program is used. (The use of MathCAD and TK-Solver is similar; the user supplies the equations to be solved and then identifies the unknown variables.) We now consider a computer program that does not require us to rewrite a subroutine for each different problem. This program will be restricted in its use to the solution of network problems, but the variables that are to be found will be specified in the input data. The program will accept differing numbers of each of the six types of unknown and known variables, so long as the total number of unknowns matches the number of independent equations. For example, an "analysis" problem could be specified, in which the discharges in all pipes and the HGL elevations at all nodes are determined and all pipe diameters, lengths, roughnesses, and nodal demands are prescribed. For analysis problems this program will not be as efficient as the programs in Chapter 4 that solved the Q-equations, the H-equations, or the ΔQ-equations because more equations are solved. The program will first read a description of the network so it will know how the pipes are connected; then it will define NJ junction continuity equations and NP pipe head loss equations; and finally it will solve these equations simultaneously for whichever variables that are identified as unknown. The input to this program must describe the network adequately in a manner that is common for pipe systems, i.e., giving data for each pipe and node in the network. Before reading further, print one of the versions of NETWEQS1 from the CD so you can study the listing as you read.

5.5.1. LOGIC AND USE OF NETWEQS1

Program NETWEQS1 will solve pipe-system problems for any of six types of unknowns, or any reasonable combination of them, so long as the number of unknowns equals the number of independent equations. The number of equations consists of the sum of the number of pipes NP and number of nodes or junctions NJ in the pipe system, or NEQS = NP + NJ. If no supply source is identified, then there exist only NJ - 1 independent equations from application of the continuity principle at the junctions.

The subroutine FUN defines these equations. The continuity equations are first evaluated in the `DO 10` loop, and then the head loss equations

$$H_i - H_j = \{ f(L/D)Q^2/(2gA^2) \}_k \tag{5.31}$$

follow in the remainder of this subroutine. The friction factor f is found by using Gauss-Seidel iteration if the flow is turbulent $(Re > 2100)$ and by using $f = 64/Re$ if the flow is laminar, but if $Re < 160$, then $f = 0.4$ to prevent f from becoming unbounded whenever $Q \to 0$. (Note RE in the program is actually $Re/7.34347283$.)

The user must provide estimates for the unknowns, as well as values for the knowns, in the input file for NETWEQS1. When NETWEQS1 is executed, the user is asked to provide three input/output unit numbers, the acceleration of gravity g, and the kinematic viscosity VISC of the fluid. The default values for these parameters are IN2 = 2, IN5 = 5, IN4 = 4, $g = 32.2$ ft/s^2 and VISC = 0.00001417 ft^2/s. To accept all these defaults, simply give / in the Fortran program after the last value that has been entered. The meaning of the three input/output units is as follows: IN2 is the input unit for the majority of the data that describes the pipe system. If IN2 = 0 or IN2 = 5, then this data must be entered from the keyboard in the proper order without any prompts. When IN2 is not 0 or 5, then a prompt will request the name of the file that contains the input data. If the user is using MS-Fortran, an alternative is to give the input file name on the "command line" (or after typing NETWEQS1 to execute the program, leave a space and list the file name). If IN4 = 0 or 6, then the output will be written to the monitor; otherwise it will go to a file.

The input data for NETWEQS1 consists of two types. The first type describes the network, and this data is read by using logical unit IN2. The second type defines the unknowns, and it is read by using logical unit IN5. If this data is placed in a file (IN5 not equal to 0 or 5), then this file provides the data that defines the unknowns. The default IN5 = 5 indicates that these data are to come from the keyboard. If IN5 is 5 or 0, then NETWEQS1 will prompt for the input that is needed to define the unknowns. These data consist of the following 6 values (on separate lines):

1. The number of HGL elevations that are unknown at nodes.
2. The number of nodal demands that are unknown.
3. The number of unknown pipe discharges.
4. The number of pipe diameters that are unknown.
5. The number of pipe roughnesses that are unknown.
6. The number of unknown water surface elevations.

After these six lines that give the number of each type of unknown, the next lines give lists of node or pipe numbers that identify the individual unknowns. The number of these lines will match the number of categories (a maximum of six) that are given nonzero numbers. These lists of numbers can consist of individual values or a range of values separated by a minus sign (-). The subroutine RLINE will allow ranges of integers to be intermixed with single integers. The argument NUM returns the number of integers in the list in the Fortran program. For example, if a pipe system consists of 6 pipes and 5 nodes, and the unknowns are to be the discharges in all pipes and the HGL elevations at all nodes, then the input specifications should consist of the following numbers:

5; 0; 6; 0; 0; 0; 1 2 3 4 5; 1 2 3 4 5 6

In these files the semicolon (;) indicates that a new line should be used. If the keyboard is used to supply the input data, then a prompt appears in place of each semicolon and automatically separates the data. Alternatively this input could be the following:

5; 0; 6; 0; 0; 0; 1-5; 1-6

In place of the HGL elevation at node 5, if it were desirable to determine the demand at node 5, then the input file could be the following:

4; 1; 6; 0; 0; 0; 1 2 3 4; 5; 1 2 3 4 5 6

An alternative listing of this input might be this set:

4; 1; 6; 0; 0; 0; 1-4; 5; 1-6

To reiterate, if IN5 = 5 or 0, then NETWEQS1 will prompt the user for the next expected piece of information. If IN2 and IN5 are given the same value so both types of data are in the same file, then the data read under IN5 (the data that defines the unknowns) is given after the data read by IN2 which defines the configuration of the pipe system.

The unit defined by IN4 is the Fortran unit number that will write the problem solution as output data. This output will go to the terminal/monitor if IN4 = 0, 5, or 6. Otherwise it will be written to a file. A prompt will request the file name, unless it is included on the command line. The program calls subroutine SOLVEQ (see Appendix A) to solve the linear system of equations that is obtained by implementation of the Newton method. In this solution the elements of the Jacobian matrix of derivatives are evaluated numerically, as described in Chapter 4.

5.5.2. DATA TO DESCRIBE THE PIPE SYSTEM

Most of the data that describe the system is normally placed in a file that will be read on Fortran logical unit IN2. These data consist of the following:

Line 1: Four integers; number of pipes NP, number of nodes NJ, number of reservoirs NRES, number of pumps NPUMP.

Line 2: Pairs of values; each pair consists of the pipe number that connects a reservoir to the network and the water surface elevation of this reservoir. Each pair can be on a separate line.

Line 3: If pumps exist (NPUMP>0), then seven values are required for each pump: the number of the pipe containing the pump, followed by three pairs of discharge and pump head which define the pump characteristic curve. The data for each pump is a separate line.

Next NP lines: These lines contain the pipe data, six items per pipe. The pipes must be numbered consecutively from 1 through NP. There is one line per pipe, sequenced by pipe number, since the pipe number itself is omitted. Each line contains the following:

1. The upstream node.
2. The downstream node.
3. The pipe length.
4. The pipe diameter.
5. The pipe roughness.
6. The discharge in the pipe.

The pipe diameters and wall roughnesses must have the same units, e.g., feet for ES units or meters for SI units. These values are only estimates if the variable is an unknown, for they then become initial values for the Newton method in the solution process.

Next NJ lines: These lines contain the node data, three items per node. There is one line per node, sequenced as the nodes are numbered because the node number is not included. Each line consists of the following:

1. The demand at the node.
2. The HGL elevation at the node.
3. The ground elevation of the node.

All values must be listed in consistent units, e.g., ft^3/s and ft for ES units or m^3/s and m for SI units.

To see how this input scheme works, consider the small network in Fig. 5.24. The input data set would take the form shown in Fig. 5.25.

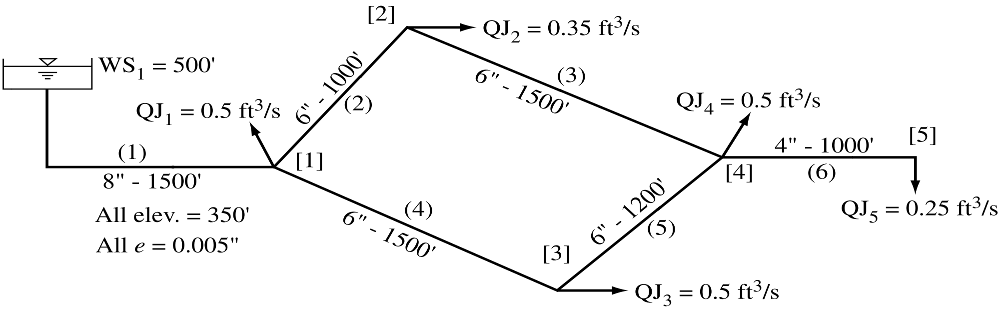


Figure 5.24 A small network to study with NETWEQS1.

```
6 5 1 0
1  500.
0  1  1500.  .667  .000417  2.1
1  2  1000.  .5   .000417   .82
2  4  1500.  .5   .000417   .47
1  3  1500.  .5   .000417   .78
3  4  1200.  .5   .000417   .28
4  5  1000.  .333   .000416667   .25
.5  476.05   350.
.35  464.8  350.
.5  460.7  350.
.5  458.9  350.
.25  452.8  350.
```

Figure 5.25 The input data set for the small network in Fig. 5.25.

Assume the data in Fig. 5.25 have been placed in a file FIG1.DAT, which is the file name for it on the CD. Upon execution of NETWEQS1, the following three values could be given from the keyboard in response to the first prompt: 2 5 4/ Next a file name is requested. The name FIG1.DAT would be given in reply. Since the second input logical unit has been given as 5, the user is then asked to define the number and types of the unknown variables. Upon supplying 5 0 6 0 0 0, the user is requested to give the numbers associated with items 1 and 3. The two responses could be 1-5 and 1-6. Next the output file name is requested. The solution in this output file consists of the following two tables:

PIPE DATA

PIPE NO.	NODES FROM	NODES TO	L ft.	DIA. in	e x10^4 in	Q ft^3/s	HEAD LOSS ft.
1	0	1	1500	0.667	4.17	2.100	23.95
2	1	2	1000	0.500	4.17	0.824	11.39
3	2	4	1500	0.500	4.17	0.474	5.98
4	1	3	1500	0.500	4.17	0.776	15.21
5	3	4	1200	0.500	4.17	0.276	2.17
6	4	5	1000	0.333	4.17	0.249	10.94

Figure 5.26 The output from NETWEQS1.

NODE DATA

NODE	DEMAND ft^3/s	ELEV. ft.	HEAD ft.	PRESSURE lb/in^2	HGL ELEV. ft.
1	0.500	350	126.0	54.6	476.0
2	0.350	350	114.7	49.7	464.7
3	0.500	350	110.8	48.0	460.8
4	0.500	350	108.7	47.1	458.7
5	0.250	350	97.7	42.4	447.7

Figure 5.26, concluded. The output from NETWEQS1.

5.5.3. COMBINATIONS THAT CAN NOT BE UNKNOWNS

We have noted that NP + NJ independent equations exist, and therefore we might regard this many variables as unknowns that we may seek to find. However, there are combinations of these variables that cannot be selected as unknowns because mathematical problems then arise, for which there is no solution. The difficulty is that the equations are mathematically inconsistent. If an impossible problem is specified, then either convergence to a solution will not occur or the Jacobian matrix that is used in implementing the Newton method will be singular. Let us look further at the elements that create a problem for which it is impossible to find a solution. It has become obvious to us for a single pipe that it is not possible to specify its diameter and roughness, the discharge in it and the heads at both ends. By specifying D, e, Q, H_i, and H_j, the relation between discharge and head loss is fully defined, and the need for a frictional head loss equation has been eliminated. When these variables are all prescribed, a problem is defined which has no solution, and the equations are inconsistent. Therefore, if any pipe in a network problem has D, e, and Q given and the heads at both ends are selected as known values, the problem has no solution. One of these variables must be unknown.

There are many less obvious combinations of known and unknown variables that can cause a problem to be impossible to solve. To illustrate some of the possibilities, consider the three-pipe looped network in Fig. 5.27. For this network there are five basic equations, two junction continuity equations and three pipe head loss equations. Assume all demands are known, and we decide to specify Q_1 in pipe 1. Now it is no longer possible to specify the heads at both ends of either pipe 2 or 3 along with their diameters and roughnesses, because the specification of Q_1 has also fixed Q_2 and Q_3, since $Q_2 = Q_1 - QJ_2$ and $Q_3 = QJ_3 - Q_2 = QJ_3 + QJ_2 - Q_1$. We would therefore be deceiving ourselves if we placed either Q_2 or Q_3 in the list of unknowns. Regardless of which

Node	Demand ft^3/s	HGL Elev. ft.
1	- 2.2	200
2	1.2	198
3	1.0	195

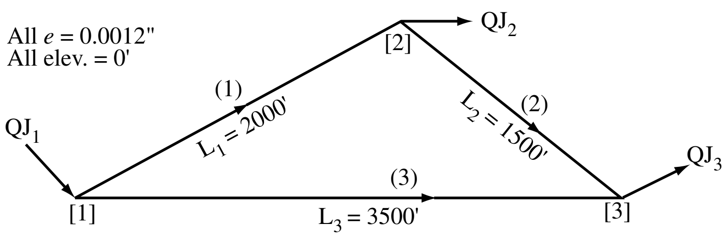


Figure 5.27 A three-pipe looped network.

discharge is specified, the other two have also been fixed if any two of the three demands are specified.

Table 5.8 lists 20 combinations of known and unknown variables and the solution for each combination that leads to a set of consistent simultaneous equations. The lengths of pipes are L_1 = 2000 ft., L_2 = 1500 ft. and L_3 = 3500 ft. To limit the entries in the table it is assumed that all roughnesses are the same, $e_1 = e_2 = e_3 = 0.0012$ in, and that the elevation of the hydraulic grade line at node 1 is H_1 = 200 ft. For cases 10, 13, and 14 no solution is possible, i.e., an impossible problem has been specified. These three

Table 5.8
Combinations of Variables as Unknowns, With the Remaining Variables Specified.

Case	Unknown Variables					Specified Variables				
1	H_2	H_3	Q_1	Q_2	Q_3	D_1	D_2	D_3	QJ_2	QJ_3
	196.55	196.43	1.257	0.057	0.943	10	6	10	1.2	1.0
2	QJ_2	H_3	Q_1	Q_2	Q_3	D_1	D_2	D_3	H_2	QJ_3
	1.636	195.30	1.537	- 0.096	1.097	10	6	10	195	1.0
3	H_2	QJ_3	Q_1	Q_2	Q_3	D_1	D_2	D_3	QJ_2	H_3
	195.91	1.315	1.380	0.180	1.135	10	6	10	1.2	195
4	H_3	QJ_3	Q_1	Q_2	Q_3	D_1	D_2	D_3	QJ_2	H_2
	192.15	1.797	1.539	0.339	1.792	10	6	10	1.2	195
5	QJ_2	QJ_3	Q_1	Q_2	Q_3	D_1	D_2	D_3	H_2	H_3
	0.584	1.484	0.933	0.0491	1.135	10	6	10	198	195
6	QJ_2	QJ_3	D_1	Q_2	Q_3	Q_1	D_2	D_3	H_2	H_3
	0.651	1.135	0.855	0.349	1.135	1.0	6	10	198	195
7	QJ_2	QJ_3	Q_2	D_2	Q_3	D_1	Q_2	D_3	H_2	H_3
	0.433	1.635	0.933	6.863	1.135	10	0.5	10	198	195
8	QJ_2	QJ_3	Q_1	Q_2	D_3	D_1	D_2	Q_3	H_2	H_3
	1.349	0.584	0.933	0.349	0.795	10	6	1.0	198	195
9	QJ_2	QJ_3	D_1	D_2	Q_3	Q_1	Q_2	D_3	H_2	H_3
	1.635	0.5	10.26	6.86	1.135	1.0	0.5	10	198	195
10	QJ_2	QJ_3	D_1	Q_2	D_2	Q_1	Q_3	D_3	H_2	H_3
	Inconsistent (no solution)					1.0	1.0	10	198	195
11	QJ_2	QJ_3	D_1	Q_2	D_3	Q_1	Q_3	D_2	H_2	H_3
	0.651	1.349	10.26	0.349	9.54	1.0	1.0	6	198	195
12	QJ_2	QJ_3	D_1	D_2	D_3	Q_1	Q_2	Q_3	H_2	H_3
	1.5	0.5	10.26	6.86	9.54	1.0	0.5	1.0	198	195
13	QJ_2	QJ_3	D_1	D_3	Q_3	Q_1	Q_2	D_2	H_2	H_3
	Inconsistent (no solution)					1.0	0.5	10	198	195
14	QJ_2	QJ_3	D_2	D_3	Q_3	Q_1	Q_2	D_1	H_2	H_3
	Inconsistent (no solution)					1.0	0.5	10	198	195
15	H_2	H_3	D_1	Q_2	Q_3	Q_1	D_2	D_3	QJ_2	QJ_3
	198.45	197.35	12.28	0.2	0.8	1.4	6	10	1.2	1.0
16	H_2	H_3	Q_1	D_2	Q_3	D_1	Q_2	D_3	QJ_2	QJ_3
	198.27	197.35	1.4	6.23	0.8	12	0.2	10	1.2	1.0
17	H_2	H_3	Q_1	Q_2	D_3	D_1	D_2	Q_3	QJ_2	QJ_3
	198.27	197.17	1.4	0.2	9.87	12	6	0.8	1.2	1.0
18	H_2	H_3	D_1	D_2	Q_3	Q_1	Q_2	D_3	QJ_2	QJ_3
	Inconsistent (no solution)					1.2	0.2	10	1.2	1.0
19	H_2	H_3	Q_1	D_2	D_3	Q_2	Q_3	D_1	QJ_2	QJ_3
	Inconsistent (no solution)					0.2	1.0	12	1.2	1.0
20	H_2	H_3	D_1	Q_2	D_3	Q_1	Q_3	D_2	QJ_2	QJ_3
	Inconsistent (no solution)					1.2	1.0	10	1.2	1.0

A note on units: All heads are in ft; all discharges are in ft^3/s; all diameters are in in.

inconsistent combinations are all created by specifying the head at both ends of one of the pipes while trying also to specify its diameter and discharge. In case 13 this over-specification is obvious. For pipe 2 Q_2, D_2 and its head loss $h_{f2} = H_2 - H_3$ are all known. In case 10 this overspecification is not so obvious until it is realized that, since $H_1 = 200$ ft. is given, then Q_3, D_3 and $h_{f3} = H_1 - H_3$ are in effect given. And for case 14 the same type of overspecification for pipe 1 occurs; the discharge Q_1, diameter D_1, and head loss $h_{f1} = H_1 - H_2$ are all specified. NETWEQST detects such inconsistencies by finding that the Jacobian matrix in the Newton iteration is singular; it reports this and then stops.

Let us examine cases 15, 16, and 17. For this particular network the specification of any one discharge and the demands QJ_2 and QJ_3 is equivalent to a specification of the other two discharges. This situation occurs because the continuity equation at node 2 requires, if Q_1 is given, that $Q_2 = Q_1 - QJ_2$ (or if Q_2 is given, then $Q_1 = Q_2 + QJ_2$). At node 2 $Q_2 + Q_3 = QJ_3$, so with Q_2 found from the continuity equation at node 2, we find that the continuity equation at node 3 then requires $Q_3 = Q_2 + QJ_3$. However, the fact that the specification of any one discharge also fixes the other two discharges (with QJ_2 and QJ_3 known) does not in itself result in an inconsistent problem because the junction continuity equations are part of the system of equations. It does mean that we are making the problem more computationally intensive than is necessary. In case 15 a mental computation with the continuity equations would give $Q_2 = 0.2$ ft^3/s and $Q_3 =$ 0.8 ft^3/s. Next the head losses in pipes 2 and 3 could be computed, from these H_2 and H_3 could be determined, and finally D_1 could be found. Similar steps requiring the solution of only one equation at a time (and the Colebrook-White equation) can be used for cases 16 and 17. For all other cases in the table two simplifications in the solution process also exist. Treating each consistent case as a system of NP + NJ simultaneous equations will always be successful, even if it leads to more arithmetic than is necessary.

Another selection of known variables that produces inconsistent equations is the specification of all of the pipe discharges that join at a node while simultaneously giving the demand at this node. By doing so, that junction continuity equation no longer defines a relation between the pipe discharges and the demand there. The inconsistency will occur whether or not the junction continuity equation is satisfied by the given discharges. Cases 18, 19, and 20 are problems for which no solutions exist because a junction continuity equation can not be used. In case 18 the overspecification is obvious because Q_1, Q_2, and QJ_2 are all knowns, and yet these three variables are the only variables in the continuity equation at node 2: $Q_1 - Q_2 = QJ_2$. (We have actually reduced the network to a one-pipe network.) That case 19 gives all variables in the junction continuity equation at node 3 is now obvious. However, case 20 is not quite so obvious. Since the inflow at node 1 (the magnitude of the negative demand there) must equal $QJ_2 + QJ_3$, we note that giving Q_1 and Q_3 along with QJ_2 and QJ_3 results in a specification of all variables in the junction continuity equation at node 1.

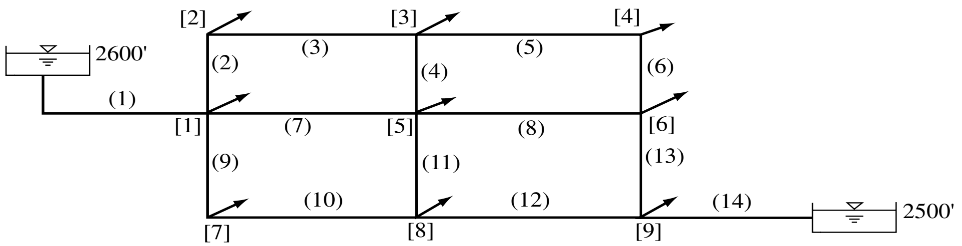
The foregoing situations will always result in a failure to find a solution. There are other specifications that will cause NETWEQS1 (or NETWEQST) to seek a solution but fail. Here are two more examples: (a) a situation requires a reservoir to supply a flow to the network, but the water surface elevation of the reservoir is given a value that is lower than the head at the other end of the connecting pipe; (b) consider a junction where two pipes join to meet a positive demand, but at the same time specify the heads at the opposite ends of the pipes so the flow must leave the junction. So we see that an unthinking specification of known values can, and often will, create a problem for which there is no solution, and the likelihood of this occurring increases with the size of the network because it then becomes increasingly difficult to identify situations for which there is no solution. An inconsistent problem will often become evident with NETWEQS1 when a message from the linear algebra solver SOLVEQ says that the Jacobian matrix is singular, which usually means that the linear system of equations, consisting of the Jacobian matrix and the equation vector as the known vector, is not an independent system of equations.

Example Problem 5.5

The pipe lengths and other data for a 14-pipe network supplied by two reservoirs are given in the file EXP5_5.IN on the CD. Obtain a solution from NETWK using this input, and then prepare input data for NETWEQS1 and obtain solutions therefrom for the following cases:

1. The heads at all nine nodes, as well as the discharges in all 14 pipes, will be regarded as unknown. (This is the problem solved by NETWK.)
2. At node 5 the demand QJ_5 will be considered unknown, and the head H_5 will be specified as $H_5 = 2504.3$ ft.
3. All of the demands are considered unknown, and the heads are as given in the input data to NETWEQS1.
4. The heads at all nine nodes and the discharges in all 14 pipes are considered unknown.
5. The heads at all nine nodes are unknown, and the diameters of pipes 2, 7, and 10 are unknown with the discharges in these three pipes specified.

Repeat the five cases with the reservoir attached to pipe 14 having a water surface elevation of 2450 ft. and with a pump in this pipe that has the following operating characteristic data pairs: $Q_1 = 1$ ft^3/s, $H_1 = 55$ ft.; $Q_2 = 2$ ft^3/s, $H_2 = 50$ ft.; $Q_3 = 3$ ft^3/s, $H_3 = 43$ ft.



The input data, to be read on logical unit IN5, are shown below for the five cases. For case 4 NETWEQS1 reports a "singular matrix," which indicates that at least one redundant equation was included in the system of equations. It should be clear that a solution to case 4 was not possible because we can not specify the heads at both ends of all pipes while simultaneously specifying all of the demands. These same cases are solved by using NETWEQS1 with a pump in line 14 and the water surface of this reservoir lowered to 2450 ft. These solutions follow those from NETWEQS.

<u>Data file for NETWEQS1 using input IN2 ≠ 5</u>

```
14 9 2 0
1 2600 14 2500
0 1 1500 1. .000166667 9.7
1 2 1000 .66666667 .000166667 2.9
2 3 2000 .66666667 .000166667 1.7
5 3 1000 .66666667 .000166667 0.2
3 4 2000 .5 .000166667 0.9
6 4 1000 .5 .000166667 0.5
1 5 2000 .66666667 .000166667 2.7
5 6 2000 .5 .000166667 0.8
1 7 1200 .66666667 .000166667 2.9
7 8 2000 .66666667 .000166667 1.7
5 8 2000 .66666667 .000166667 0.8
8 9 1200 .66666667 .000166667 1.4
9 6 1200 .66666667 .000166667 1.2
0 9 1500 .66666667 .000166667 1.3
1.3 2548.6 2410
1.2 2522.8 2405
1.0 2504.1 2400
1.4 2481.1 2340
0.9 2504.3 2405
1.5 2485.4 2350
1.2 2518.1 2405
1.0 2491.6 2400
1.5 2491.6 2370
```

Case 1 input, IN3 $\neq$ 5

9
0
14 14
0
0
0
1-9
1-14

Case 2 input, IN3 $\neq$ 5

8
1
14
0
0
0
1 2 3 4 6 7 8 9
5
1-14

Case 3 input, IN3 $\neq$ 5

0
9

0
0
0
1-9
1-14

Case 4 input, IN3 $\neq$ 5

9
0
0
14 3
0
0
1-9
1-14

Case 5 input, IN3 $\neq$ 5

9
0
11

0
0
1-9
1 3 4 5 6 8 9 11 12 13 14
2 7 10

To repeat the five cases with a pump inserted in line 14 and a specified reservoir water surface elevation, the input data file for NETWEQS1 above is modified by replacing the first two input lines with the following three input lines:

```
14 9 2 1
1 2600 14 2450
14 1 55 2 50 3 43
```

The remainder of the input file is unchanged.

The solutions in file EXP5_5.OUT on the CD were obtained for the 14-pipe network by using NETWEQS1 for the five cases. The reader should be able to obtain identical solutions. The solutions to the five cases which include a pump are in file EXP5_5.OU1 on the CD, which can be used to verify that your solutions are correct. In these solutions the pump is called device 1 when it is in pipe 14. The program output lists the change in head caused by each such device. In case 1 we find the following message:

```
Devices caused the following changes in heads:
Device 1 in pipe 14     Change in head = 53.45 ft.
```

* * *

Example Problem 5.6

For the small network below do the following:

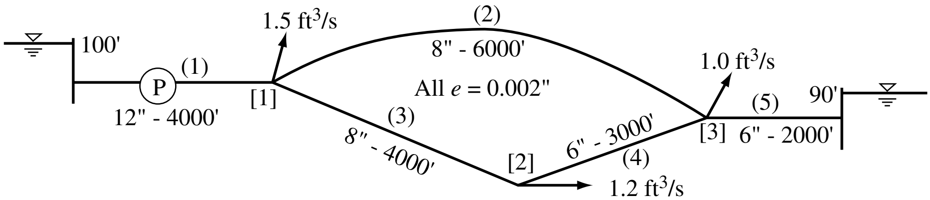
(a) Write the equations that describe the system.

(b) For the specified physical system, find the discharge in each pipe and the head at all nodes (duplicate this solution by preparing input data for NETWK).

(c) Determine the diameter of pipe 1 so the discharge through pipe 5 into the reservoir is $Q_5 = 0.5$ ft^3/s.

(d) Find the head that the pump must produce so that the discharge through pipe 5 into the downstream reservoir is $Q_5 = 1.0$ ft^3/s.

Q, ft^3/s	4.5	4.0	3.5
h$_p$, ft.	54	50	44



(a) The equations are the following:

$$F_1 = 100 + h_p - H_1 - f_1(L_1/D_1)(Q_1/A_1)^2/(2g) = 0$$

$$F_2 = H_1 - H_3 - f_2(L_2/D_2)(Q_2/A_2)^2/(2g) = 0$$

$$F_3 = H_1 - H_2 - f_3(L_3/D_3)(Q_3/A_3)^2/(2g) = 0$$

$$F_4 = H_2 - H_3 - f_4(L_4/D_4)(Q_4/A_4)^2/(2g) = 0$$

$$F_5 = H_3 - 90 - f_5(L_5/D_5)(Q_5/A_5)^2/(2g) = 0$$

$$F_6 = Q_1 - Q_2 - Q_3 - QJ_1 = 0$$

$$F_7 = Q_2 + Q_4 - Q_5 - QJ_3 = 0$$

$$F_8 = Q_3 - Q_4 - QJ_2 = 0$$

with $$h_p = -4Q_1^2 + 42Q_1 - 54$$

(b) The 8 unknowns are Q_1, Q_2, Q_3, Q_4, Q_5, H_1, H_2, and H_3. Using NETWEQS1 to solve this problem would require the following input data:

From keyboard:

```
2 5 3/
```

In a file from logical unit 2:

```
5 3 2 1
1 100
5 90
1 4.5 54 4 50 3.5 44
0 1 4000 1.0 .000167 4.2
1 3 6000 .667 .000167 1.3
1 2 4000 .667 .000167 1.5
2 3 3000 .500 .000167 0.3
0 3 2000 .500 .000167 0.5
1.5 126 0.
1.2 98 0
1.0 95 0
```

Since the second logical unit was given as 5, the keyboard data for the unknowns is

```
3
0
5
0
0
```

and then

```
1-3
1-5
```

The solution from NETWEQS1 follows.

PIPE DATA

PIPE NO.	NODES FROM	TO	L ft.	DIA. ft.	e x10⁴ ft.	Q ft³/s	HEAD LOSS ft.
1	- 1	1	4000	1.000	1.67	4.102	26.44
2	1	3	6000	0.667	1.67	1.191	29.17
3	1	2	4000	0.667	1.67	1.411	26.67
4	2	3	3000	0.500	1.67	0.211	2.49
5	- 2	3	2000	0.500	1.67	- 0.400	- 5.37

Devices caused the following changes in heads:
Device 1 in pipe 1 Change in head = 50.98 ft.

NODE DATA

NODE	DEMAND ft^3/s	ELEV. ft.	HEAD ft.	PRESSURE lb/in^2	HGL ELEV. ft.
1	1.500	0	124.5	54.0	124.5
2	1.200	0	97.9	42.4	97.9
3	1.000	0	95.4	41.3	95.4

This can be regarded as the solution to an analysis problem since all of the physical features of the network are known, and the solution describes the performance of this existing network in response to the specified demands. We could verify that this solution is the solution from NETWK by supplying this input file to NETWK:

```
Example Problem
/*
 $SPECIF OUTPU1=2 $END
PIPES
1 0 1 4000 12 .002
2 1 3 6000 8
3 1 2 4000 8
4 2 3 3000 6
5 3 0 2000
NODES
1 1.5 0
2 1.2
3 1
RESER
5 90
PUMPS
1 4.5 54 4 50 35 44 100
RUN
```

(c) The equation set is unchanged from part (a). However, here the unknowns are different. The input data file in part (b) is again supplied to NETWEQS1. When we are asked to identify the unknowns, the following keyboard input will be supplied:

```
3
0
4
1
0
0
```

followed by:

```
1-3
2-5
1
```

(In parts (b) and (c) we supply a discharge of 0.5 ft^3/s for pipe 5 in the input data file. This was merely an estimate in part (b), but now this value is the specified discharge.)

The solution from NETWEQS1 is the following:

PIPE DATA

PIPE NO.	NODES FROM	NODES TO	L ft.	DIA. ft.	e x10⁴ ft.	Q ft³/s	HEAD LOSS ft.
1	- 1	1	4000	1.041	1.67	4.200	22.09
2	1	3	6000	0.667	1.67	1.246	31.72
3	1	2	4000	0.667	1.67	1.454	28.23
4	2	3	3000	0.500	1.67	0.254	3.50
5	- 2	3	2000	0.500	1.67	- 0.500	- 8.03

Devices caused the following changes in heads:
Device 1 in pipe 1 Change in head = 51.84 ft.

NODE DATA

NODE	DEMAND ft³/s	ELEV. ft.	HEAD ft.	PRESSURE lb/in²	HGL ELEV. ft.
1	1.500	0	129.8	56.2	129.8
2	1.200	0	101.5	44.0	101.5
3	1.000	0	98.0	42.5	98.0

(d) One way to determine the required pump head so that $Q_5 = 1.0$ ft^3/s is to replace the pump and its upstream reservoir with a node having a demand of - 4.7 ft^3/s; this change will force the flow into the downstream reservoir to be 1.0 ft^3/s. The input to NETWEQS1 is as follows:

```
5 4 1 0
5 90
4 1 4000 1 .000167 4.7
1 3 6000 .667 .000167 1.3
1 2 4000 .667 .000167 1.5
2 3 3000 .5   .000167 0.3
0 3 2000 .5   .000167 1.0
1.5 126 0.
1.2 98 0
1  95 0
- 4.7 130 0
```

and the solution from NETWEQS1 will show that the pump must supply a head of 200.3 - 100 = 100.3 ft.

PIPE DATA

PIPE NO.	NODES FROM	NODES TO	L ft.	DIA. ft.	e x10⁴ ft.	Q ft³/s	HEAD LOSS ft.
1	4	1	4000	1.000	1.67	4.700	34.21
2	1	3	6000	0.667	1.67	1.536	46.90
3	1	2	4000	0.667	1.67	1.664	36.38
4	2	3	3000	0.500	1.67	0.464	10.52
5	- 1	3	2000	0.500	1.67	- 0.995	- 29.22

NODE DATA

NODE	DEMAND ft³/s	ELEV. ft.	HEAD ft.	PRESSURE lb/in²	HGL ELEV. ft.
1	1.500	0	166.1	72.0	166.1
2	1.200	0	129.7	56.2	129.7
3	1.000	0	119.2	51.7	119.2
4	- 4.700	0	200.3	86.8	200.3

* * *

Example Problem 5.7

Obtain solutions to Example Problems 5.5 and 5.6 using program NETWEQST. This program is one of the auxiliary programs in the NETWK package. It was developed to allow the user to specify the variables which are unknown, and it accepts the same input data as NETWK to define the physical features of the network. In other words it will solve the same problems as NETWEQS1 does, but it uses the same input files as NETWK. However, not all commands and options in NETWK are acceptable to NETWEQST. On the other hand it has additional options that provide the user some freedom in the way information is provided about the unknowns. If NETWEQST is to be used, then the following input files could be used, for example, to solve case 2 in Example Problem 5.5 and Example Problem 5.6, part (c), taking the default of being prompted for information that defines the unknowns. The prompts from NETWEQST are in bold type, and the responses are not.

For Example Problem 5.5, case 2, the portion of the input which defines the network is identical to that for NETWK in Example Problem 5.5 itself. Here the bold prompts from NETWKST are followed by the responses that define case 2:

Pipes = 14, Nodes = 9, Sources = 2
23 unknowns must be given. Give no. of each:
1. HGLs at nodes 8
2. Nodal demands 1
3. Pipe discharges 14
Give 8 HGLs at node numbers 1-4 6-9
Give 1 nodal demand numbers 5
Give number of nodal HGL-elevations provided 1
As pairs give 1 node number and the HGL
5 2504.3
Give number of pipe discharges provided 0

The data for Example Problem 5.6(c) is similar:

```
Example 5.6(c) using NETWEQST
/*
$SPECIF $END
PIPES
1 0 1 4000 12 .002
2 1 3 6000 8
3 1 2 4000
4 2 3 3000 6
5 3 0 2000
NODES
1 1.5 0
2 1.3
3 1
RESER
5 90
PUMPS
1 4.5 54 4 50 35 44 100
RUN
```

Pipes = 5, Nodes = 3, Sources = 2
8 unknowns must be given. Give no. of each:
1. HGLs at nodes 3
2. Nodal demands 0
3. Pipe discharges 4
4. Pipe diameters 1
Give 4 pipe discharge numbers 1-4
Give 1 pipe diameter numbers 1

Give number of nodal HGL-elevations provided 0
Give number of pipe discharges provided 1
As pairs give 1 pipe number and the discharge therein
5 - 0.5

* * *

5.6 HIGHER ORDER REPRESENTATIONS OF PUMP CURVES

The head produced by a pump has heretofore been defined as a function of the discharge by fitting a single second-order polynomial through three pairs of points. If the pump operation occurs within a relatively narrow discharge range, and these are near the normal capacity of the pump, then such a simple representation is adequate. When this is not the case, then more advanced procedures are needed to define well the pump's operating characteristics. Various interpolation procedures can be used for the mathematical representation of a pump curve. This section discusses how pump curves can be duplicated mathematically when equations are needed to define their operating characteristics.

5.6.1. WITHIN RANGE POLYNOMIAL INTERPOLATION

Any number of values might be used to define a pump characteristic curve, and a polynomial of any order might be used to interpolate the head corresponding to any given discharge if the range of the discharge values brackets the given discharge. A first-order polynomial is simply a straight line. To represent the pump head well with a first-order polynomial interpolation, we should first ensure that the smaller discharge Q_i is less than or equal to the given discharge Q, and that the larger discharge Q_{i+1} is greater than Q. The interpolating function for a first-order polynomial is

$$h_p = h_{pi} + \left(h_{pi+1} - h_{pi}\right)\left(Q - Q_i\right)/\left(Q_{i+1} - Q_i\right) \tag{5.32}$$

in which the quantities with subscripts i and $i+1$ are known, h_p is the interpolated head of the pump and $Q_i \le Q \le Q_{i+1}$. When Q becomes larger than Q_{i+1}, then the first point is dropped and the next point is added. The use of a higher-order polynomial requires more data. An nth-order polynomial requires at least $n+1$ pairs of data points since an nth-order polynomial passes through $n+1$ points, e.g., a second-order polynomial passes through three points, a third-order polynomial through four points etc. The Lagrange formula is a convenient interpolation formula to use for this purpose because the increment between consecutive values of the independent variable, the discharge Q in this case, need not be constant. Other formulas do require a constant increment of the independent variable. The Lagrange interpolation formula is

$$h_p = \sum_{i=1}^{n} F_i H_i \tag{5.33}$$

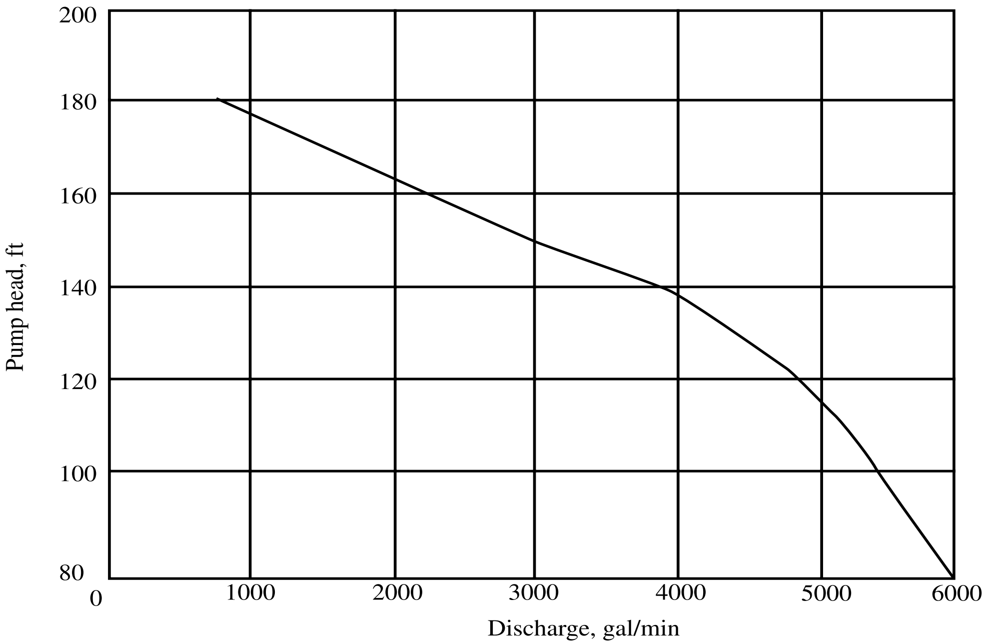
in which each H_i is the pump head at point i, and each F_i is the quotient of two products:

$$F_i = \prod_{\substack{j=1 \\ j\neq i}}^{n} \left(Q - Q_j\right) \Bigg/ \prod_{\substack{j=1 \\ j\neq i}}^{n} \left(Q_i - Q_j\right) \tag{5.34}$$

in which the two products Π include n - 1 terms, with the term $j = i$ omitted. To implement the Lagrange interpolation successfully in a computer program, two requirements must be met: (1) the discharge for which the head is wanted must lie within the range of the discharge data points (otherwise the process is extrapolation), and (2) Eqs. 5.33 and 5.34 must be properly written. The program LAGRANGE on the CD is designed to read n pairs of points for a pump curve and then provide the pump head for any specified discharge. The program can also be converted into a function subprogram which will pass (h_p, Q) pairs to the function from the main program, and an argument will specify the Q for which the head is to be determined.

Example Problem 5.8

A pump curve is shown below. Enter 10 pairs of points from this curve into a file, and then use Lagrange's formula with a third-order polynomial interpolation to obtain values of the pump head corresponding to specified discharges, i.e., find h_p for discharges of 850 gal/min, 5800 gal/min, 4200 gal/min, etc.



We start the solution by selecting 10 discharge values along the abscissa and reading the corresponding values of pump head to obtain the following:

Q gal/min	800	1600	2400	3200	4000	4500	4800	5200	5600	6000
$\mathbf{h_p}$ ft.	181.5	170.0	160.0	148.5	138.6	128.0	120.4	109.0	95.0	80.0

These data pairs now must be entered into a file that can be read by program LAGRANGE. The input from the keyboard will be 10 2 3, followed by the filename. Then provide the discharges 850, 5800, 4200, etc. in response to the prompt **Give discharge (minus to terminate)**. The heads returned by the program are the following: Q = 850 gave h_p = 181.51, Q = 5800 gave h_p = 87.53, Q = 4200 gave h_p = 135.16.

*   *   *

5.6.2. SPLINE FUNCTION INTERPOLATION

One disadvantage of using Lagrange interpolation is seen when the interpolation interval shifts to continue to bracket the discharge; then the first derivative, which is needed in the Newton method, is not continuous. An alternative is to use spline interpolation. An essential difference between spline and piecewise polynomial interpolation is that, although a given spline function interpolates only between two consecutive points, both the spline function and one or more of its derivatives are continuous across these points. We will only discuss cubic splines here, since they require roughly the same computational effort as quadratic splines and have both continuous second and first derivatives across the data points.

Cubic splines develop a third-order polynomial between each pair of consecutive points as the interpolating function, or

$$y^{(i)} = a_i x^3 + b_i x^2 + c_i x + d_i \tag{5.35}$$

in which superscript i refers to the segment of the curve before point i, the dependent variable y plays the role of the pump head h_p, and x replaces Q. (For notational simplicity let H represent h_p in the remainder of this section.) For example, if we use four (H_j, Q_j) pairs, there will be three interpolating equations of the form of Eq. 5.35. In this case the total number of unknown (a, b, c, d) coefficients is $4(n - 1)$, or for our example $4 \times 3 = 12$ unknowns. Thus $4(n - 1)$ equations are needed. By substituting the known (H, Q) pairs at points j and $j + 1$ at the ends of segment i, we obtain $2(n - 1)$ of these equations. Another $(n - 2)$ equations are developed by equating the first derivatives of the two interpolating equations that apply at each data point, and an additional $(n - 2)$ equations result from equating second derivatives at these same points. The last two required equations come from boundary or end conditions. There are two commonly used kinds of boundary conditions. One sets the second derivatives at the beginning and/or end of the global interval to zero; that is, $(d^2y/dx^2)_1 = H_1'' = 0$ and/or $(d^2y/dx^2)_n = H_n'' = 0$. These are called natural cubic splines. The other sets y_1' and/or y_n' to values calculated by assigning values to the first derivatives.

In detail, the equations for the 4-point example are the following:

$$H_1 = a_1 Q_1^3 + b_1 Q_1^2 + c_1 Q_1 + d_1 \tag{5.36}$$

$$H_2 = a_1 Q_2^3 + b_1 Q_2^2 + c_1 Q_2 + d_1 \tag{5.37}$$

$$H_2 = a_2 Q_2^3 + b_2 Q_2^2 + c_2 Q_2 + d_2 \tag{5.38}$$

$$H_3 = a_2 Q_3^3 + b_2 Q_3^2 + c_2 Q_3 + d_2 \tag{5.39}$$

$$H_3 = a_3 Q_3^3 + b_3 Q_3^2 + c_3 Q_3 + d_3 \tag{5.40}$$

$$H_4 = a_3 Q_4^3 + b_3 Q_4^2 + c_3 Q_4 + d_3 \tag{5.41}$$

$$\left(\frac{dy^{(1)}}{dx}\right)_2 = \left(\frac{dy^{(2)}}{dx}\right)_2 \quad \Rightarrow \quad 3a_1 Q_2^2 + 2b_1 Q_2 + c_1 = 3a_2 Q_2^2 + 2b_2 Q_2 + c_2 \tag{5.42}$$

$$\left(\frac{dy^{(2)}}{dx}\right)_2 = \left(\frac{dy^{(3)}}{dx}\right)_3 \quad \Rightarrow \quad 3a_2Q_3^2 + 2b_2Q_3 + c_2 = 3a_3Q_3^2 + 2b_3Q_3 + c_3 \tag{5.43}$$

$$\left(\frac{d^2y^{(1)}}{dx^2}\right)_2 = \left(\frac{d^2y^{(2)}}{dx^2}\right)_2 \quad \Rightarrow \quad 6a_1Q_2 + 2b_1 = 6a_2Q_2 + 2b_2 \tag{5.44}$$

$$\left(\frac{d^2y^{(2)}}{dx^2}\right)_3 = \left(\frac{d^2y^{(3)}}{dx^2}\right)_3 \quad \Rightarrow \quad 6a_2Q_3 + 2b_2 = 6a_3Q_3 + 2b_3 \tag{5.45}$$

The boundary conditions are either

$$\left(\frac{d^2y^{(1)}}{dx^2}\right)_1 = H_1'' = 0 \tag{5.46a}$$

$$\left(\frac{d^2y^{(3)}}{dx^2}\right)_4 = H_4'' = 0 \tag{5.46b}$$

or

$$\left(\frac{dy^{(1)}}{dx}\right)_1 = \textit{specified} = 3a_1Q_1^2 + 2b_1Q_1 + c_1 \tag{5.47a}$$

$$\left(\frac{dy^{(3)}}{dx}\right)_4 = \textit{specified} = 3a_3Q_4^2 + 2b_3Q_4 + c_3 \tag{5.47b}$$

In these equations y has been used as the continuous dependent variable. For use in interpolating a point from a pump curve, the dependent variable will be called the pump head H, and in subsequent equations it will be used in place of y.

One obvious continuation is simply to solve the above equations for a_i, b_i, c_i and d_i, $i = 1, 2, 3$ and then use the appropriate equation to compute H for a given Q. However, an alternative that requires less arithmetic is the following interpolation equation:

$$H^{(i)} = a_iH_j + b_iH_{j+1} + c_iH_j'' + d_iH_{j+1}'' \tag{5.48}$$

The coefficients $a, b, c,$ and d are now obviously different than before. The coefficients a and b are weighting functions that are applied to the dependent variable H at points j and $j + 1$, and c and d are weighting functions applied to the second derivatives at these same points. In the finite element method a and b are the shape, basis or interpolation functions that are associated with a linear one-dimensional element. It can easily be shown that $a_i = (Q_{j+1} - Q)/(Q_{j+1} - Q_j)$ and $b_i = (Q - Q_j)/(Q_{j+1} - Q_j)$ with $a_i + b_i = 1$. We see that a_i and b_i are linear functions of Q. For simplicity the subscripts and superscripts will be deleted in many of the following equations; just keep in mind that the interpolating function provides values of Q within the interval $[Q_j, Q_{j+1}]$. Since c and d are functions of a and b, the number of additional unknowns that are introduced with each new segment is two rather than four. Thus the total number of equations for n intervals will be $2(n - 1)$ rather than $4(n - 1)$. Since $b = 1 - a$, only one new unknown appears for each new data point, so the number of required equations is only n - 1. The relations between c and d and a and b are

$$c = (a^3 - a)(Q_{j+1} - Q_j)^2 /6 \tag{5.49a}$$

and

$$d = (b^3 - b)(Q_{j+1} - Q_j)^2 /6 \tag{5.49b}$$

Thus the dependence of the interpolating equation on Q is entirely through the linear Q-dependence of a and b. Since the derivatives are also weighted by c and d (depending on a and b), a cubic interpolating polynomial exists over the closed interval $[Q_j, \; Q_{j+1}]$. To verify these statements, we note first that c and d contain terms involving Q^3 and Q^2 since the definitions of c and d contain a^3 and b^3. Thus the interpolating equation is a third-order polynomial. And we see also that $da/dQ = - 1/(Q_{j+1} - Q_j)$ and $db/dQ = 1/(Q_{j+1} - Q_j) = - da/dQ$. Now we compute the derivative of H itself to obtain

$$H' = \frac{dH}{dQ} = \frac{H_{j+1} - H_j}{Q_{j+1} - Q_j} - \frac{3a^2 - 1}{6}\left(Q_{j+1} - Q_j\right)H_j'' + \frac{3b^2 - 1}{6}\left(Q_{j+1} - Q_j\right)H_{j+1}'' \tag{5.50}$$

and the second derivative is

$$H'' = aH_j'' + bH_{j+1}'' \tag{5.51}$$

Since $a = 1$ at Q_j and $a = 0$ at Q_{j+1}, and also $b = 0$ at Q_j and $b = 1$ at Q_{j+1}, we have verified that the relations between c and a and between d and b are valid.

To apply Eq. 5.48 in practice, we must first determine numerical values for the second-derivative terms that appear in that equation. The required equations, ones that allow us to evaluate those terms, can be obtained by evaluating the first derivative at points 2, 3, ... , n - 2 and equating pairs from adjacent segments. We do not need the original equations or the equations that are obtained by equating second derivatives, since these are already satisfied by the interpolating polynomial. Equating first derivatives at the data points yields

$$\begin{aligned} &\frac{H_j - H_{j-1}}{Q_j - Q_{j-1}} + \frac{Q_j - Q_{j-1}}{6}H_{j-1}'' + \frac{Q_j - Q_{j-1}}{3}H_j'' \\ &\quad = \frac{H_{j+1} - H_j}{Q_{j+1} - Q_j} - \frac{Q_{j+1} - Q_j}{3}H_j'' - \frac{Q_{j+1} - Q_j}{6}H_{j+1}'' \end{aligned} \tag{5.52}$$

This equation comes directly from Eq. 5.50 with $a = 0$, $b = 1$ at Q_j for the derivative on the left side, and $a = 1$, $b = 0$ at Q_j on the right side of point j. This equation (i.e., these equations, since j is incremented) can be rewritten to display better the linear relation between the second derivatives of H, with known values on the right side, as

$$\begin{aligned} &\left(Q_j - Q_{j-1}\right)H_{j-1}'' + 2\left(Q_{j+1} - Q_{j-1}\right)H_j'' - \left(Q_{j+1} - Q_j\right)H_{j+1}'' \\ &\quad = 6\left\{\frac{H_{j+1} - H_j}{Q_{j+1} - Q_j} - \frac{H_j - H_{j-1}}{Q_j - Q_{j-1}}\right\} \end{aligned} \tag{5.53}$$

Written in matrix notation, Eq. 5.53 consists of a coefficient matrix $[A]$ multiplied by the vector of unknown second derivatives $\{H''\}$, which equals the known vector $\{B\}$, or

$$[A]\{H''\} = \{B\} \tag{5.54}$$

To make the system complete, boundary conditions must supply the first and last values. If the natural condition is used, then H_1'' and H_n'' are given zero values, which in effect starts the system at point 2 and ends the system at point $n - 1$. If first derivatives are specified, then these values provide the first and last equations in the system of equations. We note that only three consecutive values of the second derivatives are linked together in this system of equations, regardless of the choice of boundary conditions. This tridiagonal system of equations is very common, and it can be solved readily by decomposition or elimination methods. Since only one element exists in front of the diagonal, a single forward elimination pass through the rows of the matrix can convert the matrix into an upper triangular matrix with only two nonzero elements. Then a back substitution can obtain the solution for the second derivatives.

We have just seen that this alternative to the use of cubic spline interpolation requires first the solution of a tridiagonal equation system to determine numerical values for the second derivative of H at each of the points where (H_j, Q_j) pairs are given. By then applying the other interpolation relations, the head H can be found directly for any Q in the overall range of the interpolation.

The program SPLINECU implements this process. A listing of it can be obtained from the CD for study. This program is designed to read N pairs of values for (H_j, Q_j) and then determine H at M uniformly spaced values of Q, starting with Q_1 and ending with Q_n, instead of finding H for a specified Q. That is, it produces a full table of values for H. The third column in this table provides values of dH/dQ; when we compute elements of the Jacobian matrix in applying the Newton method to the solution of a network problem, this table of values is useful. The program could be modified to function in the same way as the Lagrange interpolation program, or to allow the user to provide a list of Q values for which heads are desired, and this list could be provided from a file or given individually from the keyboard. Or it could be converted into a function subprogram to supply the head for any specified discharge in solving a network problem involving a pump. Such a subprogram is on the CD under the name SPLINESU. Actually the program first reads Q and then H for each data pair, then forms and solves the tridiagonal equation system, and finally develops the new table with M entries.

Example Problem 5.9

Use program SPLINECU to obtain values of the pump head H and the derivative dH/dQ with an increment ΔQ = 100 gal/min. between 800 gal/min. and 6000 gal/min. As input data use the 10 points that are listed in the output table of Example Problem 5.8.

Solution: The following values should first be entered from the keyboard: 2 3 10 53 0. The first three and last three lines of output should be the following:

800.0	181.50	- 0.01508
900.0	179.99	- 0.01505
1000.0	178.49	- 0.01495
5800.0	87.53	- 0.03755
5900.0	83.77	- 0.03766
6000.0	80.00	- 0.03770

*　　　　*　　　　*

Example Problem 5.10

A pump having the characteristic curve of head vs. discharge given in Example Problem 5.9 is operated over six hours, as described by the following (assumed smooth) data:

Time hr.	0	1	1.5	2.0	2.8	3.8	4.9	5.2	6.0
Q gal/min	800	2500	3500	4700	5500	4900	3500	2900	2500

The pump efficiencies corresponding to the discharges in Example Problem 5.9 are

Q gal/min	800	1600	2400	3200	4000	4500	4800	5200	5600	6000
e	0.40	0.50	0.65	0.75	0.825	0.847	0.845	0.81	0.79	0.74
H ft.	181.5	170.0	160.0	148.5	138.6	128.0	120.4	109.0	95.0	80.0

Use cubic splines to define from the data pairs the relations that are needed, and determine the energy used by the pump during the six-hour period.

There are three relations that must be established by spline functions:

1. the discharge Q as a function of time t,
2. the pump head H as a function of the discharge Q, and
3. the efficiency e as a function of the discharge Q.

When these relationships have been determined, the amount of energy that is consumed can be found by numerically integrating the equation

$$Energy = \gamma \int (QH/e)dt$$

To complete this solution, it is convenient to convert the program SPLINECU into a subroutine to find the second derivatives for these relations; then a numerical integration subroutine will be used to obtain the energy. A program ELECECG to accomplish these tasks can be listed from the CD for further study as the rest of this example is read. It calls on SIMPR to complete the integration after the newly created subroutine SPLINESU has been called three times to determine the second derivatives. The arguments of SPLINESU are as follows: N = the number of data pairs, X = an array of N values for the independent variable, Y = an array of N values for the dependent variable, D2Y = an array of second derivatives returned by SPLINESU, D = a work array having N values, ITY = 0 for natural boundary conditions or ITY = 1 for prescribed first derivatives at the ends of the domain.

The program has three parts: (1) the main program that calls SPLINESU three times to obtain three sets of second derivatives and then calls the numerical integration routine SIMPR; (2) a block data subprogram to enter the data pairs rather than reading them from a file; and (3) the function subprogram EQUAT that defines the equation to be integrated. The second derivatives and sets of three data pairs are passed to EQUAT by means of the block common statements. EQUAT contains the logic that will determine, from the time, which two instants in time are to be used so that cubic spline interpolations can provide the values of the discharge as QQ, pump head as HH and efficiency as EE, and then the argument of the numerical integration *QH/e* is returned as EQUAT. The main program supplies the constant for γ and converts gal/min. to ft^3/s and energy in ft-lb to energy in kilowatt-hours.

The answer is Energy = 1.915×10^9 ft-lb or 2,600,000 kWh.

* * *

5.7 SENSITIVITY ANALYSES

We now turn to a third major type of network design. So far we have explored two design categories: the first sought to determine the size of as many pipes as possible (NJ of them since the equations would permit no more), and the second sought to determine the size of individually chosen components by considering each of them as a device that created a differential head at its location in the network. The first design category is encountered when a new network is being designed. The second type is more relevant to an existing system, for example, one in which we must determine the capacity and head of a pump to achieve a desired pressure at some point in response to some specified demands. The third design category seeks to identify the components of the network to upgrade, improve, or replace in order to increase the level of network performance most efficiently. The actual determination of unit sizes might be accomplished later, according to procedures used in the second type of design. In a sense this section describes methods that can be used to decide which system elements are most important to the improvement of system performance. For example, as a city's water use increases, the pressures may become too low during peak demand periods. Which of several pumps should be replaced by a larger one? An excellent quantitative means for making such a decision is to perform an appropriate sensitivity analysis and replace the pump with the largest pressure sensitivity. This section describes the determination of the magnitude of the sensitivity of one variable with respect to another variable in the network.

The quantification of sensitivity, which is how much one variable changes in response to a change in another variable or several variables, provides the designer a deeper understanding of network performance. Here we usually apply sensitivity analysis to identify the best component to change or replace to overcome a deficiency in the present performance of a network. A natural question is how these deficiencies can "best" or "most economically" be remedied. The answer may require a change in one or several pipe diameters, an increase in the head produced by existing pumps, an increase in the elevation of storage tanks (reservoirs), or the addition of pumps or pressure-reducing valves, etc. Normally there are a host of possible ways to correct inadequate performance. Some possibilities will be discussed in this section, but these should be regarded only as examples to stimulate thinking about alternatives. The sensitivity of one variable to another variable can be expressed by the partial derivative of the first variable with respect to the second variable. The variable(s) whose sensitivity is sought is (are) the dependent variable(s), and the variable that is the candidate for change to improve the network performance is the independent variable. There are usually several independent variables which are candidates for change. There may also be more than one dependent variable, but often one variable will be selected.

Generally it is not possible to define algebraically the partial derivative of any particular dependent variable with respect to another chosen independent variable when dealing with piping systems (there are exceptions), but these derivatives can be defined approximately by numerical methods. The mathematical definition of a partial derivative is

$$\frac{\partial f}{\partial x} = \underset{h \to 0}{limit} \frac{f(x+h, y, z) - f(x, y, z)}{h} \tag{5.55}$$

or, in more practical terms with $h = \Delta x$, $\partial f / \partial x \approx \Delta f / \Delta x$ when the other variables are unchanging. Thus, as x is changed by some small amount Δx, the corresponding change in the dependent variable (equation, system, or process) Δf is determined, and this latter difference, when it is divided by the change in the independent variable, produces an approximation of the derivative. Under conditions near those for which f is evaluated (assuming all other parameters remain constant), the sensitivity of the dependent variable is quantified as this derivative.

As this derivative becomes larger, the dependent variable f is more strongly affected by a change in the independent variable x, or the more sensitive f is to x. A negative derivative indicates that one variable decreases as the other increases. In a pipeline system there are many derivatives, or sensitivities, that can be determined and whose magnitudes provide useful information about the most effective way to change system performance. A few of many examples follow: (1) Low pressure can be corrected best by enlarging the pipe diameter that creates the largest $\partial p/\partial D$; (2) Low pressure can be corrected best by increasing the head of the pump with the largest $\partial p/\partial h_p$; (3) Too small a flow into a storage tank can be best corrected by increasing the power at the pumping station with the largest $\partial Q_{res}/\partial P$; (4) A fire demand at a node can best be augmented by the pump that has the largest $\partial QJ/\partial Q_p$; (5) Too large a pressure can best be reduced by a PRV in the pipe whose downstream head H produces the largest negative magnitude of $\partial p/\partial H$; etc.

The magnitudes of these sensitivities are generally not constant but change with problem specifications, such a peaking factors, and the largest may come from a different component (independent variable) as the total demand or demand pattern changes, or conditions under which the network is to perform change. The selection of sensitivities to compute will depend on the particular focus of each network performance study. Often several different sensitivities will provide nearly the same information.

Consider now the network shown in Fig. 5.28 to become acquainted with some of the possibilities. All of the water that is consumed in a typical daily operation must come from the two pumping stations. The tank (reservoir) at the end of pipe 11 is large and should receive water during periods of low demand so it can supply water when the demands are larger. To simplify the analyses assume the water level in the storage tank is constant at 200 m. The demands in the diagram are those that typically occur during the high de-mand period of a day. These demands are larger than those which existed when the system was designed, and now the tank does not fill sufficiently during low demand periods; the power to one of the pump stations must be increased. Which station should be upgraded (power input increased)?

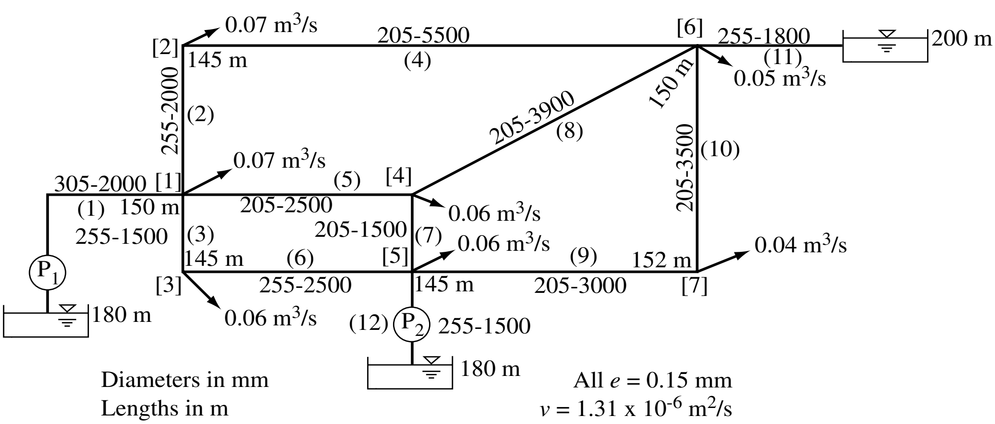


Figure 5.28 A network for sensitivity analysis.

To obtain more information on network performance, a series of solutions was obtained for peaking factors from 0.5 to 1.2. If NETWK is used to obtain these solutions, a convenient way to do this is to use the CHANGE command. The input data file to obtain such a series of solutions is presented in Fig. 5.29. (The option NETPLT = 3 in the $SPECIF list tells NETWK to write a file that can be used by program SENSITV.) The discharges in the two pipes from the pumping stations and the pipe that connects the storage tank to the network have been plotted as a function of the peaking factor and are

```
Illustration of sensitivities
/*
$SPECIF NFLOW=3,NPGPM=3,NUNIT=4,PEAKF=.5,
 NPRINT=-3,NETPLT=3 $END
PIPES
1 0 1 2000 305 .15
2 1 2 2000 255
3 1 3 1500
4 2 6 5500 205
5 1 4 2500
6 5 3 2500 255
7 5 4 1500 205
8 4 6 3900
9 5 7 3000
10 7 6 3500
11 0 6 1800 255
12 0 5 1500 255
NODES
1 .07 150
2 .07 145
3 .06 145
4 .06 140
5 .05 145
6 .05 150
7 .04 152
RESER
11 200
PUMPS
1 .15 50 .25 43 .35 35 180
12 .1 48 .15 43.25 .25 33.0 180
RUN
CHANGE
DFRAC
1.1
END
CHANGE
DFRAC
1.1
END
CHANGE
DFRAC
1.1
END
CHANGE
DFRAC
1.1
END
CHANGE
DFRAC
1.1
END
CHANGE
DFRAC
1.1
END
CHANGE
DFRAC
1.1
END
CHANGE
DFRAC
1.1
END
CHANGE
DFRAC
1.1
END
```

Figure 5.29 The input data file for the analysis of flow in the network in Fig. 5.28.

shown in Fig. 5.30. In this plot a negative flow in pipe 11 indicates a flow into the tank. This tank is seen to supply water whenever the peaking factor exceeds 0.58; the

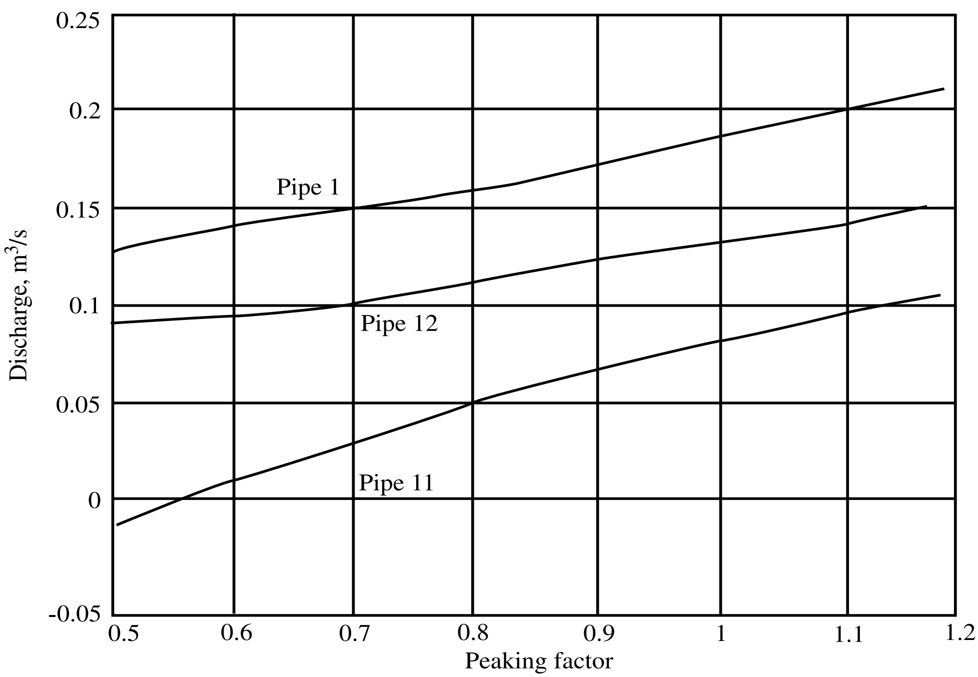


Figure 5.30 Discharge as a function of peaking factor for three pipes.

reason for the problem is clear. To choose which pumping station to upgrade, it would be useful to determine the sensitivity of the discharge from (into) the reservoir as a function of the power consumed at each of the two pumping stations, i.e., $\partial Q_{11}/\partial P_1$ and $\partial Q_{11}/\partial P_2$. (The cost associated with pumping is linearly related to the power consumption, so a properly chosen multiplier of each sensitivity will provide the increase in reservoir discharge per unit cost.) The pumping station with the larger sensitivity is the one to upgrade and is the lower cost alternative. Maximization of the sensitivity of discharge to power is the same as minimization of the cost of obtaining a desired discharge or volume of water relative to the cost of energy to pump this water.

The first two solutions from the input in Fig. 5.29 to NETWK (which can be obtained from NETWK, or another program in Chapter 4, or from NETWEQS1, to verify the values) provide the data in columns 2, 3, and 4 in Table 5.9. The first solution is for a peaking factor of 0.5 (since PEAKF = 0.5), and the second is for PF = 0.5(1.1) = 0.55 (since DFRAC under CHANGE is 1.1). Column 5 is the difference in discharge in pipe 11 (into the tank) for these two solutions. The difference in power $P = \gamma Q h_p$ from pump station 1 is given in column 6, and the difference from station 2 is in column 7. The sensitivities of the tank discharge to the power at the pumping stations are in columns 8 and 9. When the demands are 0.5 times those that are listed on the network diagram, it is best to augment the pumping at station 2 because $\partial Q_{11}/\partial P_2 = 0.0062$ is larger than $\partial Q_{11}/\partial P_1 = 0.0042$.

Table 5.9

PF (1)	Q_{11} m^3/s (2)	**Power$_1$** kW (3)	**Power$_2$** kW (4)	ΔQ_{11} (5)	ΔP_1 (6)	ΔP_2 (7)	$\Delta Q_{11}/\Delta P_1$ (8)	$\Delta Q_{11}/\Delta P_2$ (9)
0 .50	- 0.0154	64.74	42.35	0.0102	2.4 1	1.64	0.0042	0.0062
0.55	- 0.0053	66.65	43.99					

The same sensitivities were computed from the other paired consecutive solutions requested by the CHANGE command, with the results shown in Table 5.10. Over the entire range of peaking factors the sensitivity $\Delta Q_{11}/\Delta P_2 > \Delta Q_{11}/\Delta P_1$, and therefore the clear choice is to increase the input power to pump station 2.

The solutions that were used to obtain the sensitivities of the reservoir discharge Q_{11} to pump power did not directly require any of these variables to be changed from solution to solution. Instead the peaking factor was changed, which in turn caused these variables to change from solution to solution. An alternative was to obtain one series of solutions

Table 5.10

PF	0.50-0.55	0.550-0.605	0.605-0.666	0.666-0.732	0.732-0.805	0.805-0.886	0.886-0.974	0.974-1.072	1.072-1.179
$\Delta Q_{11}/\Delta P_1$ x10^3	4.2	5.7	5.3	4.8	4.1	3.6	3.3	3.1	3.0
$\Delta Q_{11}/\Delta P_2$ x10^3	6.2	8.6	7.9	6.9	5.8	5.3	4.9	4.5	4.4

in which P_1 was changed, and another in which P_2 was changed, but this would have required more effort. The fact that specifying a change in one variable (or parameter) causes changes in all other variables associated with network performance allows us to obtain many sensitivities from one series of solutions. The program SENSITV in the NETWK package is designed to allow the user to generate tables of sensitivities. Table 5.11 is the first portion of the output from SENSITV, in which the demand QJ_1 at node 1 (which is linearly related to the PF) was selected as the independent variable, and the discharge in pipe 11, or reservoir discharge Q_r, was selected as the dependent variable. In obtaining this table, the option to place the independent variables in the output table was selected. Table 5.13 then presents the final results from 10 solutions in a simpler format.

Table 5.11
Sensitivity of Discharge in Reservoir 1 to Changes in Demand at Node 1

Res.	Independent Variable at 1, Comparison between Solutions 2 and 1						
	QJ	QJ	Diff.	Q_r	Q_r	Diff.	Ratio
1	0.0350	0.0385	0.0035	- 0.0154	- 0.0053	0.0102	2.91
	Independent Variable at 1, Comparison between Solutions 3 and 2						
	QJ	QJ	Diff.	Q_r	Q_r	Diff.	Ratio
	0.0385	0.0424	0.0039	- 0.0053	0.0075	0.0128	3.32

Using SENSITV to obtain the sensitivities $\Delta Q_{11}/\Delta P_1$ and $\Delta Q_{11}/\Delta P_2$, we obtain the results that are listed in output Table 5.12. This time we chose to have only the ratios written to the output table. In this output from SENSITV the first independent variable is P_1, and the second independent variable is P_2.

Table 5.12

Sensitivity Comparison of Flow from Reservoir at Reservoir 1

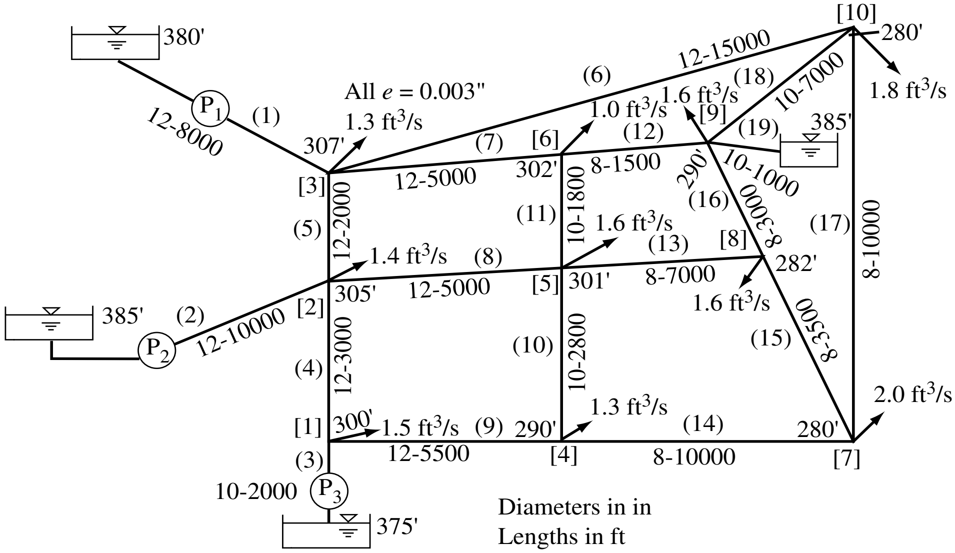
Solution: Reservoir	1-2 1	2-3 1	3-4 1	4- 5 1	5-6 1	6-7 1	7-8 1	8-9 1	9-10 1
1	0.004	0.006	0.005	0.005	0.004	0.004	0.003	0.003	0.003
2	0.006	0.009	0.008	0.007	0.006	0.005	0.005	0.005	0.004

Table 5.13

Sol.	**Indep. Variable** QJ_1	ΔQJ_1	**Dep. Variable** $Q_r = Q_{11}$	ΔQ_r	**Ratio** $\Delta Q_r/\Delta QJ_1$
1	0.03500		- 0.0154		
		0.00350		0.0101	2.89
2	0.03850		- 0.0053		
		0.00385		0.0128	3.32
3	0.04235		0.0075		
		0.00424		0.0135	3.18
4	0.04659		0.0210		
		0.00465		0.0139	2.99
5	0.05124		0.0349		
		0.00513		0.0139	2.71
6	0.05637		0.0488		
		0.00563		0.0141	2.50
7	0.06200		0.0629		
		0.00621		0.0144	2.32
8	0.06821		0.0773		
		0.00682		0.0145	2.13
9	0.07503		0.0918		
		0.00750		0.0150	2.00
10	0.08253		0.1068		

Example Problem 5.11

In the network shown below the pressures at some of the nodes near node 7 are less than desirable. Which of the three pumps should be enlarged? Use additional sensitivities to understand more completely the performance of this network.



Pump 1

Q_1, ft³/s	h_{p1}, ft
3.0	80
5.0	75
8.0	65

Pump 2

Q_2, ft³/s	h_{p2}, ft
3.0	80
5.0	75
8.0	65

Pump 3

Q_3, ft³/s	h_{p3}, ft
3.0	83
5.0	78
8.0	68

We want to determine which pump will most increase the pressure at node 7 for a given increase in the head of that pump. Since all three pumps are far from the node with the deficient pressure, it is difficult to guess which pump will most influence the pressure at that node. The table below provides a partial summary of several solutions that were obtained by using the CHANGE capability in NETWK. The input data for these solu-tions is on the CD as file EXP5_11.IN. We will find in the input file that the original solution is obtained with the pump curves that accompany the network diagram; the sec-ond solution is obtained by increasing the head of pump 1 by 10 ft; the third solution has the head of pump 2 increased by 10 ft with the head of pump 1 reset to the original value; the fourth solution has the head of pump 3 increased by 10 ft; and the fifth solu-tion is obtained by increasing the head of the reservoir by three feet. The last solution, in which the water surface elevation in the reservoir was changed by three feet, is in a different category than others in which the pumps heads were changed, since water must

Node	Sol. 1	Solution 2 Pump 1, Δh_p=10'		Solution 3 Pump 2, Δh_p=10'		Solution 4 Pump 3, Δh_p=10'		Solution 5 Res. 1, ΔH= 3'	
	Head ft	Head ft	$\Delta H/\Delta h_p$	Head ft	$\Delta H/\Delta h_p$	Head ft	$\Delta H/\Delta h_p$	Head ft	ΔH/Δh
7	360.2	361.4	0.112	361.2	0.096	361.6	0.138	362.1	0.633
10	371.2	372.2	0.104	372.0	0.084	372.3	0.115	373.2	0.680
4	390.5	392.2	0.168	392.0	0.148	392.7	0.224	391.8	0.447
5	390.2	391.9	0.168	391.7	0.147	392.4	0.212	391.6	0.460
6	390.2	391.9	0.167	391.7	0.145	392.3	0.206	391.6	0.467

be supplied by the pumps to fill the reservoir. Increasing the head of pump 3 (solution 4) is the most effective way to increase the heads at all of the nodes in the table because the derivatives $(\Delta H/\Delta h_p)_3$ are larger than these derivatives for the other two pumps. However, since all values of $\Delta H/\Delta h_p$ are not vastly different, it would be more effective to increase the head of all three pumps, particularly if the heads are deficient by more than a small amount.

Other network components have an influence on the sensitivity of dependent variables to a change in the independent variables. The table below summarizes a set of sensitivity analyses that mimic the prior table, with the one exception that pipe 3 was changed in diameter from 10 in to 8 in before obtaining the series of solutions. Now pump 3, which previously produced the largest head increments, gives the smallest head increments.

Node	Sol. 1	Solution 2 Pump 1, ΔH=10'		Solution 3 Pump 2, ΔH=10'		Solution 4 Pump 3, ΔH=10'		Solution 5 Res. 1, ΔH= 3'	
	Head ft	Head ft	$\Delta H/\Delta h_p$	Head ft	$\Delta H/\Delta h_p$	Head ft	$\Delta H/\Delta h_p$	Head ft	$\Delta H/\Delta h$
7	352.1	353.5	0.137	353.3	0.119	353.1	0.099	354.0	0.633
10	364.0	365.3	0.128	365.1	0.110	364.9	0.089	366.0	0.657
4	378.8	380.6	0.182	380.4	0.159	380.1	0.133	380.3	0.513
5	378.8	380.6	0.181	380.4	0.159	380.1	0.132	380.4	0.633
6	379.1	380.8	0.175	380.6	0.152	380.3	0.125	380.7	0.657

The reasons for this change in effectiveness are relatively clear. The 8-in pipe that contains pump 3 is just too small for this pump to cause the greatest increases in head at the downstream nodes; to increase the head, the pump must supply a larger portion of the total flow, and the head loss in the 8-in pipe increases too much as the discharge increases. We see that the interactions of network components can be complex and interwoven, and the only effective means of determining the sensitivity of selected variables with respect to others is to develop an appropriate series of solutions so these sensitivities can be estimated. These solutions must consider demands etc. that are near those for which the sensitivities are to be determined.

Consider the use of sensitivities from another perspective. We might ask which pump can be enlarged at the least cost in order to increase the head at certain nodes by a specified amount. The answer to this question is already embedded in the previous solutions. But now the independent variable is not the incremental head added by a pump but rather the power (which can be substituted for cost when only the magnitudes of the sensitivities are compared, since the cost will be in dollars per kilowatt-hour) that a pump delivers to the network. Tables containing the power consumption of each pump as three independent variables are given below. In these tables the sensitivities are in units of head per kilowatt instead of head/head, as it was in the previous tables. Each of the previous two tables is now replaced by two tables for clarity. The first of each pair of tables lists the power requirement and the incremental difference in power between a subsequent solution and the first solution. The second table of each pair divides the change in head at the listed node by the incremental power to obtain $\Delta H/\Delta P_i$ with subscript i being the pump number and P being power in kilowatts. The negative values for these sensitivities occur because the incremental power between solutions is negative, i.e., the power produced by that pump (when the head of another pump increases) is less than that of the original solution. If the negative derivatives are ignored, then the conclusion is unchanged; pump 3 will produce a larger incremental head at these nodes for a given cost than either pump 1 or pump 2 can supply if the line serving pump 3 has a 10-in diameter. This situation occurs because the positive values of $\Delta H/\Delta P_3$ are larger than either $\Delta H/\Delta P_1$ or $\Delta H/\Delta P_2$. However, if the supply line for pump 3 has an 8-in diameter, then the most cost-effective pump for increasing the head at the listed nodes is pump 1, since the second pair of tables shows that the positive values of $\Delta H/\Delta P_1$ are larger than the values of either $\Delta H/\Delta P_2$ or $\Delta H/\Delta P_3$.

Sensitivity of Nodal Head to Pump Power (Pipe 3 is 10-in dia.)

Pump	Sol. 1	Solution 2		Solution 3		Solution 4	
	P	P	ΔP	P	ΔP	P	ΔP
1	26.84	28.34	1.50	26.49	- 0.35	26.39	- 0.45
2	25.19	24.83	- 0.36	26.59	1.40	24.75	- 0.44
3	32.44	32.00	- 0.44	32.03	- 0.41	34.08	1.64
Sum			0.70		0.64		0.75

Node	Solution 1	Solution 2			
	Head, ft	ΔH	$\Delta H/\Delta P_1$	$\Delta H/\Delta P_2$	$\Delta H/\Delta P_3$
7	360.23	1.12	0.75	- 3.11	- 2.55
10	371.15	1.03	0.69	- 2.86	- 2.34
4	390.48	1.68	1.12	- 4.64	- 3.85
5	390.24	1.68	1.12	- 4.67	- 3.82
6	390.22	1.67	1.11	- 4.64	- 3.80
		Solution 3			
7		0.96	- 2.74	0.69	- 2.34
10		0.84	- 2.40	0.60	- 2.05
4		1.48	- 4.23	1.06	- 3.61
5		1.47	- 4.23	1.05	- 3.59
6		1.45	- 4.14	1.04	- 3.54
		Solution 4			
7		1.38	- 3.07	- 3.14	0.84
10		1.15	- 2.56	- 2.56	0.70
4		2.24	- 4.98	- 5.09	1.37
5		2.12	- 4.71	- 4.82	1.29
6		2.06	- 4.58	- 4.68	1.26

While the negative sensitivities were ignored above, they do present valuable information related to the network's performance, particularly if total power (or cost) is considered. In fact, to neglect negative values is to ignore potential savings. For example, when pipe 3 has a 10-in diameter, we find in the first table from the second solution that the incremental sensitivities for pumps 2 and 3 are - 0.36 and - 0.44 kW, respectively; these values indicate that the power requirements for these two pumps decrease as the power requirement for pump 1 increases by 1.50 kW. The net increase in required power is only 1.50 - 0.36 - 0.44 = 0.70 kW. Similarly, if the head across Pump 2 (see solution 3) is increased by 10 ft, then the net increase in power is slightly less, or 0.64 kW. Sometimes it is better to examine sums of differences (or just differences) rather than one difference divided by another difference, which is how we first defined "sensitivity." In this example it probably makes most sense to use a difference divided by a difference, but the difference in the denominator (or the independent variable) should be the sum of power differences. This sensitivity represents the change in head that is caused by the change in the overall or total power consumption P_t (or cost). If these are the important sensitivities, then the values in the following table should be used to decide which alternative will be the most cost-effective and/or best.

Sensitivity of Nodal Head to Pump Power (Pipe 3 is 8-in dia.)

Pump	Sol. 1	Solution 2		Solution 3		Solution 4	
	P	P	ΔP	P	ΔP	P	ΔP
1	29.23	30.55	1.32	28.90	- 0.33	28.96	- 0.27
2	27.46	27.12	- 0.34	28.70	1.24	27.22	- 0.24
3	22.59	22.28	- 0.31	22.31	- 0.28	23.78	1.94
Sum			0.67		0.63		1.43

Node	Solution 1	Solution 2			
	Head, ft	ΔH	$\Delta H/\Delta P_1$	$\Delta H/\Delta P_2$	$\Delta H/\Delta P_3$
7	352.14	1.37	1.04	- 4.03	- 4.42
10	364.00	1.28	0.97	- 3.97	- 4.13
4	378.80	1.82	1.38	- 5.35	- 5.87
5	378.81	1.81	1.37	- 5.32	- 3.61
6	379.08	1.75	1.33	- 5.15	- 5.65
		Solution 3			
7		1.19	- 3.61	0.96	- 4.25
10		1.10	- 3.33	0.89	- 3.93
4		1.59	- 4.82	1.28	- 5.68
5		1.59	- 4.82	1.28	- 5.68
6		1.52	- 4.68	1.17	- 5.18
		Solution 4			
7		0.99	- 3.67	- 4.13	0.83
10		0.89	- 3.23	- 3.74	0.75
4		1.33	- 4.93	- 5.59	1.11
5		1.32	- 4.89	- 5.50	1.11
6		1.25	- 4.63	- 5.21	1.05

We see there are many possibilities, and which is best depends upon the objective, coupled with the judgment of the engineer who is responsible for making the decision. And we must keep in mind that the magnitude of each sensitivity (and difference, or sum of differences) is not a constant but can take on quite different values as demands and other conditions change.

Node	Pipe 3, Dia. = 10 in			Pipe 3, Dia. = 8 in		
	Sol. 2	Sol. 3	Sol. 4	Sol. 2	Sol. 3	Sol. 4
	$\Delta H/\Delta P_t$	$\Delta H/\Delta P_t$	$\Delta H/\Delta P_t$	$\Delta H/\Delta P_t$	$\Delta H/\Delta P_t$	$\Delta H/\Delta P_t$
7	1.60	1.50	1.84	2.04	1.89	0.69
10	1.47	1.31	1.53	1.91	1.75	0.69
4	1.43	2.31	2.99	2.72	2.52	0.93
5	2.40	2.30	2.83	2.70	2.52	0.92
6	2.39	2.26	2.75	2.61	2.30	0.87

Another goal might be the maintenance of as large a volume of water in the storage tank (reservoir) as possible. If so, the sensitivities that should be examined are the difference in discharge in pipe 19 (which connects the reservoir to the network) divided by the sum of the pump power consumptions; rather than seek the largest value as we did before, the smallest sensitivity (the one with the largest negative magnitude) is the one we want. The reason is that our desire is to maximize $|\Delta Q_{19}|$ (the numerator) while minimizing the increase in overall pump power consumption ΔP_t (the denominator). These tables of sensitivities follow:

Item	Pipe 19, Dia. = 10 in	Pipe 19, Dia. = 8 in
Flow Q_{19} from Sol. 1, original conditions	2.30 ft^3/s	3.13 ft^3/s
Flow Q_{19} from Sol. 2, $\Delta h_p = 10$ ft at pump 1	2.17 ft^3/s	3.00 ft^3/s
ΔQ_{19}	- 0.13 ft^3/s	- 0.13 ft^3/s
$\Delta Q_{19}/\Delta P_t$	- 0.19 $ft^3/s/kW$	- 0.19 $ft^3/s/kW$
Flow Q_{19} from Sol. 3, $\Delta h_p = 10$ ft at pump 2	2.19 ft^3/s	3.01 ft^3/s
ΔQ_{19}	- 0.11 ft^3/s	- 0.12 ft^3/s
$\Delta Q_{19}/\Delta P_t$	- 0.17 $ft^3/s/kW$	- 0.19 $ft^3/s/kW$
Flow Q_{19} from Sol. 4, $\Delta h_p = 10$ ft at pump 3	2.15 ft^3/s	3.04 ft^3/s
ΔQ_{19}	- 0.15 ft^3/s	- 0.09 ft^3/s
$\Delta Q_{19}/\Delta P_t$	- 0.20 $ft^3/s/kW$	- 0.06 $ft^3/s/kW$

* * *

To compute sensitivities, we must have two solutions available in which the independent variable x has changed and the change in the dependent variable f can be obtained. Thus a numerical approximation to $\partial f/\partial x$ is obtained by dividing the change in the dependent variable Δf by the change in the independent variable Δx, or $\partial f/\partial x \approx \Delta f/\Delta x$. These paired solutions were previously obtained from NETWK by using the CHANGE command. An alternative, and for some networks a more effective, way to obtain such a series of solutions is to obtain an "Extended Time Simulation." This is a time-varying or quasi-steady solution that ignores most fluid transient effects. Extended Time Simulations, as Chapter 6 will describe further, consist of a series of steady-state solutions with different prescribed demands, water surface elevations at reservoirs, and head-discharge relations at pumps that depend upon a demand function or flow rule, storage functions, and pump rules, etc. The NETWK code allows the results from such solutions to be written in tables with time in the first column and discharges or head losses for selected pipes, and/or pressure at selected nodes, to be listed in subsequent columns. Alternative tables giving reservoir water surface elevations as a function of time can also be obtained. These tables can be used to obtain most sensitivities that may be wanted, especially if the specifications for the Extended Time Simulation dictate that some other variable is linearly related to time. The time can be used as the independent variable for the sensitivities.

Example Problem 5.12

Use the Extended Time Simulation capability of NETWK to obtain a series of steady state solutions and from these obtain the sensitivities for the 12 pipe, 7 node network diagramed in Fig. 5.28. Express the peaking factor as a linear function of time. After verifying some of the sensitivities that have already been presented, allow the elevation of the water surface in the tank to vary so its level is 198 m at time $t = 0$ (when PF = 0.5). The tank is circular with a diameter of 30 m, and its bottom elevation is 195 m, i.e., at this level there is no more water in the tank. Plot as a function of peaking factor the discharge from the two pumping stations and the discharge into and out of the reservoir.

The input file to NETWK to obtain this solution is listed on the next page. In it the linear relationship between PF and time is dictated by the DEMAND FUNCTION which applies to all nodes. The output tables are not given here but can be developed by the reader. After they are obtained, we could either use SENSITV or import the tables into a spreadsheet and then generate the sensitivities.

```
Illustration of sensitivities using Ext. Time Simulation
/*
$SPECIF NFLOW=3,NPGPM=3,NUNIT=4,PEAKF=.5,
NPRINT=-3,NODESP=0,ISIML=1,NETPLT=3,COEFRO=.15 $END
PIPE-
1 305. 2000. 1 .07 150.
2 255. 2000. 1 2 .07 145.
3 255. 1500. 1 3 .06 145.
4 205 5500. 2 6 .05 150.
5 205. 2500. 1 4 .06 140.
6 255. 2500. 5 .05 145. 3
7 205. 1500. 5 4
8 205. 3900. 4 6
9 205. 3000. 5 7 .04 152.
10 205. 3500. 7 6
11 255. 1800. 6
12 255. 1500. 5
RESER
11 200
PUMPS
1 .15 50 .25 43 .35 35 180
12 .1 48 .15 43.25 .25 33.0 180
RUN
$TDATA PRINTT=-3,HTIME=24,INCHRP=1
LINEAR=1,ISUNIT=0 $END
PIPE TABLE
ALL
NODE TABLE
ALL
RESER. TABLE
11/
END TABLES
DEMAND FUNCTION
1 0 1. 12 1.6789738 24 2.35794769/
1-7/
STORAGE FUNCTION
1 195 0 198 2120.6 205 7069/
11/
END SIML
```

To use an Extended Time Simulation to produce solutions that portray the flow at the reservoir, the size of the storage tank at the end of pipe 11, and its water surface elevation, are included in the input file by prescribing a STORAGE FUNCTION. Since the tank is circular with a diameter of 30 m, the area is $A = \pi D^2/4 = 707$ m^2; with its base at 195 m the tank will have a starting water surface elevation of 198 m when PF = 0.5. We must first change elevation 200 to 198 under the RESER command, and then we add ISUNIT=0 to the $TDATA list and finally add STORAGE FUNCTION and two lines of data before END SIML. The following (partial) tables will then be obtained:

A negative flow in pipe 11 indicates that the storage tank is filling. From the middle table of the set it can be seen between hours 2 and 3 (when the peaking factor PF is between $0.5 \times [1 + 1.358 \times (2/24)] = 0.557$ and 0.585) that the tank changes from filling to supplying the network. Shortly after hour 22, when the peaking factor PF is slightly larger than $0.5 \times [1 + 1.358 \times (22/24)] = 1.122$, the tank has emptied. (The tank base is at 195 m, at which its volume becomes 0 m^3 in the storage function.) Thereafter, all of the demand must be met by the pumps, even as the PF increases, and this is shown in the negative pressure in the last two lines of the pressure table. Obviously these pressures are

PRESSURES (kPa) AT DESIGNATED NODES AS A FUNCTION OF TIME

TIME	NODE NUMBERS						
hrs.	1	2	3	4	5	6	7
0.0	620.79	599.96	658.84	661.46	662.26	497.17	487.94
1.0	609.88	587.35	647.43	649.93	651.75	493.83	481.35
2.0	599.55	575.78	636.57	639.30	641.86	491.76	475.94
3.0	590.05	565.61	626.55	629.88	632.93	491.03	472.67
4.0	580.08	554.89	616.02	619.76	623.56	488.25	468.68
.	.	.	.	.	.	.	.
21.0	311.87	247.25	323.15	328.72	353.57	235.90	178.93
22.0	287.71	217.97	297.98	302.10	330.01	217.86	151.05
23.0	- 56.82	- 287.95	- 47.95	- 175.37	- 20.87	- 442.91	- 453.93
24.0	- 98.43	- 343.07	- 91.34	- 227.28	- 62.87	- 502.91	- 513.54

DISCHARGES IN DESIGNATED PIPES AS A FUNCTION OF TIME

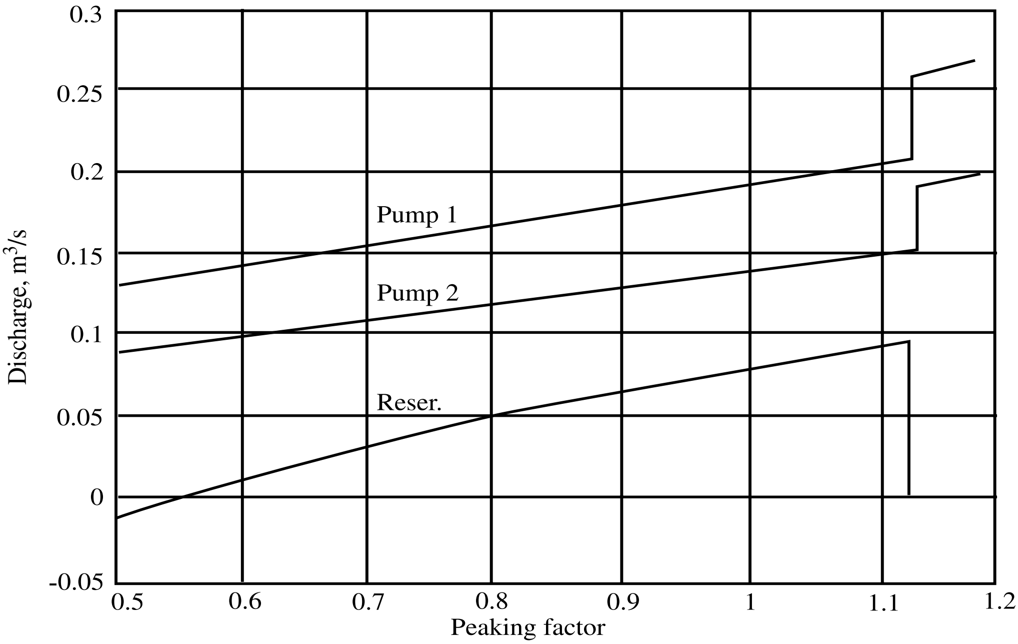
HR.	PIPE NUMBER 1	2	3	4	5	6	7	8	9	10	11	12
0.0	.127	.049	.021	.014	.022	.009	.027	.019	.028	.008	-.015	.088
1.0	.130	.050	.022	.013	.022	.010	.027	.018	.027	.006	-.010	.090
2.0	.134	.050	.023	.011	.022	.011	.027	.016	.027	.004	-.004	.093
3.0	.136	.050	.023	.009	.022	.012	.027	.015	.026	.002	.003	.094
4.0	.139	.051	.024	.008	.022	.013	.028	.013	.025	.000	.010	.098
.	.	.	.	.	.	.	.	.	.	.	.	.
21.0	.202	.063	.038	-.013	.025	.028	.033	-.008	.028	-.016	.092	.143
22.0	.207	.065	.039	-.014	.026	.029	.034	-.008	.028	-.017	.095	.147
23.0	.268	.101	.043	.020	.044	.026	.056	.031	.053	.007	.000	.192
24.0	.275	.103	.044	.021	.045	.027	.057	.032	.054	.007	.000	.197

WATER SURFACE ELEVATION IN RESERVOIR 11

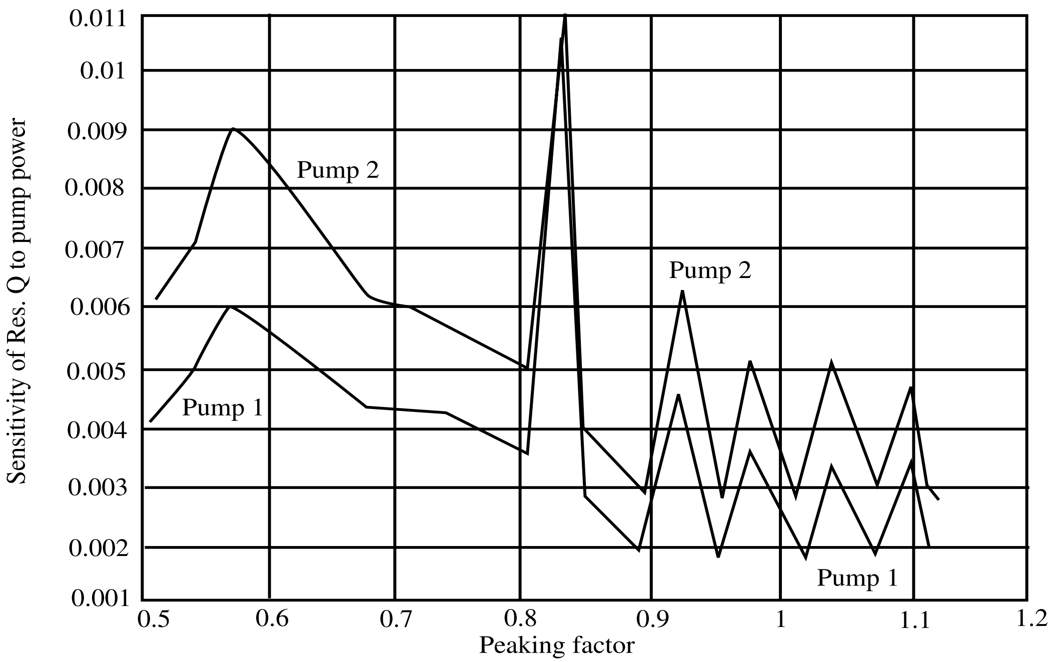
TIME hrs.	ELEVATION ft.
0.0	200.00
1.0	200.08
2.0	200.13
3.0	200.15
.	.
21.0	195.60
22.0	195.13
23.0	195.00
24.0	195.00

not real; this network cannot meet the demands with an empty tank. To prepare a plot of the discharge from the pump stations, the discharge table can be imported into a spreadsheet. The first column, which lists the time, can be changed to represent the PF by noting that time 0.0 corresponds to PF = 0.5 and time 24.0 corresponds to PF = 1.179. The plot shows the discharges in pipes 1, 12, and 11 as a function of peaking factor. We see the reservoir filling with PF < 0.58; when PF = 1.13 the reservoir has emptied. Since it can now supply no flow, the discharge from each pump station must sharply increase to satisfy the demand.

To obtain the sensitivities $\Delta Q_{11}/\Delta P_1$ and $\Delta Q_{11}/\Delta P_2$, columns in the table can be created for the power at each of the two pump stations with $P_1 = 9.806 Q_1(h_{p1}) = 9.806 Q_1(58.6 - 50Q_1 - 50{Q_1}^2)$ and $P_2 = 9.806 Q_{12}(56.8 - 82.5Q_{12} - 50{Q_{12}}^2)$. The difference of P_1 and P_2 between separate entries (rows) is the divisor of the differences in the discharge Q_{11} to obtain the sensitivity of the reservoir flow to the pump power. These sensitivities are presented in the next plot. The curves are not smooth largely because of the limited accuracy in computing the discharge in pipe 11, since the sensitivities are dependent entirely upon these values. However, the conclusion is the same as when the level of the reservoir was constant at 200 m; it is better to increase the power at pump station 2.



Discharge, m^3/s
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
Pump 1
Pump 2
Reser.
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
Peaking factor



Sensitivity of Res. Q to pump power
0.011
0.01
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
Pump 2
Pump 2
Pump 1
Pump 1
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
Peaking factor

* * *

5.8 PROBLEMS

5.1 The table below contains several pipes. Using the Darcy-Weisbach and the Hazen-Williams equations, compute the diameters of the pipes that are needed to convey the given discharge with the given head loss.

Pipe	Q	L	h_L	ex10^3	C_{HW}	Darcy-Weisbach		Hazen-Williams
No.	ft^3/s	ft	ft	in		f	D, in	D, in
1	1.0	2500	30	0.05	150			
2	2.0	400	20	20.0	95			
3	3.0	10000	105	5.0	138			

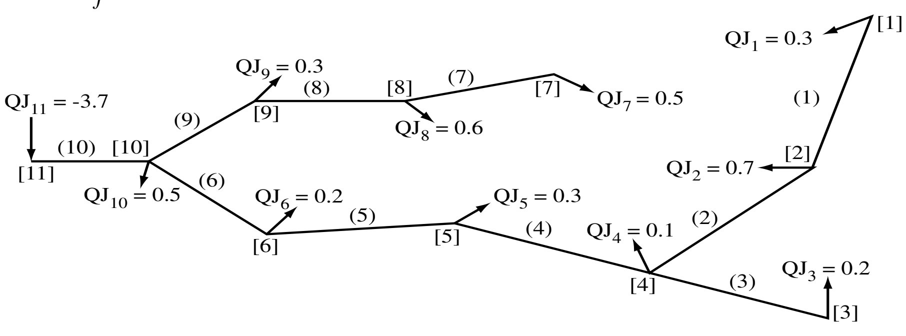
5.2 The table below contains several pipes. Using the Darcy-Weisbach and the Hazen-Williams equations, compute the diameters of the pipes that are needed to convey the given discharge with the given head loss.

Pipe	Q	L	h_L	ex10^3	C_{HW}	Darcy-Weisbach		Hazen-Williams
No.	m^3/s	m	m	cm		f	D, m	D, m
1	0.25	1500	20	0.08	150			
2	0.50	600	20	80.0	95			
3	1.50	4000	55	9.0	140			

5.3 Modify program DIAPIP so algebraic derivatives are used to evaluate the elements of the Jacobian in place of the numerical evaluation in the original listings.

5.4 Modify program DIAPIP so the two unknown variables are f and D rather than $SF = 1/\sqrt{f}$ and D.

5.5 The program SOLBRAN was used to determine the pipe diameters in a 10-pipe branched system; then the nodes and pipes were numbered by starting at the upstream end. This same branched system is shown below, but now the numbering proceeds from the downstream end. Prepare the input data for program SOLBRAN (or your own program) using this numbering, and obtain the solution. The slope of the energy line for all pipes is $S = h_f/L = 0.002$.



5.6 Retain the node numbers as in Problem 5.5, but begin the pipe numbering with 1 at the upstream end, prepare the input data for SOLBRAN and obtain the solution.

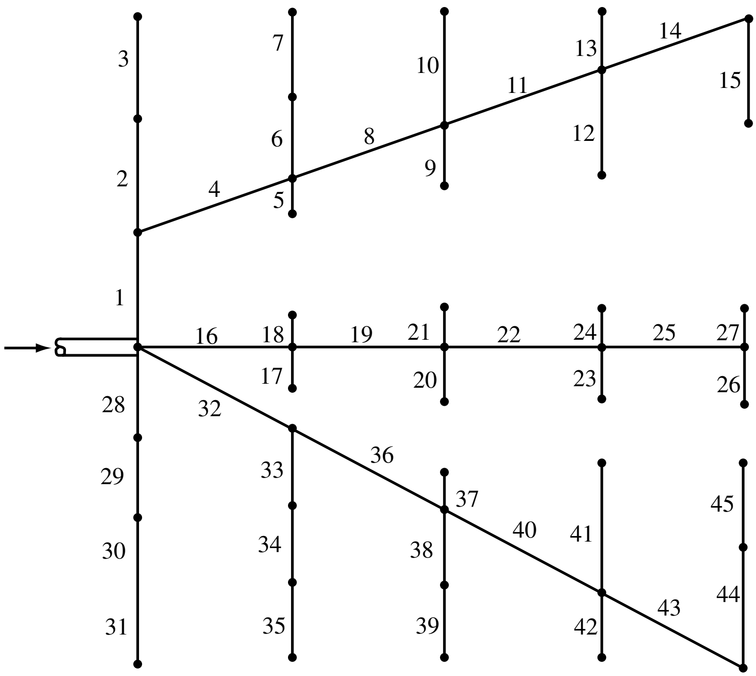
5.7 Use NETWK to solve Problem 5.5.

5.8 Develop a computer program that can determine the diameter of a pipe if the discharge, head loss, pipe length, and wall roughness are known. This program should be able to use either the Darcy-Weisbach equation (including the Colebrook-White equation) or the Hazen-Williams equation.

5.9 Modify program SOLBRAN to solve a branched system in which laminar flow exists in all pipes. Write this program so it reads from the input file for all pipes either the head losses or the diameters, and it determines either the pipe diameters or the head losses (i.e., it finds the variable that is not given) and the pipe discharges that will satisfy the specified demands.

5.10 Using the program from Problem 5.9 (or a slight modification of it), find the diameters of the tubing for the drip irrigation system shown below if each emitter (solid circle) is to supply 2 gal/min. The slope of the HGL is 0.008.

Pipe	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
L, ft	25	25	25	42	10	15	25	42	15	25	42	25	15	42	25
Pipe	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
L, ft	40	10	10	40	10	10	40	10	10	40	10	10	20	20	20
Pipe	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
L, ft	20	45	20	20	20	45	10	10	20	45	25	10	45	25	25



5.11 Determine the pipe diameter that will carry a discharge $Q = 1.8\ \text{ft}^3/\text{s}$ over a length of 4000 ft if the difference in head between the beginning and end of the line is to be 65 feet. The wall roughness for this pipe is $e = 0.005$ in.

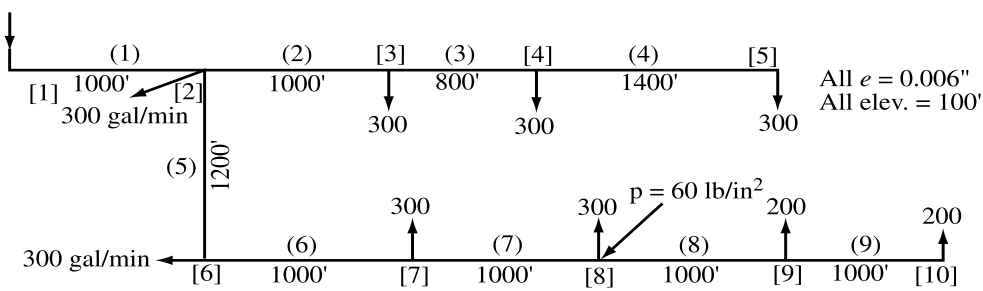
5.12 Find the pipe diameter in Problem 5.11 by using the Hazen-Williams equation with $C_{HW} = 145$.

5.13 A 3000 ft long pipeline carries a discharge of 2.0 ft^3/s over 2000 ft of its length, at which point an unknown amount of water is withdrawn. The drop in head from the beginning to the end of the pipe is 30 ft. The pipe is 8-inch-diameter PVC pipe, and the kinematic viscosity of the water is $\nu = 1.2 \times 10^{-5}$ ft^2/s. Determine the amount of the demand at the intermediate point in the pipeline.

5.14 Determine all pipe diameters in the branched piping system in the sketch below so that the slope of the HGL is 0.008. All pipes have a roughness $e = 0.006$ inches. Also determine the pressure, pressure head, and elevation of the HGL at each node of this network, so that the pressure at node 8 is 60 lb/in^2. Then select the closest standard pipe diameter for each pipe from the list below and again obtain a solution for the pressure, pressure head, and elevation of the HGL at all nodes. What head should a pump in pipe 1 produce if its supply water surface elevation is 100 ft? If the combined motor-pump efficiency is 73 percent, what is the cost per day to pump continuously if electricity costs $0.10/kWh? Also determine the cost of the pipe. The standard pipe sizes and costs per unit length follow:

Dia. in.	4	6	8	10	12	15
Cost $/ft	3.67	5.33	7.67	10.67	16.67	24.00

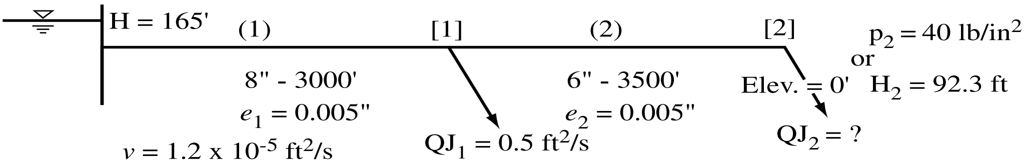
Dia. in.	18	20	24	30	42
Cost $/ft	43.33	56.67	80.00	100.00	145.00



5.15 Develop a spreadsheet solution for the branched piping system in Problem 5.14. Use the closest standard pipe diameters that you determined in that problem and give the pressure and head at every node. In developing the spread sheet solution use the Hazen-Williams equation with $C_{HW} = 150$.

5.16 Modify program SOLBRAN so different HGL slopes can be specified for individual pipes or groups of pipes, and use it to solve Example Problem 5.3.

5.17 In the pipeline system shown atop the next page the pressure at the downstream node has been measured as $p_2 = 40$ lb/in^2. Compute the demand at this node twice by using the Darcy-Weisbach equation and the Hazen-Williams equation. Assume C_{HW} = 145 for the Hazen-Williams roughness coefficient.



5.18 In the piping system of Problem 5.17, determine the diameter of pipe 2 so the discharge to node 2 is 0.6 ft^3/s.

5.19 Solve Problem 5.18 using the Hazen-Williams equation with $C_{HW} = 145$.

5.20 Analyze the 16-pipe, 9-node network shown in Fig. 5.6 and modified in Fig. 5.13. In the paragraph which follows Fig. 5.13, a design solution determines a diameter for all pipes except pipes 1, 3, and 16; adjust those diameters to the nearest standard pipe sizes, and assign a diameter of 150 mm to pipes 1, 3, and 16. To obtain the solution, you must first select appropriate pump characteristic curves and the number of pumps that should be in parallel and/or series. The first analysis should be based on the demands that were used in determining the pipe sizes, namely twice the average demand. Also obtain an analysis based on the average demands, and then obtain a third solution for which the demands are half of the average demands. Under this last demand condition, what discharge will be entering the storage tank when it is half full, i.e., when the water surface elevation in the tank is 119.5 m? (For these analyses assume the high-cost water from pipes 1 and 3 is shut off. Assume a fire flow of 0.08 m^3/s is needed at node 4 during the time of the largest hourly demand and both pipes 1 and 3 are open. What pressure will exist at node 4 to fight the fire, and how much flow will come from the four supply sources using the pump chosen earlier?

5.21 The 16-pipe, 9-node network was converted into a branched network by omitting the 7 pipes numbered 1, 3, 9, 10, 12, 13, and 16, as shown in Fig. 5.13. If pipe 16 were included and pipe 8 were omitted, would a branched system be formed? Since pipe 16 is the pipe to the storage tank, it generally would be considered to be part of the main transmission system. In fact, if pipes 2, 7, 12, and 16 are retained, the most direct path between the pump and storage tank exists to fill the tank during periods of low demand. Delete other pipes so this path exists, and determine the size of each pipe.

5.22 In Fig. 5.15 pipe 1 was given a diameter of 18 in, and pipe 2 was given a diameter of 15 in. For the pump characteristics given with this network, and for elevations of the HGL at nodes 2 and 3 of $H_2 = 645$ ft and $H_3 = 640$ ft, respectively, compute the discharges that the two pumps will supply. What discharge must the reservoir therefore supply? Verify your results by comparing them with the NETWK solution.

5.23 If the diameter of pipe 1 in the 30-pipe, 16-node network is changed from 18 in to 24 in, compute as in Problem 5.22 the discharge supplied by the two pumps. Why does this change create an impossible situation? What specification(s) could be changed to allow a solution?

5.24 In the 30-pipe, 16-node network assign to pipe 6 a diameter $D_6 = 6$ in but find the diameter D_{10} of pipe 10. To obtain this solution, use NETWK by appropriately modifying the input given in file FIG5_15.IN.

5.25 In the 30-pipe, 16-node network give pipes 6 and 9 the diameters $D_6 = 6$ in and $D_9 = 6$ in but compute the diameters D_{10} and D_{12} of pipes 10 and 12. To obtain this solution, use NETWK by modifying the input given in file FIG5_15.IN.

5.26 In the 30-pipe, 16-node network assign a diameter $D_{30} = 6$ in to pipe 30 that connects the reservoir to the network but determine the diameter D_1 of pipe 1 through which source pump 1 supplies the network.

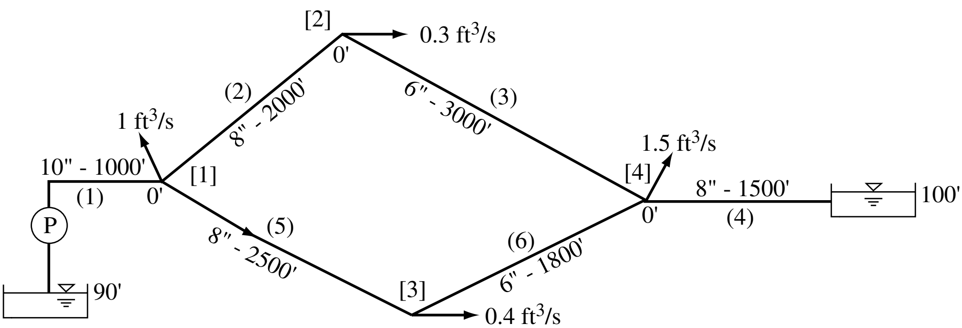
5.27 In the 30-pipe, 16-node network specify a diameter $D_{30} = 6$ in for pipe 30 that connects the reservoir to the network but compute the diameter D_2 of pipe 2 through which source pump 2 supplies the network. Initially retain 18 in for the diameter of pipe 1. Why is a solution not possible? Increase the diameter of pipe 1 to 24 in and obtain a solution.

5.28 The pressure can not become negative anywhere in a network, even though the mathematics of solving a network problem can produce negative pressures. Often 40 lb/ft^2 is the lowest pressure that is permitted. Determine the water surface elevation of the reservoir that supplies the 30-pipe, 16-node network via pipe 30 so the pressure at node 16 is 40 lb/ft^2 if the pipe diameters are determined by solving Problem 5.27 with D_1 = 24 in. First obtain this solution with $D_{30} = 6$ in, and then increase the diameter to $D_{30} = 12$ in. What feature is not realistic in the use of 6 in for D_{30}?. (Hint: use a differential head device in pipe 30.)

5.29 In the 9-pipe, 6-node network of Example Problem 5.4, indicate whether a solution is possible, or why a solution is not possible, for the following combinations of three pipes with their diameters specified as 6 in. If a solution is possible, solve the problem for the remaining six pipe diameters. Use the heads given in Example Problem 5.4, but in the last case modify the head at node 1 to $H_1 = 97$ ft.

Case	Pipe Numbers with Specified Diameters
1	1, 2, 5
2	1, 5, 7
3	3, 8, 9
4	1, 6, 7

5.30 In this small network you are to determine the head and discharge of the pump in pipe 1 so no flow will enter or leave the reservoir that is connected to the network by pipe 4 in response to the nodal demands shown on the diagram.



5.31 Retaining the head that was determined for the pump in Problem 5.30, but not the same discharge, determine the discharge that must be supplied by the reservoir if the demands are all increased to 1.5 times the values shown on the figure. In solving this problem, replace the pump by a DHEAD device of type 1, i.e., one that produces the specified differential head.

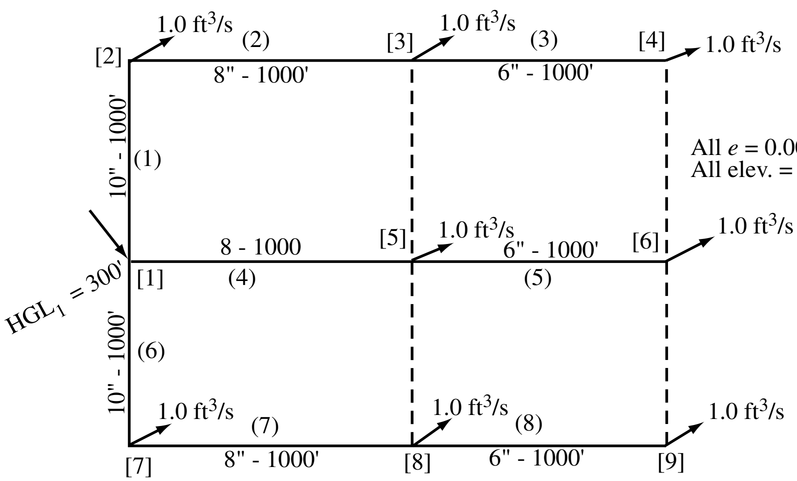
5.32 Rework Problem 5.31 with demands that are 0.8 times those in the diagram; in this case determine the discharge into the reservoir. Are the equations for this problem different from those of Problem 5.31? If so, what changes?

5.33 The 8-pipe network shown below was built to supply demands of 1.0 ft^3/s at each of eight nodes. Over the years the demands have doubled, and the network is now unable to supply 2.0 ft^3/s at these nodes. The 10-in pipes, numbers 1 and 6, are to be replaced by 12-in pipes, and the network is to be looped by adding the 4 pipes listed in the table:

Pipe	Node 1	Node 2
9	5	3
10	5	8
11	6	4
12	6	9

First analyze the original network for the original demands. Next analyze the same network again, but with the eight nodal demands each increased to 2.0 ft^3/s. At how many nodes is the present network unable to supply a pressure of at least 40 lb/in^2? Obtain a design solution for this network to determine the sizes of the four additional pipes; since eight diameters must be found in such a solution, also determine the sizes of pipes 2, 4, 5, and 7. The nodal HGL elevations that might be specified are listed in this table:

Node	1	2	3	4	5	6	7	8	9
H, ft.	300	291	276	264	290	280	291	276	264



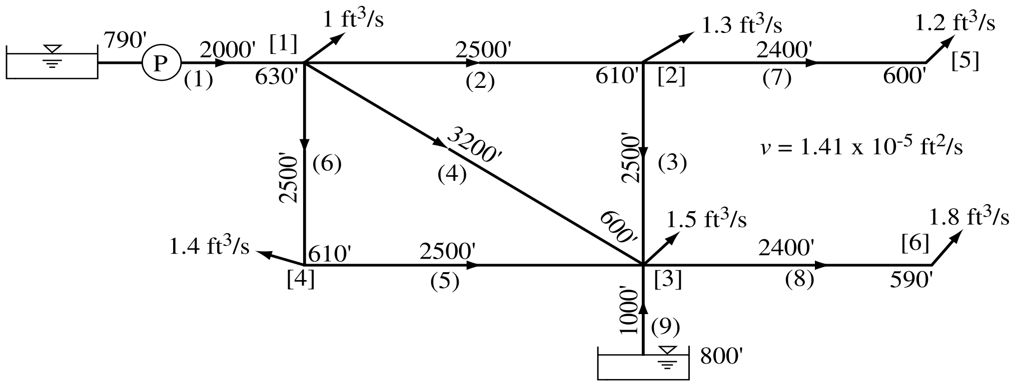
5.34 In the network shown in Fig. 5.23 a total of 15 combinations of three pipes exist and are candidates to have their diameters specified. Obtain a solution for each of these groupings using NETWK. In obtaining these solutions, also obtain an analysis solution for each case by using the nearest standard pipe sizes. Try specifying an 8-in diameter for pipe 6 while prescribing the diameters of pipes 3, 5, and 6, and note the message that the program returns to inform the user that inappropriate specifications have been made. Rather than increasing the diameter of pipe 6, adjust the nodal HGL-elevation specifications to specify a problem for which a solution is possible.

5.35 In Problem 5.33 each of eight nodal demands was 2 ft^3/s. Solve the same problem under the assumption that the new nodal demands are each 2.5 ft^3/s. Before you seek a design solution to this network, select appropriate diameters for pipes 1 and 2 (and all other pipes having specified diameters).

5.36 Design the looped network shown below. The target HGL-elevations at the nodes should be near those given in the head table for the demands shown on the sketch. Assume $e = 8.0\times10^{-6}$ in for pipes 1, 2, and 3 and $e = 6.0\times10^{-6}$ in for the other pipes.

Node No.	1	2	3	4	5	6
HGL, ft	832	805	798	815	795	785

Q ft^3/s	**Head** ft.
6	65
9	61
12	55

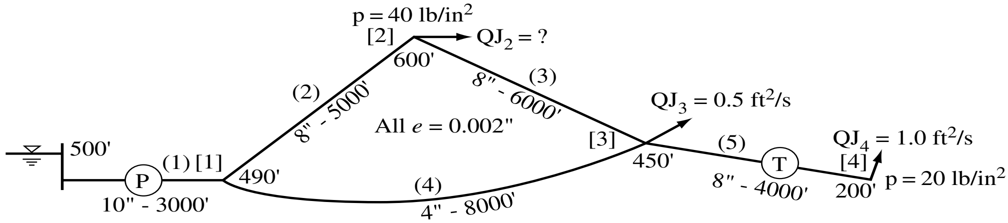


To complete this design, do the following: (1) Assign diameters to pipes 1, 3, and 6 as 13 in, 6 in, and 8.5 in, respectively, and determine the six diameters D_2, D_4, D_5, D_7, D_8, and D_9 to produce the specified HGLs. Obtain this design solution using NETWK. (2) Verify the results from NETWK with hand calculations by first finding the discharges in the three pipes using the specified diameters. Then find Q_3 and Q_6 from the Darcy-Weisbach and Colebrook-White equations. Next fit the given data to determine the polynomial for the pump curve and solve for the three unknowns Q_1, f_1, and h_p. With Q_1, Q_3, and Q_6 known, reduce the network and determine the other discharges and head losses. (3) Identify other pipes that are candidates to have their diameters specified, and identify specifications that would make a solution impossible.

5.37 Water is pumped from a reservoir with a water surface elevation of 500 ft over a hill crest of elevation 600 ft by means of the piping system shown in the next figure. The primary questions that need to be answered are: (a) what demand QJ_2 can be supplied at the top of the hill with a pressure of 40 lb/in^2, and (b) how much power can be extracted by the turbine in pipe 5 if 1.0 ft^3/s at 20 lb/in^2 is to be delivered at node 4? Write and then solve the system of equations that will provide these answers.

Pump

Q ft^3/s	h_p ft
2.0	290
2.5	285
3.0	272



5.38 Two pumps, pump a and pump b, have the operating characteristics given by the three (Q, h_p) pairs listed in the two tables below. At what rotational speed ratios $N_{ra} = (N_2/N_1)_a$ and $N_{rb} = (N_2/N_1)_b$ should each of these pumps be operated if the required combined discharge is $Q_{tot} = 3.5$ ft^3/s? Since the required discharge is well beyond the values in the tables for either pump, the two pumps must be placed in parallel. Assume that the middle point in each table represents the normal operating condition for each pump, and at their new rotational speeds the pumps should be operating at their maximum efficiencies.

Pump a (N_{a1} = 800 rev/min)

Q_a ft^3/s	h_{pa} ft.
0.75	43.00
1.10	38.75
1.50	32.20

Pump b (N_{b1} = 1000 rev/min)

Q_b ft^3/s	h_{pb} ft.
1.5	44.00
2.0	38.25
2.5	30.00

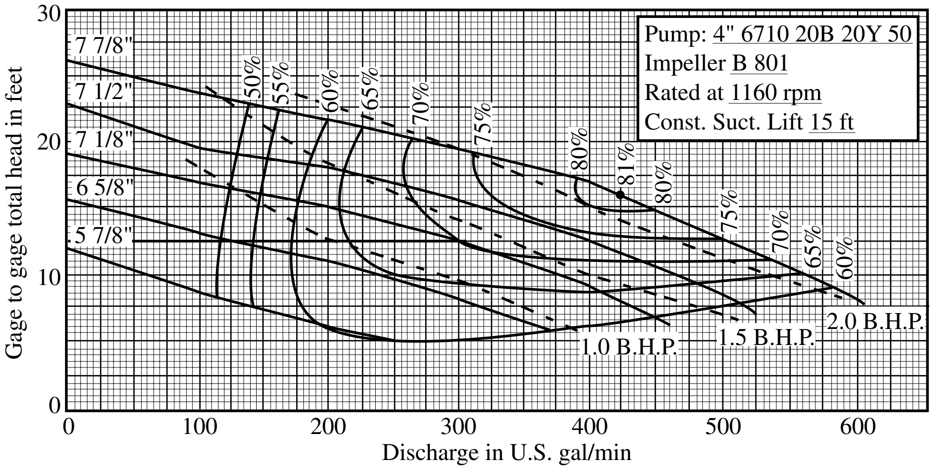
5.39 Modify program SPLINECU so that natural boundary condition is always used; it is desired not to give the user the option of specifying either natural boundary conditions (second derivatives set to zero at the ends of the domain) or the end slopes.

5.40 Program SPLINECU fits pairs of head vs. discharge data with a cubic spline and provides M interpolated values with equal increments. Convert this main program into a subroutine (function) that (1) receives one pair of values as arguments from the main program, and (2) provides to the main program the values of the second derivatives so that the main program can carry out cubic spline interpolations.

5.41 Modify program SPLINECU so it acts in the same way as program LAGRANGE, i.e., it provides the interpolated value for the pump head for any value of discharge that is supplied to it.

5.42 Modify program ELECENG so it computes energy consumption over any period of time. The program should perform the following tasks: (1) read the number of pairs of Q vs. time data, and then read these pairs; (2) read the number of pairs of Q vs. head H and Q vs. efficiency η; and then read these two sets of data pairs; and (4) compute the energy consumed by integrating the pump power over time.

5.43 The operation of a pump with a 7 7/8 in diameter impeller is described by the pump characteristic curves given below. Take 7 pairs of points from this curve, starting with 0 gal/min and ending with 600 gal/min in increments of 100 gal/min, and use a second-order polynomial between three consecutive pairs of points to interpolate values. Using this interpolation, obtain values of pump head for the following discharges, and compare the interpolated values with the corresponding values that are read from the pump curve itself: 50 gal/min, 120 gal/min, 190 gal/min, 250 gal/min, 330 gal/min, 410 gal/min, 460 gal/min, 550 gal/min.



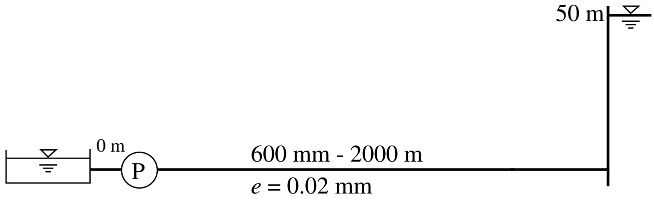
5.44 Repeat Problem 5.43 but use a cubic spline in place of the second-order polynomial for the interpolation.

5.45 For the pump whose characteristic curve is given in Problem 5.43, obtain the energy consumed by the motor when the discharge varies over a 24-hour period as the table describes:

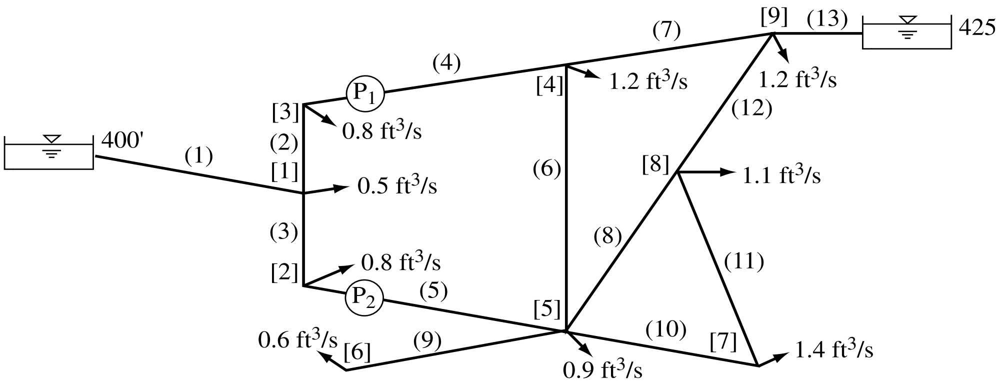
Time hr.	***Q*** ft^3/s
0.0	0.20
2.0	0.40
4.5	0.70
6.0	1.10
8.2	1.40
10.3	1.50
12.3	1.60

Time hr.	***Q*** ft^3/s
14.0	1.50
16.5	1.30
18.0	1.10
20.1	0.80
21.0	0.60
22.5	0.40
24.0	0.15

5.46 A pump is attached to a pipeline that has a length of 2000 m and a diameter of 600 mm (with $e = 0.02$ mm). The downstream reservoir has a water surface elevation that is 50 m above the supply reservoir water surface elevation. The pump characteristic curves show that the efficiency variation is essentially linear between $Q = 0.0\ m^3/s$ and $Q = 1.2\ m^3/s$. At $Q = 0.0\ m^3/s$ the efficiency is zero, and at $Q = 1.2\ m^3/s$ the efficiency is 85%. And as the discharge increases from $1.2\ m^3/s$ to $2.1\ m^3/s$, the efficiency varies linearly with discharge from 85% to 30%. Plot the power supplied by the pump to the fluid, and the power required by the pump from its motor, for discharges from $0.2\ m^3/s$ to $2.1\ m^3/s$.



5.47 In the network shown below two booster pumps supply all of the water for the system, and this water must come from the reservoir on the left, which is extremely large. The reservoir on the right is a storage tank that receives water during periods of low demand and supplies some water during periods of higher demand. The pipe sizes, their lengths, etc., are defined in the input data file for NETWK. It has been decided to increase the head of one of the pumps so that pressures are larger at the downstream end of the network and larger flows enter the reservoir during periods of low demand. For the demands in the diagram, determine the increases in pressure and the discharge into the tank if the head of pump 1, or the head of pump 2, were increased by 5 ft. Which solution is more cost effective? Why is this the case? List some other options in improving the cost effectiveness of the system.



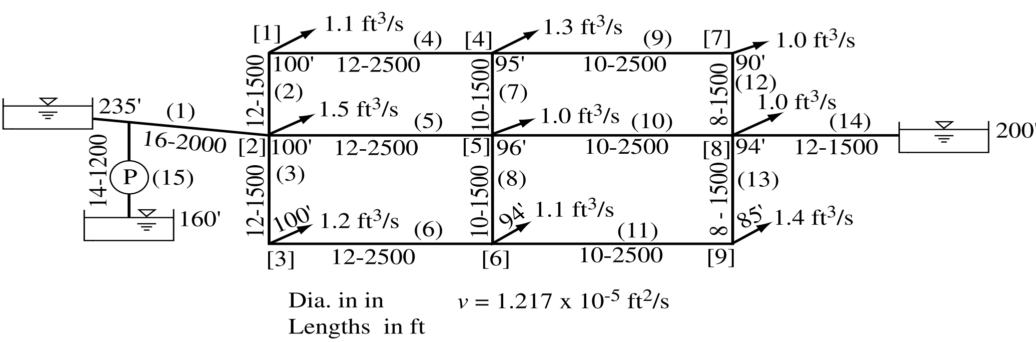
Chapter 5, Problem 5.47.
/*
$SPECIF OUTPU1=2,NPSERI=0 $END
PIPES
1 0 1 1000 14 .0008
2 1 3 700 12
3 1 2
4 3 4 1500 8
5 2 5 1500 12
6 5 4 1600
7 4 9 1400 8
8 5 8 1000
9 5 6 1200 6
10 5 7 1000 8
11 7 8
12 8 9 1200
13 0 9 500

NODES
1 .5 293
2 .8 285
3 .8 290
4 1.2 340
5 .9 345
6 .6 340
7 1.3 338
8 1.1 335
9 1.2 335
RESER
1 400
13 425
BOOSTER
4 1.5 55 3. 50 4.5 42/
5 3 63 5 60 7 55/
RUN

5.48 The network shown below has a pump in pipe 15 that obtains its water supply from ground water with a constant water surface elevation of 160 ft, and it pumps into a circular tank with a diameter of 185 ft. The bottom of the tank is at elevation 225 ft, and its top is at elevation 245 ft. The demands on the sketch are average values. The reservoir that is connected to the network by pipe 14 is water that is bought from an outside water agency for $ 0.35 per thousand cubic feet, and it is received from a conduit under a constant pressure that produces a HGL of 200 ft. The costs of the pump, well, tank and the connection to the outside water agency have been fully paid, so they should no longer be considered in economic analyses. Do the following:

1. Obtain a series of solutions in which the peaking factor (demand function) varies for all nodes from 1.5 to 0.5. In this series of solutions assume that the water surface elevation of the tank is at 235 ft when PF = 1.5; start with this PF and assume it decreases linearly over a 24 hr time period to 0.5.
2. Plot the discharge variation in pipes 1, 14, and 15 with the demand function.
3. Compute the cost of pumping the water from the well. For these costs assume that the combined pump-motor efficiency can be defined by a second-order polynomial function of the discharge, with the efficiencies related to the discharges in the pump characteristic table as follows: 0.70, 0.75, and 0.58. The cost of electrical energy is $ 0.08/kWh. Show how the pumping cost varies as the peaking factor changes, and how the average cost of pumping compares with the price of water purchased from the agency.
4. Show that the cost of water is a constant times the reciprocal of the sensitivity of the discharge to the pump power, i.e. the cost is equivalent to a constant times the sensitivity of power to discharge, which is *ΔP/ΔQ*.
5. Compute and plot the sensitivities of the pressure at nodes 4, 5, and 6 to the discharge in pipes 1 and 14.

Q ft^3/s	**h_p** ft
8.5	105.0
9.5	95.6
10.5	85.0



CHAPTER 6

EXTENDED TIME SIMULATIONS AND ECONOMICAL DESIGN

6.1 INTRODUCTION

This chapter looks primarily at two topics that are important to the design of looped networks, which includes networks for water distribution to numbers of "on demand" users, as occur in large cities. These systems do not operate under steady-state conditions. First we introduce and describe "extended time simulations" to simulate the performance of these systems as they respond to demands which vary with time, and which may have pumps turned on or off, depending upon those demands. The chapter will also describe some useful elements of engineering economic analysis. Both of these topics will then be applied to the design of large looped networks. Thereafter, subsequent chapters explore methods to analyze unsteady flows, including inertial and/or elastic effects. For networks of pipes the analysis of unsteady flow requires the simultaneous solution of combined systems of ordinary differential equations and algebraic equations. The reader will be introduced in Chapter 7 to such analyses. In Chapters 8-11 progressively more comprehensive systems will be studied. In Chapter 12 true transients in looped networks will be examined.

Extended time simulations consist of a series of steady-state solutions based on changing demands and reservoir water levels, the number of operating pumps etc. This type of time-dependent solution is obtained by solving a system of simultaneous nonlinear algebraic equations, as was done in Chapters 4 and 5. Another term for this type of unsteady flow problem is "quasi-steady," since inertia is ignored and the equation of motion is a steady-state form, even though individual terms in the equation do change with time. Section 7.2 will offer additional perspective on this class of flows.

Time-dependent analyses that account for inertia require the simultaneous solution of a combined system of ordinary differential and algebraic equations. Transient analyses that also account for elastic effects must use partial differential equations in place of ordinary differential equations since the pressure and velocity now vary not only with time in each pipe but also with the position along the pipe. Not only does this mean that the computational effort increases dramatically for a solution, but also that the amount of information (numbers) that is required to describe the network behavior increases correspondingly, since it is necessary to provide pressures and velocities at a number of positions along each pipe in the network to describe the hydraulic transient after each successive time increment. For example, if the flow in a network is described by the HGL and pressure (two unknowns) at each node and the discharge and velocity (two unknowns) in each pipe in a 100-pipe, 80-node network, then the description consists of 360 numbers at an instant in time. A typical extended time simulation would use a one hour time increment, and therefore the analyst must examine these 360 values for each of 24 time steps if the simulation were for a 24-hour period. If inertia were included, then the time increment must be on the order of seconds (or less if the pipes are relatively short, as in a fire-fighting sprinkler system in a building). One would prefer not to have to conduct such an investigation over a full day. But if a solution that accounted for inertia were to be performed for only 100 time increments, then the solution consists of 36,000 values. For a transient analysis with elastic effects, instead of just two values (discharge and velocity) for each pipe, there will be two values for each pipe increment (these space increments must be compatible with the time increment), so if 20 increments are used for each pipe, the number of values in the network description jumps to 416,000. As the comprehensiveness of the network description increases, the amount of data that is needed to describe the solution adequately expands rapidly, and it becomes clear that compromises are needed.

When will an extended time simulation that ignores elastic effects be adequate? The answer is obviously subjective. For the operation of most municipal water systems the changes in demands are normally slow enough to cause the effects of inertia to be relatively minor, and certainly the elastic effects can be ignored. Furthermore, in a large network the effect of a very rapid change in flow in a single pipe, which may have a valve at one end closed rapidly, will soon be dissipated in the network of pipes. Thus it is sufficient to recognize that a high-pressure transient wave may propagate though this pipe and possibly affect a few pipes near it. There may be a few times in the operation of many water distribution systems, and other liquid distribution systems, when the neglect of inertia will cause a simulation to produce results that are notably different than those that actually occur. Such conditions may occur when major flows are changed in seconds, or perhaps minutes. For shorter pipes these changes may be more rapid without creating a significant change in pressures and discharges that is attributable to inertial effects.

6.2 EXTENDED TIME SIMULATIONS

This section describes a type of time-dependent solution that has become known as an "extended time simulation." These solutions are for pipe networks rather than single pipes. Since this type of solution ignores both elastic and inertial effects, the solutions are actually a series of steady-state solutions in which a past solution is updated over a time increment in response to changes in time-dependent parameters to the new solution for the new instant in time. Thus these time-dependent solutions are quasi-steady solutions. The following six items commonly change in extended time simulations:

1. Demands at nodes. The nodal demands will change in almost all extended time simulations, and a typical means of specifying these changes is to provide peaking factors as functions of time for selected groups of nodes. Such changes in demand patterns over time might be thought of as demand schedules.
2. Storage versus elevation relations for reservoirs. Some reservoirs may have constant water surface elevations, but most are storage tanks with a water surface elevation that varies with time as water is withdrawn from, or added to, the tank. Typically a storage versus water surface elevation function is constructed to describe changing reservoir water surface elevations. When this function is described by data pairs for water surface elevation and volume in storage, then the bottom water surface elevation will be the lowest operating level of the tank, and the largest water surface elevation will be the top of the tank.
3. Pump schedules. A pump schedule states how many pumps must operate in parallel or in series at a given station at any time. In other words, a schedule specifies the number of pumps that are turned on for each time step. An alternative is to specify the rotational speed of a pump as a function of time.
4. Pump rules. A pump rule relates the number of operating pumps to either the magnitude of the pressure (or HGL) at a selected node, or the water level in a reservoir. Rules are distinguished from schedules by a condition that dictates the number of pumps in operation rather than having pumps start or stop at a specified time. Instead of specifying the number of operating pumps, the rule might give the rotational speed of a pump.
5. Flow rules. The difference between flow rules and demand functions (schedules) is the same as between pump rules and pump schedules. That is, the demand at selected nodes is determined by the pressure at some node or by the water surface

elevation in a reservoir. Flow rules would typically be given for negative demands, which are external flows coming into the network.

6. Discharge rules. Specify the discharge that must exist in selected internal pipes in the network. Internal pipes are distinguished from dead end pipes and pipes that connect supply sources to the network.

There are many additional items that might be a part of the specifications that describe the time-dependent solution, such as the following:

7. Schedules for valves. These schedules may specify the valve setting (percent open) as a function of time, which may in turn employ a relation between valve position and head loss to determine how the valve restricts the flow, or specify the valve loss coefficient as a function of time.
8. Rules for valves. The rules can either prescribe the valve setting (percent open) or give its loss coefficient as a function of the pressure at a node. In place of pressure, the rule may be based on the water surface elevation in a reservoir.
9. Differential head devices. These devices may specify the amount of differential head (positive or negative) in selected pipes as a function of time, i.e., a schedule of head losses in pipes, or the amount of the differential head may be computed so that a specified HGL (or pressure) is achieved at a selected node, and the HGL may vary with time.
10. Tank level or pressure control algorithms. Such algorithms simulate controllers that may activate valves etc. to maintain the water levels in reservoirs at or between specified limits, or to maintain a pressure at a designated value, or between specified limits, by changing the flow into the network or adjusting a valve setting.

It is common to implement these items, which prescribe changes in network behavior over the next time step, and which are rules based on pressure or water surface elevation, in terms of values that are taken from the solution for the current time instant. In other words, the implementation of the rule lags the solution itself by one time step. To do otherwise would require an iterative approach.

We will not describe any implementation details for these rules. However, as they act to change the network behavior over each time step, it is generally not necessary to redefine the equations that govern the mathematical problem as if a new network problem were being solved. Instead the existing equations are simply modified to reflect the conditions that apply to the new time step. For example, to change nodal demands we simply change the values of those demands. But when a pump is turned off or a pressure reduction valve opens fully, the type and/or number of equations that describes the system must be altered.

Example Problem 6.1

Obtain an extended time simulation for the 30-pipe, 16-node network described in Chapter 5 and shown in Fig. 5.15, using the diameters (for all pipes $e = 0.004$ in) found there by using DESIGN=1 that are listed in the pipe data table. The following specifications control this simulation: (1) The storage tank attached to the network by pipe 30 is circular with a diameter of 115 ft, and its bottom is at elevation 590 ft; at the beginning of the simulation its water surface elevation is 605 ft. (2) Two different demand functions are described on the graph which follows; the first applies to the north portion of the network at nodes 1, 2, 5, 6, 9, 10, 13, and 14, and the second applies to nodes 3, 4, 7, 8, 11, 12, 15, and 16. (3) Initially three pumps are in parallel at each pump station, and the tables give pump characteristics that apply to all three operating pumps. The number of operating pumps is given by the pump schedule.

PUMP SCHEDULE

	Pump Station 1					Pump Station 2				
Time, hr.	0	8	10	15	17	0	5	8	15	20
Number operating	3	2	1	2	3	3	2	1	2	3

NODE DATA

No.	Demand ft^3/s	Elevation ft
1	1.2	500
2	1.2	490
3	0.8	485
4	1.6	480
5	1.4	495
6	1.2	494
7	1.0	490
8	0.8	483

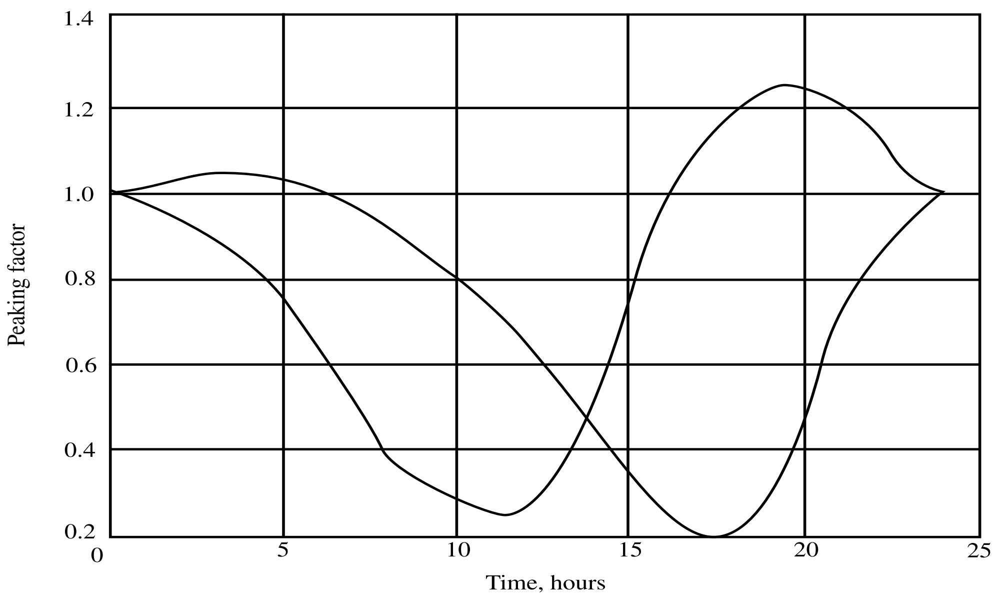
No.	Demand ft^3/s	Elevation ft
9	2.0	493
10	2.0	492
11	3.6	488
12	2.8	484
13	4.0	480
14	2.0	478
15	1.8	475
16	2.0	470

PIPE DATA

No.	Length ft	Diameter in
1	500	18
2	500	15
3	800	12
4	800	6
5	800	12
6	1800	12
7	1800	12
8	1800	12
9	1800	10
10	800	6

No.	Length ft	Diameter in
11	800	6
12	800	6
13	1600	12
14	1600	12
15	1600	12
16	1600	12
17	800	6
18	800	6
19	800	6
20	1600	12

No.	Length ft	Diameter in
21	1600	8
22	1600	10
23	1600	6
24	800	6
25	800	6
26	800	6
27	2500	6
28	2500	6
29	2500	6
30	1000	10



We begin the solution of this problem by reading key demand function data from the two plots of peaking factor at points where the curves break; this step allows us to digitize the demand functions (each function can have a separate set of times), as listed in this table:

Hour:	0	2	3.5	6.5	10	11.5	14	17	19	20	22	24
DF(1)	1.0	0.95	0.875	0.625	0.30	0.25	0.50	1.10	1.23	1.25	1.10	1.0
DF(2)	1.0	1.05	1.06	1.00	0.80	0.68	0.50	0.20	0.30	0.50	0.85	1.0

A solution from NETWK can be obtained by first adding the option ISIML=1 to the $SPECIF list of options and supplying input that would describe the network appropriately, and then adding the additional lines that describe the extended time simulation that is desired. The input file follows:

```
Extended time simulation for example 30 pipe network.
/*
$SPECIF ISIML=1,NODESP=0 $END
PIPES
1 0 2 500 18 .004
2 0 3 500 15
3 2 1 800 12
4 2 3 800 6
5 3 4 800 12
6 1 5 1800 12
7 2 6 1800 12
8 3 7 1800 12
9 4 8 1800 10
10 6 5 800 6
11 6 7 800 6
12 7 8 800 6
13 5 9 1600 12
14 6 10 1600 12
15 7 11 1600 12
16 8 12 1600 12
17 10 9 800 6
18 10 11 800 6
19 11 12 800 6
20 9 13 1600 12
21 10 14 1600 8
22 11 15 1600 10
23 12 16 1600 6
24 14 13 800 6
25 14 15 800 6
26 15 16 800 6
27 2 5 2500 6
28 2 7 2500 6
29 3 8 2500 6
30 0 14 1000 10
RESER
30 605
PUMPS
1 2.67 157 5 152 7.33 144 500
2 2 152 4 147 6 139 500
PARALLEL
1 3
2 3
NODES
1 1.2 500
2 1.2 490
3 0.8 485
4 1.6 480
5 1.4 495
6 1.2 494
7 1.0 490
8 0.8 483
9 2.0 493
10 2.0 492
11 3.6 488
12 2.8 484
13 4.0 480
14 2.0 478
15 1.8 475
16 2.0 470
RUN


$TDATA ALTV=0,HTIME=24,INCHR=1,ISUNIT=0,LINEAR=1,NPUNOD=2,PRINTT=3 $END
PIPE TABLE
ALL
NODE TABLE
ALL
RESER. TABLE
ALL
END TABLES
STORAGE FUNCTION
1 590 0 600 103870 605 155805/
30/
```

```
DEMAND FUNCTION
1 0 1 2 .95 3.5 .875 6.5 .625 10 .3 11.5 .25 14 .5 17 1.1 19 1.23 20 1.25 22 1.1 24 1./
1 2 5 6 9 10 13 14/
2 0 1 2 1.05 3.5 1.06 6.5 1 10 .8 11.5 .68 14 .5 17 .2 19 .3 20 .5 22 .85 24 1/
3 4 7 8 11 12 15 16/
PUMP SCHEDULES
1 2 0 3 8 2 10 1 15 2 17 3/
2 2 0 3 5 2 8 1 15 2 20 3/
END SIMULATION
END
```

The input after the RUN command provides specifications for the time-dependent solution. A brief explanation of this part of the file (see the NETWK manual for more detail) follows:

1. The $TDATA line sets options associated with the extended time simulation: (a) ALTV=0 tells NETWK to extrapolate the volume-elevation data that is provided for the storage tank beyond the given limiting values; if ALTV=1, then the tank will no long supply water when the elevation falls to the smallest elevation in the data, nor will it fill further if the water surface elevation reaches the largest elevation in the data; (b) HTIME=24 indicates the simulation is to cover 24 hours, the default; (c) INCHR=1 indicates one-hour increments and is also the default; (d) ISUNIT=0 indicates that storage volumes will be given in ft^3; (e) LINEAR=1 specifies a linear interpolation (or extrapolation if necessary) of given data; (f) NPUNOD=2 indicates that source pumps and reservoirs will be referenced by pipe number; (g) NPRINTT=3 tells NETWK to write special tables, with time in the first column, for pressure at designated nodes and discharges in designated pipes.
2. The ALL after PIPE TABLE and NODE TABLE indicates all pipes and nodes are to be in these special tables; similarly, all reservoirs are to have their water surface elevations reported in the tables.
3. The individual demand functions are described next under the command DEMAND FUNCTION. Each separate demand function consists of two lines; the first value on the first line is a number the user chooses to assign to this demand function as an identifier, which is followed by time and peaking factor data pairs. The second line indicates the nodes at which this demand function applies.
4. After the PUMP SCHEDULES command the second value on each line, a 2 after the number of the pump station has been given, indicates parallel pump operation, and the times and numbers of operating pumps are given thereafter as pairs.

The special tables follow; the varying discharges in pipes 1, 2, 4, 18, 25, and 30 are plotted in a figure, and the pressures at nodes 1, 2, and 16, plus the water surface elevation in the storage tank, are plotted in the other figure. From this simulation we note that the storage tank initially has a water surface elevation of 605 ft and ends the 24-hour period with a water surface elevation of 603 ft. In other words the tank will not be full at the beginning of the next day; hence the capacity of either one pump or both pumps should be increased, or the lengths of time intervals when pumps are in operation should be increased. The discharge reverses direction in several pipes over the 24-hour period, including pipe 30 connecting the storage tank to the network. For the first 7 hours the storage tank supplies water to the network; then it fills until 18 hours, and thereafter it again supplies water. If the middle point used to define the pump curves is the normal capacity, then the discharge at maximum efficiency for station 1 is 15 ft^3/s at the start of pump operation, and for station 2 the discharge is 12 ft^3/s. When the number of operating pumps is reduced from 3 to 2 and then to 1 during the period of lower demand, the pumps are then producing flows that are considerably above their normal capacities, as seen in the plot of discharges in pipes 1 and 2 in relation to the normal

Pressure (lb/in^2) at Nodes as a Function of Time

Hours	Node Number								
	1	2	3	4	5	6	7	8	9
1	57.67	67.26	66.95	65.71	52.12	53.53	54.89	55.53	46.51
2	57.79	67.29	66.81	65.47	52.31	53.53	54.62	55.09	46.79
3	57.91	67.33	66.67	65.23	52.49	53.55	54.34	54.63	47.06
4	58.36	67.54	66.74	65.29	53.21	54.08	54.54	54.75	48.00
5	58.98	67.85	66.93	65.56	54.17	54.90	55.05	55.25	49.22
6	59.40	67.97	65.09	63.99	54.85	55.39	54.41	54.56	50.11
7	60.16	68.37	65.47	64.50	56.02	56.47	55.20	55.40	51.60
8	61.04	68.84	66.03	65.27	57.40	57.82	56.34	56.66	53.41
9	59.10	66.38	59.94	59.75	56.19	56.41	52.59	52.92	52.95
10	60.49	67.32	61.46	61.53	58.21	58.34	54.67	55.25	55.40
11	55.80	61.76	60.70	60.94	55.04	55.27	53.84	54.97	53.46
12	57.17	62.97	62.36	62.88	56.65	56.90	56.05	57.46	55.23
13	57.32	63.16	63.24	63.95	56.79	57.16	57.17	58.85	55.32
14	56.44	62.49	63.48	64.35	55.71	56.20	57.40	59.47	53.96
15	55.70	61.99	63.64	64.65	54.75	55.68	57.46	59.97	52.66
16	60.64	68.04	69.64	70.75	58.12	60.18	62.97	65.93	54.38
17	59.26	67.45	69.76	71.06	55.91	58.91	63.47	66.61	51.55
18	59.30	68.68	70.70	72.23	54.48	58.75	64.99	68.40	48.84
19	58.46	68.27	70.23	71.62	53.12	57.74	63.98	67.31	47.06
20	57.54	67.82	69.72	70.94	51.61	56.61	62.86	66.13	45.05
21	56.84	67.40	69.36	69.90	50.48	55.22	60.18	63.20	43.64
22	57.06	67.29	68.45	68.38	51.03	54.66	57.83	60.34	44.59
23	57.19	67.17	67.60	66.92	51.29	53.86	56.03	57.71	45.12
24	57.37	67.18	67.24	66.28	51.61	53.59	55.35	56.55	45.65

Pressure (lb/in^2) at Nodes as a Function of Time (cont'd)

Hours	Node Number							Reservoir Water
	10	11	12	13	14	15	16	Surface Elev., ft
1	49.07	48.63	50.56	48.83	52.34	50.69	46.73	605.00
2	49.06	48.14	50.00	49.17	52.19	50.12	45.80	604.18
3	49.07	47.62	49.43	49.49	52.05	49.52	44.84	603.40
4	49.70	47.79	49.57	50.53	52.30	49.67	44.88	602.65
5	50.66	48.42	50.17	51.77	52.77	50.31	45.61	602.00
6	51.33	48.19	49.89	52.62	53.15	50.36	45.84	601.53
7	52.56	49.18	50.88	54.16	53.39	51.30	47.09	601.24
8	54.09	50.67	52.39	56.16	53.61	52.68	49.06	601.19
9	53.38	48.18	49.51	56.30	53.64	51.17	47.83	601.41
10	55.65	50.70	52.12	59.01	54.45	53.60	51.07	601.59
11	53.69	50.47	52.06	57.75	54.44	53.75	51.83	602.05
12	55.43	53.13	54.84	59.57	55.41	56.26	55.32	602.45
13	55.71	54.58	56.39	59.58	56.01	57.40	57.34	603.04
14	54.63	55.08	57.16	57.92	55.85	57.90	58.55	603.71
15	53.99	55.45	57.85	56.34	55.65	58.49	59.69	604.28
16	57.65	60.61	63.90	56.84	56.49	63.40	65.37	604.75
17	56.11	61.46	64.93	53.95	55.48	64.67	66.83	605.36
18	55.41	63.44	67.04	50.65	55.30	67.04	69.39	605.61
19	54.25	62.07	65.76	48.71	55.09	65.51	67.74	605.60
20	52.93	60.56	64.36	46.47	54.59	63.81	65.98	605.39
21	51.22	56.41	60.45	44.98	53.65	58.86	60.54	605.02
22	50.49	53.10	56.71	46.24	53.07	55.19	55.58	604.44
23	49.25	50.74	53.37	47.03	52.11	52.93	51.14	603.80
24	48.93	49.62	51.88	47.72	51.79	51.70	48.88	603.03

Discharges (ft^3/s) in Pipes as a Function of Time

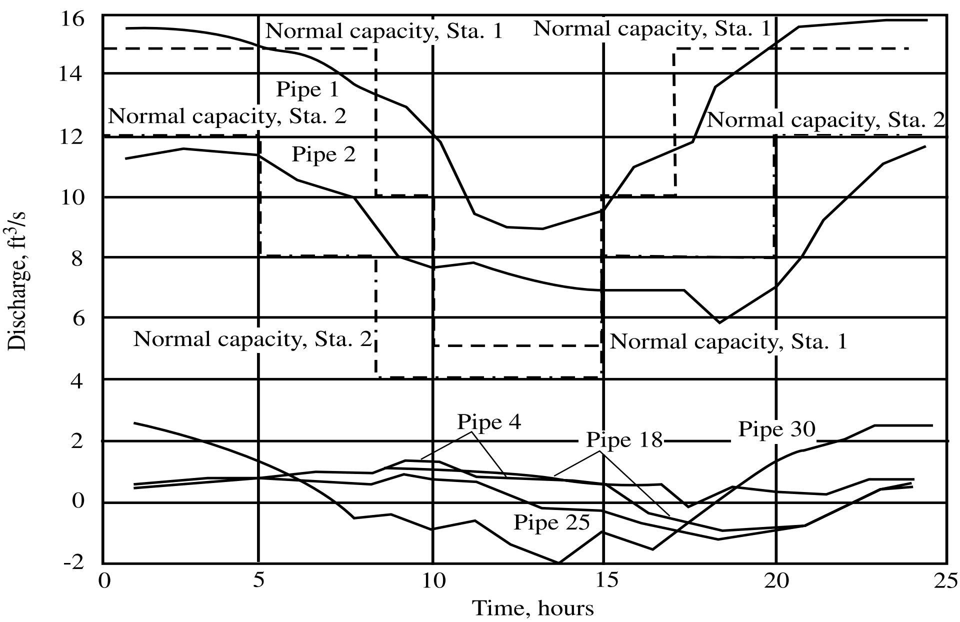
Hours	Pipe Number									
	1	2	3	4	5	6	7	8	9	10
1	15.71	11.33	6.05	0.66	4.83	4.85	6.10	5.52	3.23	0.40
2	15.66	11.46	6.00	0.68	4.91	4.83	6.11	5.56	3.27	0.36
3	15.62	11.59	5.95	0.70	4.99	4.81	6.11	5.60	3.31	0.31
4	15.33	11.53	5.80	0.72	4.99	4.72	6.03	5.56	3.30	0.26
5	14.90	11.35	5.61	0.74	4.93	4.61	5.89	5.47	3.25	0.21
6	14.73	10.52	5.42	0.95	4.73	4.52	5.79	5.11	3.09	0.12
7	14.15	10.26	5.17	0.96	4.64	4.37	5.60	4.98	3.02	0.04
8	13.43	9.86	4.88	0.95	4.47	4.19	5.34	4.79	2.92	- 0.04
9	12.91	8.07	4.49	1.26	3.99	3.90	5.02	3.95	2.53	- 0.18
10	11.98	7.58	4.12	1.22	3.76	3.65	4.70	3.72	2.39	- 0.21
11	9.33	7.83	3.29	0.75	3.59	2.93	3.77	3.74	2.31	- 0.17
12	8.82	7.27	3.12	0.70	3.31	2.80	3.60	3.51	2.16	- 0.16
13	8.74	6.96	3.16	0.60	3.10	2.80	3.57	3.40	2.07	- 0.09
14	9.03	6.86	3.39	0.44	2.93	2.91	3.69	3.41	2.01	0.09
15	9.24	6.81	3.63	0.29	2.75	3.03	3.70	3.45	1.95	0.28
16	11.21	6.82	4.58	0.30	2.63	3.74	4.31	3.67	1.99	0.52
17	11.84	6.70	5.16	- 0.14	2.36	4.08	4.55	3.50	1.88	0.67
18	13.68	5.65	5.93	0.14	2.00	4.61	5.01	3.23	1.68	0.83
19	14.29	6.20	6.18	0.17	2.23	4.79	5.20	3.49	1.83	0.86
20	14.94	6.75	6.45	0.20	2.47	4.98	5.40	3.74	1.99	0.91
21	15.52	8.77	6.61	0.17	3.29	5.11	5.68	4.62	2.49	0.88
22	15.67	9.82	6.42	0.41	3.88	5.01	5.80	5.09	2.80	0.75
23	15.82	10.70	6.29	0.54	4.40	4.97	5.99	5.38	3.04	0.61
24	15.81	11.05	6.18	0.60	4.63	4.92	6.06	5.47	3.15	0.51

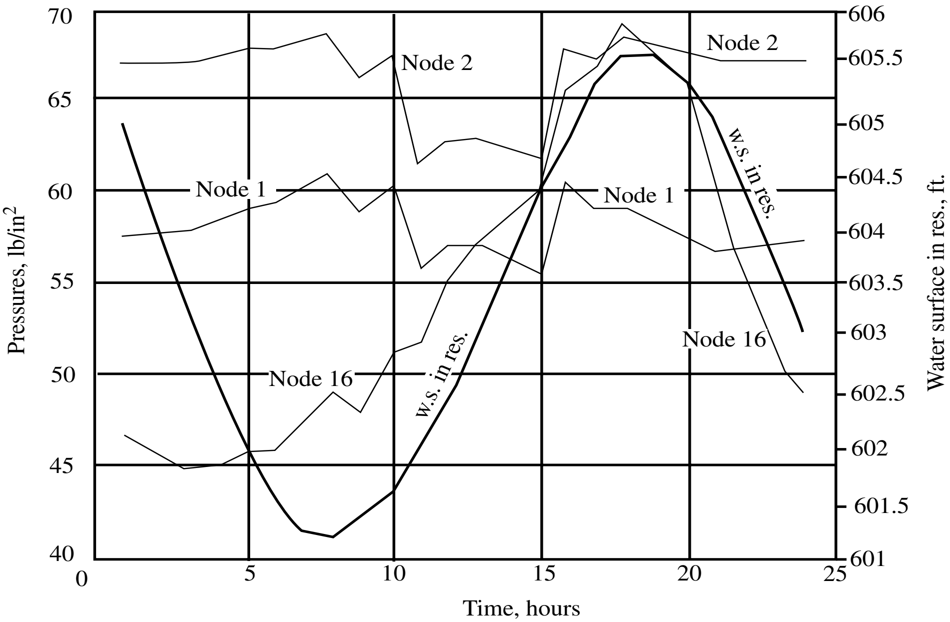
Discharges (ft^3/s) in Pipes as a Function of Time (cont'd)

Hours	Pipe Number									
	11	12	13	14	15	16	17	18	19	20
1	0.24	0.64	4.72	4.26	4.95	3.91	0.61	0.61	- 0.17	3.32
2	0.32	0.67	4.68	4.26	5.03	3.97	0.56	0.68	- 0.13	3.29
3	0.39	0.69	4.65	4.26	5.11	4.02	0.52	0.75	- 0.10	3.26
4	0.46	0.70	4.56	4.23	5.12	4.01	0.46	0.80	- 0.07	3.22
5	0.52	0.70	4.46	4.16	5.08	3.97	0.41	0.84	- 0.04	3.20
6	0.69	0.71	4.37	4.09	4.94	3.78	0.35	0.94	0.06	3.22
7	0.73	0.70	4.24	4.03	4.86	3.71	0.29	0.96	0.06	3.19
8	0.75	0.69	4.06	3.94	4.73	3.59	0.19	0.96	0.05	3.09
9	1.00	0.69	3.72	3.61	4.24	3.14	- 0.01	1.13	0.25	2.73
10	0.99	0.65	3.51	3.45	4.05	2.98	- 0.16	1.10	0.21	2.56
11	0.75	0.57	2.83	2.84	3.78	2.85	- 0.17	0.95	0.14	2.06
12	0.67	0.52	2.74	2.77	3.56	2.67	- 0.18	0.85	0.05	2.02
13	0.54	0.48	2.77	2.76	3.40	2.57	- 0.08	0.71	- 0.10	2.09
14	0.29	0.40	2.94	2.83	3.26	2.46	0.18	0.46	- 0.23	2.32
15	- 0.08	0.29	3.13	2.90	3.08	2.33	0.38	0.20	- 0.33	2.51
16	- 0.42	0.10	3.95	3.36	3.28	2.26	0.70	- 0.45	- 0.51	3.25
17	- 0.70	- 0.12	4.22	3.50	3.08	1.98	0.86	- 0.80	- 0.54	3.28
18	- 0.90	- 0.24	4.72	3.76	2.81	1.69	1.06	- 1.07	- 0.56	3.58
19	- 0.90	- 0.21	4.88	3.83	3.03	1.87	1.11	- 1.05	- 0.58	3.66
20	- 0.90	- 0.18	5.06	3.92	3.25	2.06	1.17	- 1.03	- 0.60	3.77
21	- 0.76	0.04	5.16	4.06	3.96	2.76	1.14	- 0.78	- 0.63	3.80
22	- 0.49	0.29	5.02	4.13	4.37	3.26	0.99	- 0.38	- 0.57	3.66
23	- 0.26	0.48	4.92	4.32	4.59	3.62	0.81	0.19	- 0.38	3.53
24	- 0.05	0.56	4.84	4.34	4.76	3.78	0.71	0.42	- 0.29	3.45

Discharges (ft^3/s) in Pipes as a Function of Time (cont'd)

Hours	Pipe Number									
	21	22	23	24	25	26	27	28	29	30
1	1.04	2.14	0.94	0.68	0.72	1.06	0.86	0.84	0.84	2.36
2	1.07	2.16	0.96	0.61	0.77	1.09	0.86	0.85	0.85	2.26
3	1.10	2.18	0.98	0.54	0.83	1.12	0.85	0.86	0.86	2.16
4	1.16	2.19	0.99	0.38	0.84	1.13	0.83	0.86	0.86	1.86
5	1.25	2.19	0.98	0.13	0.82	1.12	0.81	0.86	0.85	1.37
6	1.30	2.10	0.96	- 0.22	0.85	1.10	0.79	0.88	0.80	0.83
7	1.45	2.12	0.94	- 0.53	0.77	1.08	0.76	0.87	0.79	0.13
8	1.63	2.15	0.92	- 0.78	0.62	1.02	0.72	0.84	0.76	- 0.63
9	1.53	1.82	0.83	- 0.79	0.82	1.00	0.67	0.89	0.66	- 0.53
10	1.72	1.85	0.79	- 0.99	0.61	0.92	0.62	0.85	0.63	- 1.32
11	1.46	1.71	0.75	- 0.86	0.58	0.85	0.50	0.66	0.60	- 1.14
12	1.57	1.77	0.70	- 0.95	0.26	0.74	0.47	0.62	0.56	- 1.72
13	1.52	1.89	0.67	- 0.89	- 0.11	0.62	0.47	0.57	0.53	- 1.92
14	1.39	1.88	0.64	- 0.72	- 0.35	0.51	0.50	0.53	0.51	- 1.66
15	1.32	1.81	0.60	- 0.51	- 0.51	0.40	0.53	0.49	0.49	- 1.34
16	1.71	1.90	0.63	- 0.45	- 1.01	0.17	0.66	0.52	0.50	- 1.77
17	1.65	1.74	0.60	0.32	- 1.20	0.00	0.73	0.46	0.46	- 0.72
18	1.58	1.59	0.56	0.82	- 1.39	- 0.16	0.83	0.44	0.41	0.05
19	1.45	1.65	0.59	1.00	- 1.30	- 0.09	0.86	0.48	0.45	0.59
20	1.32	1.73	0.62	1.15	- 1.21	- 0.02	0.90	0.52	0.49	1.08
21	1.20	2.01	0.72	1.20	- 0.83	0.28	0.92	0.63	0.62	1.67
22	1.17	2.13	0.80	1.04	- 0.36	0.55	0.90	0.73	0.71	1.86
23	1.12	2.10	0.86	0.87	0.27	0.84	0.89	0.80	0.78	2.22
24	1.12	2.13	0.90	0.75	0.48	0.95	0.88	0.82	0.81	2.21



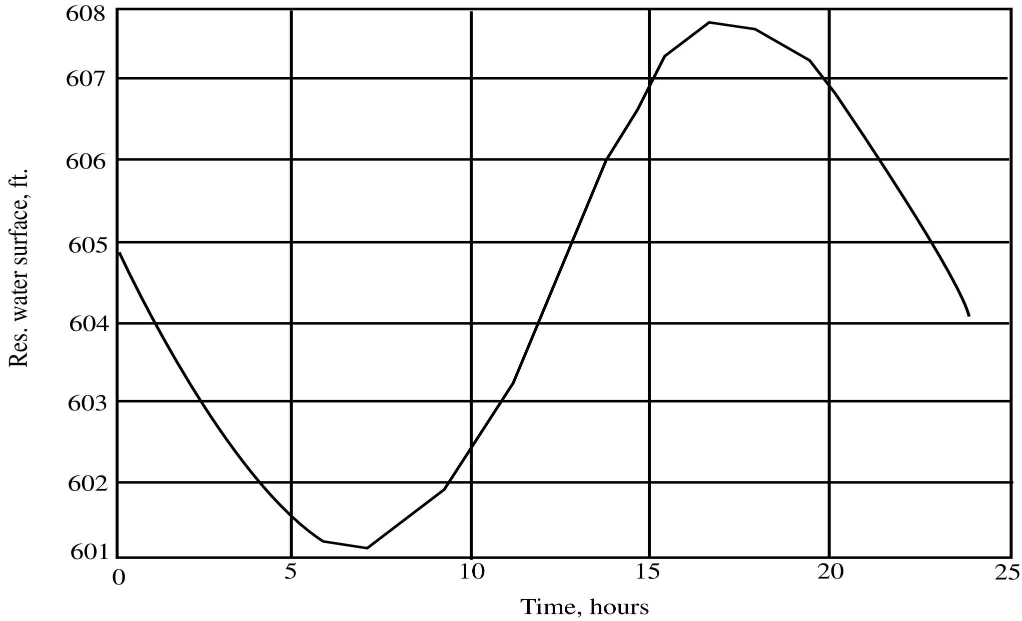


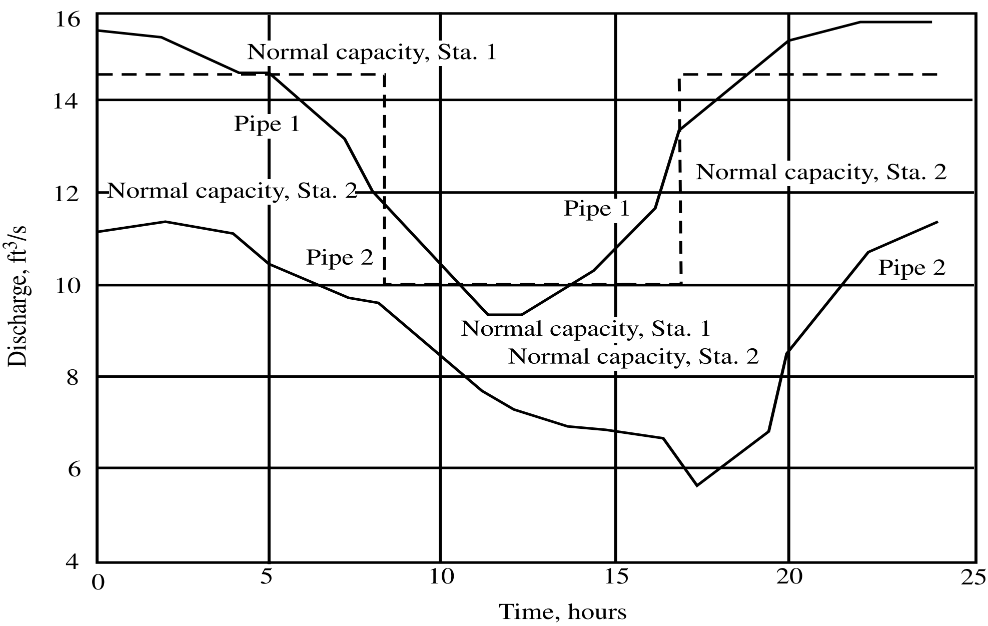
capacity lines on the figure, with accompanying reductions in efficiency. In fact, it would appear that the pump schedule should never reduce the number of pumps in operation at either station to one; then the tank would be full at the end of the 24-hour period.

The simulation can be run again with the following changes to the input data:

PUMP SCHEDULES
1 2 0 3 8 2 17 3/
2 2 0 3 5 2 20 3/

The solution then shows that the tank ends the simulation period with a water surface at elevation 604.03 ft, and the discharges in pipes 1 and 2 are more nearly at their normal capacities, as the graphs below show. Can the reader develop an operating scenario that would cause the tank to end the period with a water surface elevation of 605 ft?


608
607
606
605
604
603
602
601
Res. water surface, ft.
0
5
10
15
20
25
Time, hours


16
14
12
10
8
6
4
Discharge, ft³/s
Normal capacity, Sta. 1
Pipe 1
Normal capacity, Sta. 2
Pipe 2
Normal capacity, Sta. 2
Pipe 1
Pipe 2
Normal capacity, Sta. 1
Normal capacity, Sta. 2
0
5
10
15
20
25
Time, hours

* * *

6.3 ELEMENTS OF ENGINEERING ECONOMICS

As with most engineering endeavors, the design process for water distribution systems explores alternative solutions to a given situation, analyzes these alternatives and then relies on the designer's engineering experience and judgment to select the best alternative. An important element of this process is economics, wherein the total cost of the delivery of water at the pressures and in the quantities that are required is examined. A brief review of engineering economics is given here as a base for the consideration of economics in the selection of pipe and pump sizes. This review may repeat parts of the reader's first course in engineering economics or another economics course; this background is essential in understanding some of the material that follows.

In engineering economics we seek the least cost solution, that is, the one that calls for the smallest overall expenditure over its expected life, taking into account the time value of money. The costs can be divided into two major categories, those needed now to build the system and start its operation, and the recurring annual costs to keep the system in operation. The first category is commonly called the capital investment, and the second category contains the operating costs. An alternative way to view these two categories (but with yearly income) is to consider the present value of an annuity (a recurring payment) over some time interval as equivalent to the capital investment, and the amount of the annual annuity as the operating cost. If these two categories of costs are to be combined to provide the total cost, the two must be put on an equivalent basis, considering that there is interest (value) associated with the use of money. To develop a fair comparison, we work with the terms **present worth** and **series payment**. A capital investment cost adds directly to the present worth, but a recurring cost must be multiplied by a **present worth factor**, **pwf**, before it is added to the capital investment to compute the overall present worth. Similarly, recurring costs are added directly to give the series payment amount, but capital investment amounts must be multiplied by a **capital recovery factor**, **crf**, before being added together to obtain the total series payment amount. In other words, the two alternatives are either (1) to convert recurring or annual operating costs to the equivalent of a capital investment by multiplying by the pwf, or (2) converting capital investments to series costs by multiplying them by the crf. We assume that the individual payments are a constant amount so we are dealing with a uniform series payment.

The formula for the present worth factor pwf is

$$pwf = \frac{1}{i}\left[1 - \frac{1}{(1+i)^n}\right] = \frac{(1+i)^n - 1}{i(1+i)^n} \qquad (6.1)$$

and the capital recovery factor crf is the reciprocal thereof, or

$$crf = \frac{1}{pwf} = \frac{i}{1 - \dfrac{1}{(1+i)^n}} = \frac{i(1+i)^n}{(1+i)^n - 1} \qquad (6.2)$$

in which i is the interest rate per time period and n is the number of recurring payments or time periods. Usually the series payment is on an annual basis, i.e., once per year, and then n is the life of the project in years.

To illustrate the use of these factors, assume it will cost 1 million dollars to build a water distribution system and prepare it to begin operation. Thus a loan is taken (or a bond is issued) for $1,000,000 at a particular interest rate, and this loan is to be repaid by constant annual payments over the life of the project n = 15 years. Table 6.1 reports payment data for interest rates i = 0.06 and i = 0.10 for n = 15. The columns headed

"Payment" are obtained by multiplying the capital recovery factor by $1,000,000, i.e., for $i = 0.06$ the capital recovery factor is $crf = 0.06/[1 - 1/(1.06^{15})] = 0.102963$, and for $i = 0.10$ the factor is $crf = 0.1/[1 - 1/(1.10^{15})] = 0.131474$. The columns headed "Accum." accumulate or sum these annual payments. The column headed "Interest" is the amount of interest accrued during that year, found by multiplying the entry in the row above (in the next column) by the interest rate, and the column headed "Owing" gives the amount of the loan still outstanding. The values in this last column are obtained by subtracting the payment from, and adding the interest to, the previous entry, i.e., $957,037 = 1,000,000 - 102,963 + 60,000. In both halves of this table the payment at the end of the fifteenth year exactly equals the amount still owed plus the interest on that amount over the last year (within roundoff error), or $102,296 = 97,135 + 5828, and $131,474 = 119,521 + 11,952. Thus for an interest rate of 6% a constant annual payment of $102,963 is equivalent to a present worth of $1,000,000, and for an interest rate of 10% $131,474 paid at the end of each year for 15 years is equivalent to $1,000,000. In other words, multiplying the pwf by this annual payment will reproduce the principal amount, in this case $1,000,000.

Table 6.1
A $1,000,000 loan

	Interest Rate = 6%				Interest Rate = 10 %			
Yr	Payment	Accum.	Interest	Owing	Payment	Accum.	Interest	Owing
				1000000				1000000
1	102963	102962	60000	957037	131474	131473	100000	968526
2	102963	205925	57422	911497	131474	262947	96852	933905
3	102963	308888	54689	863224	131474	394421	93390	895822
4	102963	411851	51793	812054	131474	525895	89582	853930
5	102963	514813	48723	757815	131474	657368	85393	807849
6	102963	617776	45468	700321	131474	788842	80784	757161
7	102963	720739	42019	639378	131474	920316	75716	701403
8	102963	823702	38362	574777	131474	1051790	70140	640069
9	102963	926664	34486	506301	131474	1183264	64006	572603
10	102963	1029627	30378	433717	131474	1314737	57260	498389
11	102963	1132590	26022	356777	131474	1446211	49838	416754
12	102963	1235553	21406	275221	131474	1577685	41675	326956
13	102963	1338515	16513	188771	131474	1709159	32695	228178
14	102963	1441478	11326	97135	131474	1840632	22817	119521
15	102963	1544441	5828	0	131474	1972106	11952	0

To carry this illustration further, assume that a solution to the network problem indicates that the annual cost of electrical energy to operate the pumps is $120,000, and the maintenance department will require an average of $50,000 per year to operate and repair the system. The total annual costs are then obtained as 120,000 + 50,000 = $170,000. The two alternative approaches for comparing the capital investment and operating costs are the following: 1. Add pwf x (annual cost) to the capital investment; or 2. Add crf x (capital investment) to the annual cost. These two alternatives are listed in Table 6.2. As the interest rate increases, we see in the table that the total cost decreases if the present worth basis is used, but when a series payment is used to obtain the total cost we find that the total cost increases with interest rate. These differences occur because the pwf decreases with interest rate, but the crf increases with interest rate.

Table 6.2 Total Costs

As Present Worth		As a Series Payment	
For $i = 0.06$		For $i = 0.06$	
1. Capital investment	= 1,000,000	1. Cap. invest., crfx$1,000,000	= 102,963
2. Operating, pwfx$170,000	= 1,651,082	2. Operating	= 170,000
Total	$2,651,082		$272,963
For $i = 0.10$		For $i = 0.10$	
1. Capital investment	1,000,000	1. Cap. invest., crfx$1,000,000	= 131,474
2. Operating, pwfx$170,000	= 1,293,034	2. Operating	170,000
Total	$2,293,034		$301,474

Example Problem 6.2

Compare the cost of using a 6 in, 8 in, or 10 in pipe line (wall roughness e = 0.005 in) to pump 1.5 ft^3/s of water ($\nu = 1.217 \times 10^{-5}$ ft^2/s) for 200 days per year from a groundwater elevation of 1500 ft to an elevation of 1550 ft with a delivery pressure of 50 lb/in^2. The total pipe length is 4000 ft. Energy costs $0.11/kWh; the capital investment cost of the well and pumps is $50,000; the combined efficiency of the pump-motor is $\varepsilon = 0.70$, and the pipe costs are $30/ft for 6-in-pipe, $45/ft for 8-in-pipe, and $55/ft for 10-in-pipe. The interest rate is $i = 0.10$, and the project life is 30 years.

The solution is summarized in two tables. We begin the solution by computing the capital recovery and present worth factors as crf = $0.1/[1 - 1/1.1^{30}] = 0.10608$ and pwf = $1/0.10608 = 9.4268$. The pump must supply a head that is the sum of the frictional head loss, the elevation difference, and the delivery pressure head, or $h_p = h_f + 50 + 50(144)/62.4 = 165.38 + h_f$ (ft). The power is $P = Q\gamma h_p(0.746)/(550\varepsilon) = 0.18137 h_p$ kW. The Darcy-Weisbach and Colebrook-White equations can be solved for the frictional head loss that would occur in each of the three pipes. These head losses are listed in column (2) in the tables. Columns (3) and (4) in these tables list the computed pump heads that must be supplied and the power requirement of the pumps in kW. The annual operating cost, in $/yr in column (5), is found by multiplying this power requirement by 24 hr/day times 200 days times the $0.11/kWh unit cost of energy. In the first table the total cost is stated in terms of annual amounts; hence column (6), which contains the entire capital investment cost (the cost of the pipe plus $50,000 for the pumps and well), is multiplied by the crf to obtain an equivalent annual cost which is in column (7), and this amount is added to the annual energy cost to find the total annual cost in column (8). We note that the use of 8-in pipe leads to an annual cost of $ 43,411 over the 30-year life of the project, which is the lowest cost of the three alternatives. It is the higher energy costs that cause the total cost with 6-in pipe to be larger, and the higher cost with 10-in pipe is caused by the expense of the pipe itself. A present worth computation is presented in the second table. The total cost is the present worth; here the cost of energy is converted to a present value by use of the pwf before it is added to the capital cost.

Cost on an annual payment basis

Pipe in (1)	h_f ft (2)	h_p ft (3)	P kW (4)	 $/yr (5)	Cap. Cost $ (6)	x crf $ (7)	Cost $ (8)
6	144.46	309.90	56.194	29,670	170,000	18,023	47,703
8	33.17	198.55	36.009	19,013	230,000	24,398	43,411
10	10.72	176.10	31.938	16,863	270,000	28,641	45,504

Cost on a present worth basis

Pipe in (1)	h_f ft (2)	h_p ft (3)	P kW (4)	 $/yr (5)	x pwf $ (6)	Cap. Cost $ (7)	Cost $ (8)
6	144.46	309.89	56.194	29,670	279,695	170,000	449,695
8	33.17	198.55	36.009	19,013	179,233	230,000	409,233
10	10.72	176.10	31.938	16,863	158,965	270,000	428,965

* * *

6.3.1. ECONOMICS APPLIED TO WATER SYSTEMS

To apply economics to the evaluation of piping systems, it is productive to classify each system according to whether (1) the demands are constant or variable, (2) it is gravity feed or pump feed, and (3) branched or looped. When the demands are constant, the discharge in each pipe is fixed, and an optimal design can be achieved by minimizing the total cost over the life of the system. When the demands vary, the changes usually have one or more random components, so in real life a repeating pattern of change with time does not occur, and the identification of an optimal design becomes a matter of interpretation. Gravity-feed systems have no energy costs that can change with the head losses and the desired pressures; if the demands are also fixed, then the optimal design process will simply select the smallest possible set of pipes that will deliver the required pressures and discharges. If the system is branched, then the flow in each pipe is fixed regardless of its diameter, while in looped systems the discharges are dependent upon all of the pipe diameters as well as the demands.

Some irrigation systems fall into the classifications that are simplest to analyze, e.g., branched systems with constant demands. Municipal water distribution systems generally are among the most complex types. They are looped for redundancy to allow individual components to be taken out of operation; they must be able to respond to emergency flows, they have varying demands, and at least some of the water is pumped. Even when the supply enters the network from a reservoir (storage tank), this water often has been pumped into that tank at a previous time. For these complex systems the completion of a formal optimal design cannot be achieved by an application of mathematics and the use of a computer program. Sound engineering judgment based on experience and a thorough understanding of the system's vital components is needed to achieve even a "good," let alone a "near optimal," design. The issues that contribute to the complexity of such systems include the following:

1. Reliability considerations: standby power, manual versus automatic control, types of storage, and the extent and type of monitoring of operations.
2. Demand: spatial and temporal variations in use, types of users, costs for failure to deliver, fire flow requirements, future trends.
3. Storage requirements: groundwater storage with pumping versus elevated or ground-level tanks, tank volumes for reserves and/or peaking, location and variations of water levels.

4. Maximum and minimum pressure requirements: residential areas, high value business districts, industrial districts, future areas that will be served.
5. Population distribution and future trends.
6. Topographic changes: pressure controls (PRV's, BPV's, valves) versus separate systems.
7. Separate systems for irrigation and/or fire fighting and other uses versus a single system etc.

There is no attempt to handle all these issues in this Chapter. Optimization techniques won't even be used. But basic cost considerations will be applied to insure that a system is cost effective.

6.3.2. LEAST COST

In each design we want to select the pipe size which will produce the lowest overall cost, considering both capital recovery and annual operating costs. Of course these two costs must be put on a common basis before they are summed. One way to do this is to solve the problem for different pipe sizes, compute the cost for each and plot the costs on a graph as a function of pipe size. If the water is pumped, a graph of these costs might look like Fig. 6.1, in which the annual capital recovery cost for the pipe and other facilities will increase with the pipe diameter, but the costs associated with pumping will decrease with pipe size. These two opposite trends cause the total cost to decrease with pipe diameter to a minimum and then increase. The nearest standard pipe diameter to this minimum is the diameter to select.

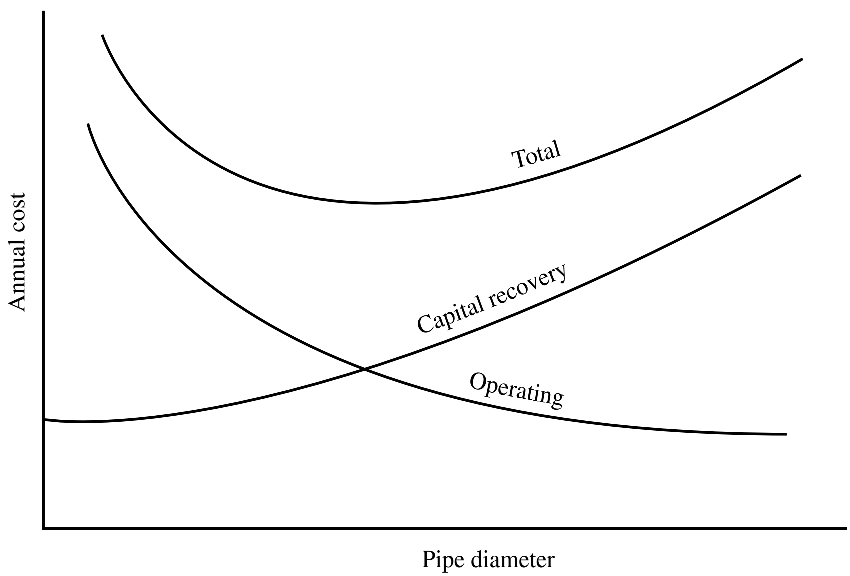


Figure 6.1 System cost as a function of pipe diameter.

For a network of pipes the discharges will usually differ in each pipe, and it would be nonsense to design a network with all pipes of the same diameter, compute the cost and make a graph like Fig. 6.1 to select a pipe size that produces the least total cost. An alternative that does make sense, however, is to replace the pipe diameter, which is the independent variable, by the slope of the HGL. Since the slope of the HGL is the head loss divided by the pipe length over which this loss occurs, it also makes sense to rescale the ordinate to be the cost per unit length of pipe. Solving the Darcy-Weisbach equation for the diameter gives

$$D = \{ fQ^2/(2gS(\pi/4)^2 \}^{0.2} \tag{6.3}$$

in which S is the slope of the HGL, and f is given by the Colebrook-White equation. The cost is then computed in the following way: (1) for a given S, solve the Darcy-Weisbach and Colebrook-White equations for the pipe diameter; (2) with this pipe diameter determine the cost of pipe per unit length C_p from a table of costs for different pipe diameters (this cost might be obtained by interpolation or by using the standard diameter that is closest to the computed value); (3) compute the energy cost for pumping, per unit length of pipe, from

$$C_e = \gamma QRt(S + \Delta H/L)/(eC) \tag{6.4}$$

in which R is the unit cost of energy ($/kWh), ΔH is difference in total head (elevation plus pressure) between the ends of the pipe, t is the time that the pump operates, e is the combined motor-pump efficiency, and C is a unit constant. It is important for C and the energy costs to be on the same time basis. For example, if t is in days, then we have $C = (550/0.746)/24 = 30.72$ for ES units, and $C = 1000/24 = 41.7$ for SI units.

Example Problem 6.3

Prepare a graph of total cost as a function of the slope of the HGL for several different discharges, and then determine the least cost pipe diameter to convey this water in 2000 ft of horizontal pipe with a delivery pressure of 40 lb/in^2. The water is pumped from a reservoir with a water surface elevation that is one foot below the pipe elevation. A table provides the cost per unit length for installing different pipe sizes. Other economic data are as follows: energy costs $0.10/kWh; project life = 30 years; the operating period is 365 days per year; the pipe wall roughness is $e = 0.005$ ft.

Dia., in.	4	6	8	10	12	15	18
$/ft	3.67	5.33	7.67	10.67	16.67	24.00	43.33

Dia., in.	20	24	30	36	42	48	54
$/ft	56.67	80.00	100.00	120.00	145.00	170.00	200.00

We begin the solution by computing the difference in total head between the reservoir water surface and the end of the pipe as $\Delta H = 40(144)/62.4 + 1.0 = 93.3$ ft; upon dividing this result by the pipe length, we obtain $\Delta H/L = 0.0466$. The program MCOST will generate the cost data as a function of S. The input consists of the following: RATE = energy cost per kWh, LIFE = project life in years, Q = discharge, DZ = $\Delta H/L$, DAYS = number of days per year of system operation, G = acceleration of gravity, E = pipe wall roughness (inches for ES units, m for SI units), EFF = combined pump-motor efficiency, N = the number of pairs of (pipe diameter, cost per unit length) to follow. The next input line lists these data pairs; each pipe diameter is in inches for ES units or in m for SI units; the cost is the installation cost per unit length for that diameter. The `DO 80` loop within the program repeats the computations for the 13 values of S in the data statement. Within this loop the Darcy-Weisbach and Colebrook-White equations are solved simultaneously for D and SF = $1/f^{1/2}$. After the diameter is obtained, a second-order polynomial interpolation, using Lagrange's formula, obtains the capital cost per unit length associated with this diameter from the table of data pairs (D, $/L). This value is multiplied by the crf, the energy cost of pumping is computed, and these costs, and their sum, are printed to an output file.

```
C.    PROGRAM MCOST.FOR
      REAL D(20),CP(20),S(13)
      DATA S/.0001,.00025,.0005,.001,.002,.003,
     &.004,.005,.006,.007,.008,.009,.01/
      READ(2,*) RATE,LIFE,Q,DZ,DAYS,G,E,EFF,N
      READ(2,*)(D(I),CP(I),I=1,N)
      IF(G.GT.20.) THEN
      POF=2.031290182
      VIS=1.217E-5
      DO 20 I=1,N
   20 D(I)=D(I)/12.
      E=E/12.
      ELSE
      POF=235.344
      VIS=1.31E-6
      ENDIF
      CRF=RATE*(1.+ RATE)**LIFE/((1.+ RATE)**LIFE - 1.)
      SF=8.
      G2=1.23370055*G
      DD=.8
      K2=2
      DO 80 I=1,13
   30 DD1=DD
   40 SF1=SF
      SF=1.14-2.*ALOG10(E/DD1+7.343472826*VIS*
     &DD1*SF/Q)
      IF(ABS(SF-SF1).GT. 1.E-6) GO TO 40
      DD=((Q/SF)**2/(S(I)*G2))**.2
      IF(ABS(DD1-DD) .GT. 1.E-5) GO TO 30
   50 IF(DD.LT.D(K2+1).OR. K2.GT.N-2) GO TO 60
      K2=K2+1
      GO TO 50
   60 IF(DD.GE.D(K2) .OR. K2.EQ.2) GO TO 70
      K2=K2-1
      GO TO 60
   70 K1=K2-1
      K3=K2+1
      C1=CP(K1)/((D(K1)-D(K2))*(D(K1)-D(K3)))
      C2=CP(K2)/((D(K2)-D(K1))*(D(K2)-D(K3)))
      C3=CP(K3)/((D(K3)-D(K1))*(D(K3)-D(K2)))
      AC=C1+C2+C3
      BC=-C1*(D(K2)+D(K3))-C2*(D(K1)+D(K3))-C3*(D(K1)+D(K2))
      CC=C1*D(K2)*D(K3)+C2*D(K1)*D(K3)+C3*D(K1)*D(K2)
      COST=(AC*DD+BC)*DD+CC
      CPIP=CRF*COST
      CENE=POF*DAYS*(S(I)+DZ)*Q*RATE/EFF
   80 WRITE(3,100) S(I),DD,COST,CPIP,CENE,CPIP+CENE
  100 FORMAT(F10.7,5F10.4)
      END
```

An example of the input data file follows:

```
.1 30 1 .0466 365 32.2 .005 .7 14
4 3.67 6 5.33 8 7.67 10 10.67 12 16.67 15 24 18 43.33 20 56.67 24 80 30 100
36 120 42 145 48 170 54 200
```

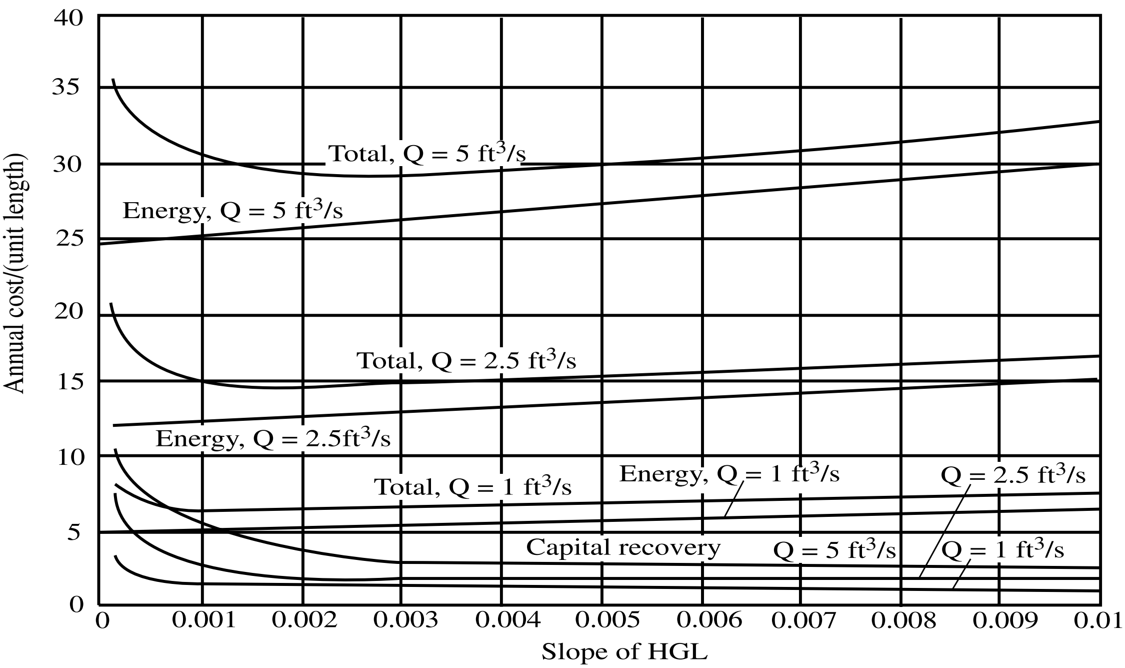
For $Q = 1.0$ ft^3/s the following output table is created:

S	D, ft	C_p	$C_p$xcrf	C_e	Total Cost	
0.0001	1.3869	33.0954	3.5107	4.9463	8.4571	
0.0002	1.1506	21.3257	2.2622	4.9622	7.2244	
0.0005	0.9999	16.6665	1.7680	4.9887	6.7567	
0.0010	0.8697	11.7235	1.2436	5.0417	6.2853	
0.0020	0.7571	9.2161	0.9776	5.1476	6.1252	
0.0030	0.6984	8.1907	0.8689	5.2535	6.1224	*Min. Cost
0.0040	0.6597	7.5582	0.8018	5.3594	6.1612	
0.0050	0.6312	7.1147	0.7547	5.4653	6.2201	
0.0060	0.6088	6.7811	0.7193	5.5712	6.2906	
0.0070	0.5906	6.5178	0.6914	5.6772	6.3686	
0.0080	0.5753	6.3026	0.6686	5.7831	6.4517	
0.0090	0.5621	6.1223	0.6494	5.8890	6.5384	
0.0100	0.5506	5.9681	0.6331	5.9949	6.6280	

Results from these cost calculations for the discharges Q = 1.0 ft^3/s, 2.5 ft^3/s and 5.0 ft^3/s are plotted in the graph on the next page. The HGL slope that led to the lowest total cost, the costs themselves, and the associated pipe diameters are given in the next table:

Minimum Total Cost

Q, ft^3/s	1.0	1.5	2.0	2.5	3.0	3.5	4.0	5.0
Cost	6.12	8.97	11.88	14.84	17.69	20.47	23.31	29.02
S	0.003	0.003	0.003	0.003	0.002	0.002	0.002	0.003
D, ft	0.698	0.814	0.908	0.988	1.147	1.216	1.279	1.285
D, in	8.38	9.77	10.89	11.85	13.76	14.59	15.35	15.42



* * *

As the discharge increases, the pipe size with the least total cost increases, as one would expect. However, the slope S is nearly constant; for discharges of 3 to 4 ft^3/s the slope S = 0.002 led to the smallest total cost, but for the other discharges S = 0.003 gave the smallest total cost.

Example Problem 6.4

Augment the program MCOST so it (1) determines the slope S of the HGL that results in minimum total cost, and (2) writes this total cost per unit pipe length and the corresponding pipe diameter to an output file.

In this project there a several approaches that could be taken. As it is, MCOST generates a table that lists in the last column the total cost per unit length as a function of the HGL-slope S (listed in column 1). As this table is being generated, we monitor the last column to see if the total cost has increased from the previous line. If so, we then pass a second-order polynomial through the last three data pairs of (S, \$) with the Lagrangian interpolation formula. The polynomial is of the form $\$ = aS^2 + bS + c$. The minimum cost can be found by taking the derivative of this equation with respect to S, setting it to zero, and solving for S as $S_{min} = - b/(2a)$. Upon substituting S_{min} into the polynomial equation, we find $\$_{min}$. In a similar way the corresponding pipe diameter D_{min} can be obtained with a similar polynomial $D = a_1S^2 + b_1S + c_1$, with S_{min} substituted for S. This procedure is implemented by replacing `80 WRITE ...` with the statements given below. The arrays CT(3), SS(3), DS(3) have also been added to the REAL declaration, and I1 = 1 and CT(1) = 1.E20 initialize these variables.

```
      IF(CTO.LT.CT(I1).
     &OR.I.LT.3) THEN
      IF(START) THEN
      CT(1)=CTO
      SS(1)=S(I)
      DS(1)=DD
      START=.FALSE.
      ELSE
      IF(NFIRST) THEN
      CT(1)=CT(2)
      SS(1)=SS(2)
      DS(1)=DS(2)
      ENDIF
      CT(2)=CTO
      SS(2)=S(I)
      DS(2)=DD
      I1=2
      NFIRST=.TRUE.
      ENDIF
      ELSE
      CT(3)=CTO
      SS(3)=S(I)
      DS(3)=DD
      AA=0.
      DA=0.
      BB=0.
      DB=0.
      CC=0.
      DC=0.
      DO 76 J=1,3
      CR=CT(J)
      DR=DS(J)
      DO 74 K=1,3
      IF(K.EQ.J) GO TO 74
      CR=CR/(SS(J)-SS(K))
      DR=DR/(SS(J)-SS(K))
   74 CONTINUE
      AA=AA+CR
      DA=DA+DR
      SUM=0.
      PRO=1.
      DO 75 K=1,3
      IF(K.EQ.J) GO TO 75
      SUM=SUM+SS(K)
      PRO=PRO*SS(K)
   75 CONTINUE
      BB=BB-CR*SUM
      DB=DB-DR*SUM
      DC=DC+DR*PRO
   76 CC=CC+CR*PRO
      SHMIN=-0.5*BB/AA
      CTMIN=(AA*SHMIN+BB)*SHMIN+CC
      DMIN=(DA*SHMIN+DB)*SHMIN+DC
      GO TO 90
      ENDIF
   80 CONTINUE
   90 WRITE(6,110) Q,SHMIN,
     &CTMIN,DMIN
  110 FORMAT(' Q =',F8.2,', HGL-S
     &=',F9.6,', $/L =',F8.2,
     &', Dmin =',F8.3)
      END
```

* * *

6.4 ECONOMIC NETWORK DESIGN

6.4.1. ONE PRINCIPAL SUPPLY SOURCE

The method in Section 6.3 can be used to select a single pipe that will yield the least cost. If a network of pipes exists, then the method must be modified. In a real water distribution system the capital costs associated with installing pipes might vary widely from location to location for the same pipe size. In a new subdivision the only costs in addition to the purchase of the pipe may be the costs associated with the operation of a trenching machine or backhoe. In a highly developed business district there will be costs associated with replacement of roads, replacement and/or relocation of other utilities, rerouting traffic, acquisition of right-of-way, etc., and these costs may be enormous in comparison with the pipe costs, so there is then no significant difference in cost with pipe size. Annual costs might also vary considerably, depending upon whether the pipe is fed by gravity and whether the water comes directly from a pump or from a storage facility that receives its water via pumps. For the latter there is the capital recovery cost associated with the storage facility in addition to the energy cost of pumping. Obviously the variability of costs is dependent upon the city and/or location of the water system, and to describe procedures to follow in the design of a "near least-cost" system we will use only the simplest case, in which no variability of either capital recovery or operating costs occurs. The same principles apply, however, for the more complex real water system, but they must be applied on an individual basis.

Let us assume that the layout, or a proposed layout, of the system is given. It includes the supply sources as well as the lengths and locations of pipes, some of which may later be eliminated. For design purposes the demands are known and fixed. Such design demands may be the maximum demands that are expected, without emergency flows for fire fighting etc. From Example Problem 6.3 we have seen that the slope of the HGL that is related to the least total cost does not vary greatly with the discharge. Considering the uncertainty of costs over the life of the network and the variability of discharges, one HGL slope for all pipes in the network might be used, rather than finding the least cost HGL slope for each pipe. A practical approach to the design of an economical pipe network might in general follow these eight steps:

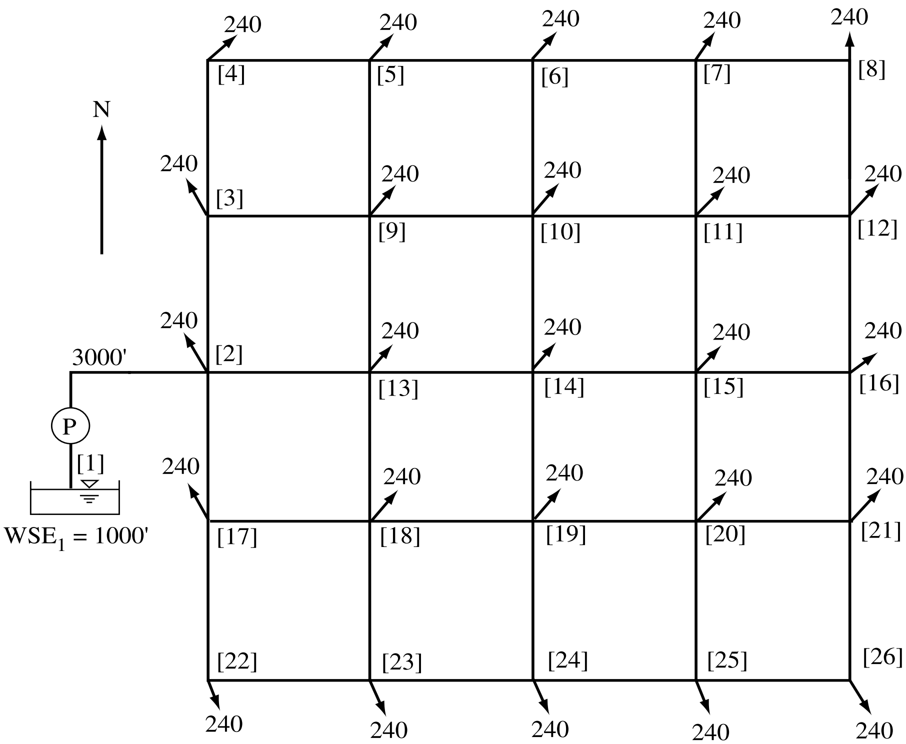
1. Remove pipes that will carry the smaller discharges so a branched network is created.
2. Determine the demands for which the network is to be designed. These design demands normally are the peak hourly demands, but they do not include fire, or other emergency, flows.
3. Use the demands from step two to determine the discharge in every pipe.
4. Compute the pipe diameters, based on a selected HGL slope that is consistent with the satisfaction of the desired operating pressures for the system. After finding the pipe diameters, replace them with the nearest (or with the next larger, depending on one's judgment) standard pipe size and obtain by analysis a solution for this system.
5. Compute the costs associated with the pipe sizes that were found in step four, as well as the pumping cost and other operating costs.
6. Repeat steps four and five with a set of different HGL slopes, until the minimum cost can be identified.
7. Each pipe that was removed in step one can now be re-installed as a pipe having a minimum diameter or, as judgment dictates, a larger size, especially if the pipe is located so its flow will be important for fire fighting, or if it will be needed when another key component of the system is out of operation. We now analyze this piping system for several demand levels and/or fire flow requirements and attempt to identify any deficiencies in the network's performance. If deficiencies are found, we must exercise judgment in deciding how they might best be corrected.
8. Based on a knowledge of network performance that was obtained in step seven, formulate logical schedules and/or rules for the operation of the network. Select the

elevation, sizes and heights of storage tanks that will produce a "good" daily operation that will fully meet the anticipated varying demands, maintain pressures above the minimum and not create any excessive pressures. Test these choices by obtaining an extended time simulation of the proposed network and its operation. From an examination of results from this time-dependent solution, make appropriate changes to the network and/or its operation, and repeat the extended time simulations until the designer concludes that no further significant cost reductions can be made, especially in view of the uncertainty of the demand data and some of the other data that are the foundation of the computation.

Example Problem 6.5

A water distribution system is to be designed to serve a new community development. The land is level where the development is located and has an elevation of 1100 ft. As a consulting hydraulic engineer you are responsible for the design of the water system. The following requirements exist:

1. The entire water supply must come from a well that has recently been drilled. All indications, including pumping tests, suggest the well will provide enough water for the community for the coming 20 years.
2. The skeletonized pipe layout for design should coincide with the proposed network of primary streets with locations on a square grid that is one-quarter mile on each side, so all pipes, with the exception of the supply line from the well, are 1320 ft long. (Secondary streets and individual service connections to buildings will exist in the interior of indi-vidual blocks, but they are omitted in the skeletonization.) All pipes have e = 0.005 in.



All nodal demands in gal/min

3. A survey of future water consumption indicates that the peak hourly demand will be 240 gal/min at each node at the street intersections.
4. For this region of the country it is anticipated that the peak hourly demand is 2.3 times the average daily demand.
5. For a town of this size and the anticipated industrialization, the National Bureau of Fire Underwriters recommends that the system be able to supply an emergency fire flow of 2000 gal/min at any node at a pressure of at least 20 lb/in^2.

In estimating costs for the project, the following information is provided to you by the firm's economist: interest rate = 9 percent; project life = 20 years; electricity costs \$0.090/kWh; overall cost for the well, buying the pump and installing it is \$180,000; the cost of construction C for a water storage tank is $C = \$15000V^{0.5}$, in which V is the tank capacity in thousands of cubic feet; the cost of the pipe is divided into (a) the purchase price of the pipe and (b) the costs associated with its installation. These costs are given in the following table:

Pipe Costs per Foot of Length

Diameter of Pipe, in	**6**	**8**	**10**	**12**	**15**	**18**	**24**	**36**
Purchase price, \$	4.00	6.00	12.00	18.00	25.00	45.00	60.00	100.00
Installation, \$	8.00	8.20	8.40	8.60	9.00	9.20	9.40	9.60

As the hydraulic engineer that is responsible for the design of the water system, you are to (1) specify the size of each pipe in the system, (2) specify the pump(s) to be installed (this includes the characteristics, i.e., the discharge and head that the pump should produce), (3) specify where storage tanks should be located, and give their elevations and capacities, (4) provide an engineering economic analysis of the proposed water system, and suggest what price should be charged for the water to recover the costs associated with the construction and maintenance of the system.

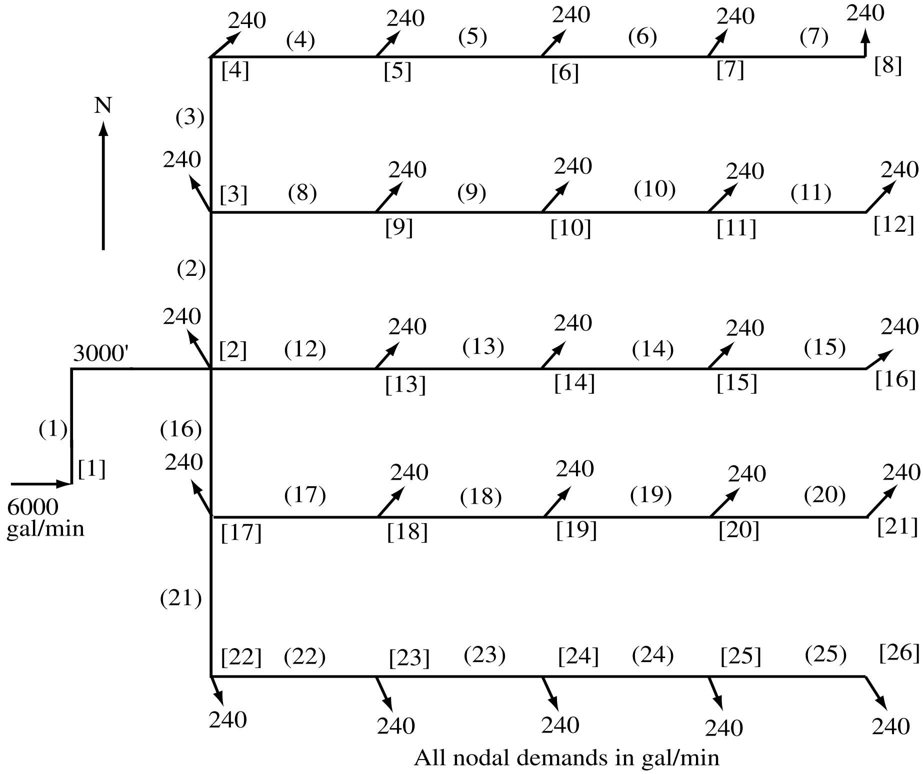
The solution will proceed according to the steps outlined above.

Step 1. To create a branched system from the proposed layout, the pipes along north-south streets have been removed, except the one that is nearest to the pump, to create the network shown atop the next page. There are many alternative branched networks that could be used. This one assumes that the north-south pipes closest to the pump will be the main transmission lines. An alternative which would assume that the primary north-south transmission line is nearer the center of the system would remove pipes 2, 3, 16, and 21, and put in their place pipes connecting node pairs 14 and 10, 10 and 6, 14 and 19, and 19 and 24.

Step 2. The problem statement provides the demands to be used for the design, i.e., 240 gal/min = 0.535 ft^3/s at each node of the network. In practice, obtaining these design demands requires studies of current water demands in the area and projections of future trends over the life of the network, all tempered with a sound interpretation of these data and judgment.

Steps 3, 4, 5, and 6. These steps will be accomplished together. In fact, these steps will be completed via two alternative pathways. The first alternative will determine the most economical HGL-slope for each pipe by using MCOST1, the modified computer program from Example Problem 6.4. The second alternative is to use special input allowed by NETWK to define a branched network.

In the first alternative the program SOLBRAN, which was described in Chapter 5, will be modified by replacing subroutine DIAPIP, which finds f and D when the head loss for the pipe is given, by MCOST1 as a subroutine that determines the least cost HGL-slope, diameter, and cost per unit length of pipe, based on economic data on pipe cost and the energy cost of pumping. The program is called MCBRAN. The first portion of MCBRAN, almost to the `DO 90` loop, is the same as SOLBRAN with a COMMON



/COSTB/ statement and arrays H (nodal heads), QJJ (nodal demands), NOP (pipe numbers), L1 (upstream pipe node) and L2 (downstream pipe node), added. Aftert the DO 85 loop an added READ statement enters, as triples, the pipe number NOP and its upstream and downstream nodes, L1 and L2. The sequence of pipes in this input establishes the order in which the least cost pipes are to be found with subroutine MCOSTS. This sequence must start at the end of one branch and proceed to the point where another branch departs from a node and then downstream along the next branch, so an HGL-elevation is known at one end of each new pipe that is processed. In this example the order that is selected is 7, 6, 5, 4, 3, 8, and so on. The upstream and downstream node numbers are used to compute the elevations of the HGL at each node.

The next read statement contains (1) the node number for which a beginning HGL-elevation will be given, e.g. the downstream node of the first pipe in the previous list, (2) this HGL-elevation, and (3) the elevation slopes DZ that MCOSTS should use. The information in group (3) contains first the number of pairs that will be given, and then as pairs the pipe number and the DZ that should be used until the next pipe in the list is given. Since the slope of the HGL is the sum of the frictional slope and DZ for our example problem, we note the elevation difference of 100 ft between node 8 and the source and the length of the pipes between these two points is 6x1320 + 3000 = 10920 ft; hence DZ = 100/10920 = 0.0091575. This elevation gradient is constant, so the input consists of only 3 values: 1 7 0.0091575. Since this DZ will add a constant to the energy cost for all pipes, it will not change the least cost slope of the HGL.

The last input set contains the economic data. The first line consists of the following: RATE = cost of electricity in $/kWh; LIFE = life expectancy of the project in years; DAYS = the number of days per year that the pump will operate; EFF = the combined pump-motor efficiency; N = the number of data pairs to follow. This last data, consisting of pairs of pipe diameter and its cost per unit length, is read in subroutine MCOSTS when it is first called. The input for our network problem follows the program listings.

In determining the least-cost HGL-slope for each pipe in a network, the energy lost through the demands at the nodes must be included in the overall energy cost. To account for this energy, the program must determine the head at each node. The power loss is the product of the demand QJ, the specific weight γ and the difference between the nodal head H and the nodal elevation. Since the head at the upstream pipe node equals the head at the downstream node plus the product of HGL-slope and pipe length, the program passes the demand QJN and the difference between the known head and the nodal elevation, divided by the length of pipe, to subroutine MCOSTS. A positive slope S(I) is used, independent of direction, since the pressure at the end will be computed by the process when we move in the downstream direction, rather than being given. Program MCBRAN follows:

```
************************************************************************
*    PROGRAM NO. 6.1, MCBRAN, FORTRAN
*    THIS PROGRAM HAS BEEN INCLUDED FOR THE CONVENIENCE OF THE READER.
*    THE AUTHOR ACCEPTS NO RESPONSIBILITY FOR ITS CORRECTNESS.
*    USERS OF THIS PROGRAM DO SO AT THEIR OWN RISK.
************************************************************************
*    FINDS THE LEAST-COST HGL-SLOPE FOR ALL PIPES IN A BRANCHED NETWORK
*
      PARAMETER (N4=4)
      REAL L[ALLOCATABLE](:),E[ALLOCATABLE](:),Q[ALLOCATABLE](:),
     &QJ[ALLOCATABLE](:),H[ALLOCATABLE](:),ELEV[ALLOCATABLE](:),
     &QJJ[ALLOCATABLE](:),DHS(10)
      INTEGER*2 IPP(11),NUM[ALLOCATABLE](:),JN[ALLOCATABLE](:,:),
     &NOP[ALLOCATABLE](:),L1[ALLOCATABLE](:),L2[ALLOCATABLE](:)
      LOGICAL*2 LNODE[ALLOCATABLE](:)
      CHARACTER*38  F110
     &/'(I5,F8.1,F8.6,F10.3,F8.3,3F10.3,F10.6)'/
      COMMON CI4,G2,DI4,S,SF,CFDIA
      COMMON /COSTB/CRF,DZ,G,VISC,FK2,POF,N
      WRITE(*,*)' GIVE (1) NO. OF PIPES, (2) INPUT UNIT,'
     &,' (3) OUTPUT UNIT '
      READ(*,*) NP,INPUT,IOUT
      NJ=NP+1
      CONV=12.
      ALLOCATE (L(NP),E(NP),Q(NP),QJ(NJ),NUM(NJ),JN(NJ,N4),
     &LNODE(NJ),H(NJ),ELEV(NJ),QJJ(NJ),NOP(NP),L1(NP),L2(NP))
      READ(INPUT,*) G,VISC
      FK2=1.3757
      CFDIA=(5.01137E-4/0.003)**0.2053
      IF(G.LT.30.) THEN
      CONV=1.
      FK2=1.62613
      CFDIA=(1.13437E-3/0.003)**.2053
      ENDIF
      G2=2.*G
      SF=8.
      CI4=7.3434712828*VISC
      DI4=0.785398163
      DO 8 I=1,NP
    8 L(I)=0.
      READ(INPUT,*) L
      DO 9 I=1,NP
      IF(L(I) .LT. 0.001) L(I)=L(I-1)
    9 CONTINUE
      DO 10 I=1,NP
```

```
10 E(I)=0.
   READ(INPUT,*) E
   DO 20 I=1,NP
   IF(E(I).GT.0.) THEN
   IF(E(I).LT.10.) E(I)=E(I)/CONV
   ELSE
   E(I)=E(I-1)
   ENDIF
20 CONTINUE
   DO 22 I=1,NJ
22 ELEV(I)=-10001.
   READ(INPUT,*) ELEV
   DO 25 I=2,NJ
   IF(ELEV(I).LT.-10000.) ELEV(I)=ELEV(I-1)
25 CONTINUE
   SUM=0.
   DO 40 I=1,NJ
   DO 30 J=1,N4
30 JN(I,J)=0
   READ(INPUT,*) QJ(I),(JN(I,J),J=1,N4)
   SUM=SUM+QJ(I)
   DO 35 J=N4,1,-1
   IF(JN(I,J).EQ.0) GO TO 35
   NUM(I)=J
   IF(J.EQ.1) THEN
   IF(FLOAT(JN(I,1))*QJ(I).LE. 0.) GO TO 40
   WRITE(*,*)' DIRECTION OF FLOW IN PIPE',JN(I,1),
  &' IS NOT CONSISTENT WITH DEMAND NODE',I,QJ(I)
   GO TO 99
   ENDIF
   GO TO 40
35 CONTINUE
40 LNODE(I)=.FALSE.
   DO 41 I=1,NJ
41 QJJ(I)=QJ(I)
   IF(ABS(SUM).GT. 1.E-4) THEN
   WRITE(*,*)' DEMANDS DO NOT SUM TO ZERO, SUM =', SUM
   GO TO 99
   ENDIF
   NNJ=NJ
45 DO 65 I=1,NNJ
   IF(NUM(I).GT.1) GO TO 65
   LNODE(I)=.TRUE.
   IJ1=IABS(JN(I,1))
   Q(IJ1)= - FLOAT(JN(I,1)/IJ1)*QJ(I)
   DO 60 J=1,NNJ
   IF(J.EQ.I .OR. NUM(J).LT.2) GO TO 60
   K=1
48 IF(IABS(JN(J,K)).EQ.IJ1) THEN
   QJ(J)=QJ(J)+Q(IJ1)*FLOAT(JN(J,K)/IJ1)
   DO 50 KK=K+1,NUM(J)
50 JN(J,KK-1)=JN(J,KK)
   NUM(J)=NUM(J)-1
   GO TO 60
   ENDIF
   K=K+1
   IF(K.LE.NUM(J)) GO TO 48
60 CONTINUE
65 CONTINUE
```

```
      JJ=0
      DO 80 I=1,NNJ
      IF(LNODE(I)) GO TO 80
      JJ=JJ+1
      QJ(JJ)=QJ(I)
      LNODE(JJ)=.FALSE.
      DO 70 J=1,NUM(I)
   70 JN(JJ,J)=JN(I,J)
      NUM(JJ)=NUM(I)
   80 CONTINUE
      IF(JJ.GE.NNJ) THEN
      WRITE(*,*)' NOT A BRANCHED NETWORK.  NO ADDITIONAL'
     &,' DEAD END PIPES'
      GO TO 99
      ENDIF
      NNJ=JJ
      IF(NNJ.GT.0) GO TO 45
      IF(G.LT.30.) THEN
      D=SQRT(.85*Q(1))/.8
      ELSE
      D=SQRT(.25*Q(1))/.8
      ENDIF
      WRITE(IOUT,100)
  100 FORMAT(' PIPE  LENGTH ROUGHNESS    DIA.    AREA',
     &' DISCHARGE  VELOCITY  HEAD LOSS HGL-SLOPE'
     &,/,1X,78('-'))
      DO 85 J=1,NJ
   85 H(J)=-1001.
      READ(INPUT,*)(NOP(I),L1(NOP(I)),L2(NOP(I)),I=1,NP)
      READ(INPUT,*) K,H(K),NSTART,(IPP(I),DHS(I),I=1,NSTART)
      READ(INPUT,*) RATE,LIFE,DAYS,EFF,N
      IF(G.GT.20.) THEN
      POF=2.03*DAYS*RATE/EFF
      ELSE
      POF=235.344*DAYS*RATE/EFF
      ENDIF
      CRF=RATE*(1.+RATE)**LIFE/((1.+RATE)**LIFE-1.)
      JJ=2
      DZ=DHS(1)
      IPP(NSTART+1)=10000
      COSTE=POF*QJJ(K)*(H(K)-ELEV(K))
      COSTP=0.
      DO 90 J=1,NP
      I=NOP(J)
      IF(I.EQ.IPP(JJ)) THEN
      DZ=DHS(JJ)
      JJ=JJ+1
      ENDIF
      IF(H(L1(I)).LT.-1000.) THEN
      IF(H(L2(I)).LT.-1000.) THEN
      WRITE(*,*)' CANNOT CONTINUE SINCE NO H FOR PIPE',I,
     &' IS KNOWN'
      GO TO 99
      ELSE
      QJN=QJJ(L1(I))
      HDL=(H(L2(I))-ELEV(L2(I)))/L(I)
      ENDIF
      ELSE
      QJN=QJJ(L2(I))
```

```
      HDL=(H(L1(I))-ELEV(L1(I)))/L(I)
      ENDIF
      IF(QJN.LT.0.) QJN=0.
      CALL MCOSTS(Q(I),E(I),QJN,HDL,S,D,CL,CEL,CPL)
      IF(H(L1(I)).LT.-1000.) THEN
      H(L1(I))=H(L2(I))+S*L(I)
      ELSE
      H(L2(I))=H(L1(I))-S*L(I)
      ENDIF
      COSTE=COSTE+L(I)*CEL
      COSTP=COSTP+L(I)*CPL
      D12=CONV*D
      COEF=CONV*E(I)
      A=0.7853982*D*D
   90 WRITE(IOUT,F110) I,L(I),COEF,D12,A,Q(I),Q(I)/A,S*L(I),S
      WRITE(IOUT,130) COSTE,COSTP,COSTE+COSTP
  130 FORMAT(/' COST OF ENERGY    =  $',F10.2,/
     &' COST OF PIPE       =   ',F10.2,/
     &' TOTAL COST/YEAR    =   ',F10.2,/)
      WRITE(IOUT,*)' HGL-ELEVATIONS AT NODES, IN FT'
      WRITE(IOUT,120)(J,H(J),J=1,NJ)
  120 FORMAT(6(I4,F9.2))
   99 DEALLOCATE(L,E,Q,QJ,NUM,JN,LNODE,H,NOP,L1,L2)
      STOP
      END
      SUBROUTINE MCOSTS(Q,EE,QJN,HDL,SHMIN,DMIN,CTMIN,CEMIN,CPMIN)
      REAL D(20),CP(20),S(13),CT(3),SS(3),DS(3)
      LOGICAL*2 NFIRST/.FALSE./,START/.TRUE./,SREAD/.TRUE./
      COMMON /COSTB/CRF,DZ,G,VISC,FK2,POF,N
      DATA S/.0001,.00025,.0005,.001,.002,.003,.004,.005,
     &.006,.007,.008,.009,.010/
      IF(SREAD) THEN
      READ(2,*)(D(I),CP(I),I=1,N)
      IF(G .GT. 20.) THEN
      DO 1 I=1,N
    1 D(I)=D(I)/12.
      ENDIF
      SREAD=.FALSE.
      ENDIF
      I1=1
      CT(1)=1.E20
      SF=8.
      G2=1.23370055*G
      DIA=DMIN
      K2=2
      DO 80 I=1,13
      IF(EE.GT.10.) THEN
      DIA=FK2*(Q/(EE*S(I)**.54))**.380228
      ELSE
   30 DIA1=DIA
   40 SF1=SF
      SF=1.14-2.*ALOG10(EE/DIA1+7.343472826*VISC*DIA1*SF/Q)
      IF(ABS(SF-SF1).GT. 1.E-6) GO TO 40
      DIA=((Q/SF)**2/(S(I)*G2))**.2
      IF(ABS(DIA1-DIA) .GT. 1.E-5) GO TO 30
      ENDIF
   50 IF(DIA.LT.D(K2+1).OR. K2.EQ.N-1) GO TO 60
      K2=K2+1
      GO TO 50
```

```
60 IF(DIA.GE.D(K2) .OR. K2.EQ.2) GO TO 70
   K2=K2-1
   GO TO 60
70 K1=K2-1
   K3=K2+1
   C1=CP(K1)/((D(K1)-D(K2))*(D(K1)-D(K3)))
   C2=CP(K2)/((D(K2)-D(K1))*(D(K2)-D(K3)))
   C3=CP(K3)/((D(K3)-D(K1))*(D(K3)-D(K2)))
   AC=C1+C2+C3
   BC=-C1*(D(K2)+D(K3))-C2*(D(K1)+D(K3))-C3*(D(K1)+D(K2))
   CC=C1*D(K2)*D(K3)+C2*D(K1)*D(K3)+C3*D(K1)*D(K2)
   COST=(AC*DIA+BC)*DIA+CC
   CPIP=CRF*COST
   CENE=POF*((S(I)+DZ)*Q+QJN*(HDL+S(I)))
   CTO=CPIP+CENE
   IF(CTO.LT.CT(I1).OR.I.LT.3) THEN
   IF(START) THEN
   CT(1)=CTO
   SS(1)=S(I)
   DS(1)=DIA
   START=.FALSE.
   ELSE
   IF(NFIRST) THEN
   CT(1)=CT(2)
   SS(1)=SS(2)
   DS(1)=DS(2)
   ENDIF
   CT(2)=CTO
   SS(2)=S(I)
   DS(2)=DIA
   I1=2
   NFIRST=.TRUE.
   ENDIF
   ELSE
   CT(3)=CTO
   SS(3)=S(I)
   DS(3)=DIA
   AA=0.
   DA=0.
   BB=0.
   DB=0.
   CC=0.
   DC=0.
   DO 76 J=1,3
   CR=CT(J)
   DR=DS(J)
   DO 74 K=1,3
   IF(K.EQ.J) GO TO 74
   CR=CR/(SS(J)-SS(K))
   DR=DR/(SS(J)-SS(K))
74 CONTINUE
   AA=AA+CR
   DA=DA+DR
   SUM=0.
   PRO=1.
   DO 75 K=1,3
   IF(K.EQ.J) GO TO 75
   SUM=SUM+SS(K)
   PRO=PRO*SS(K)
```

```
   75 CONTINUE
      BB=BB-CR*SUM
      DB=DB-DR*SUM
      DC=DC+DR*PRO
   76 CC=CC+CR*PRO
      SHMIN=-0.5*BB/AA
      CTMIN=(AA*SHMIN+BB)*SHMIN+CC
      DMIN=(DA*SHMIN+DB)*SHMIN+DC
      CEMIN=CTMIN*CENE/CTO
      CPMIN=CTMIN*CPIP/CTO
      RETURN
      ENDIF
   80 CONTINUE
      END
```

The input data file (DESIGMU1.DAT) for MCBRAN for this problem follows:

```
32.2 1.41E-5
3000 1320/
0.005/
1100./
-13.36809 1/
.5347236 -1 2 12 16/
.5347236 -2 3 8/
.5347236 -3 4/
.5347236 -4 5/
.5347236 -5 6/
.5347236 -6 7/
.5347236 -7/
.5347236 -8 9/
.5347236 -9 10/
.5347236 -10 11/
.5347236 -11/
.5347236 -12 13/
.5347236 -13 14/
.5347236 -14 15/
.5347236 -15/
.5347236 -16 17 21/
.5347236 -17 18/
.5347236 -18 19/
.5347236 -19 20/
.5347236 -20/
.5347236 -21 22/
.5347236 -22 23/
.5347236 -23 24/
.5347236 -24 25/
.5347236 -25/
7 7 8 6 6 7 5 5 6 4 4 5 3 3 4 8 3 9 9 9 10 10 10 11 11 11 12
2 2 3 12 2 13 13 13 14 14 14 15 15 15 16 16 2 17 17 17 18 18 18 19
19 19 20 20 20 21 21 17 22 22 22 23 23 23 24 24 24 25 25 25 26 1 1 2
8 1192.3 1 7 .0091575
.09 20 365 .7 8
6 12 8 14.2 10 20.4 12 26.6 15 31 18 54.2 24 69.4 36 109.6
```

The table provides part of the solution, and additional information follows:

Pipe	Length ft	ex10^3 in	Dia. in	Area ft^2	Q ft^3/s	V ft/s	h_L ft	HGL-Slope
7	1320	5.0	7.02	0.268	0.535	1.992	2.966	0.00225
6	1320	5.0	8.45	0.389	1.069	2.746	4.363	0.00331
5	1320	5.0	9.57	0.499	1.604	3.214	5.076	0.00385
4	1320	5.0	11.16	0.680	2.139	3.147	4.028	0.00305
3	1320	5.0	13.77	1.034	2.674	2.586	1.979	0.00150
8	1320	5.0	11.16	0.680	2.139	3.147	4.028	0.00305
9	1320	5.0	9.57	0.499	1.604	3.214	5.076	0.00385
10	1320	5.0	8.45	0.389	1.069	2.746	4.363	0.00331
11	1320	5.0	7.02	0.268	0.535	1.992	2.966	0.00225
2	1320	5.0	15.43	1.298	5.347	4.120	4.523	0.00343
12	1320	5.0	11.16	0.680	2.139	3.147	4.028	0.00305
13	1320	5.0	9.57	0.499	1.604	3.214	5.076	0.00385
14	1320	5.0	8.45	0.389	1.069	2.746	4.363	0.00331
15	1320	5.0	7.02	0.268	0.535	1.992	2.966	0.00225
16	1320	5.0	15.43	1.298	5.347	4.120	4.523	0.00343
17	1320	5.0	11.16	0.680	2.139	3.147	4.028	0.00305
18	1320	5.0	9.57	0.499	1.604	3.214	5.076	0.00385
19	1320	5.0	8.45	0.389	1.069	2.746	4.363	0.00331
20	1320	5.0	7.02	0.268	0.535	1.992	2.966	0.00225
21	1320	5.0	13.77	1.034	2.674	2.586	1.979	0.00150
22	1320	5.0	11.16	0.680	2.139	3.147	4.028	0.00305
23	1320	5.0	9.57	0.499	1.604	3.214	5.076	0.00385
24	1320	5.0	8.45	0.389	1.069	2.746	4.363	0.00331
25	1320	5.0	7.02	0.268	0.535	1.992	2.966	0.00225
1	3000	5.0	25.49	3.544	13.37	3.772	4.386	0.00146

COST OF ENERGY = \$ 248,987.70
COST OF PIPE = 87,081.89
TOTAL COST/YEAR = 336,069.60

The listed energy costs in this output do not include the capital recovery cost for the pump, which is crf(180,000) = 0.10955(180,000) = \$19,719.

HGL-ELEVATIONS AT NODES, IN FT

1	1219.62	2	1215.23	3	1210.71	4	1208.73
5	1204.70	6	1199.63	7	1195.27	8	1192.30
9	1206.68	10	1201.61	11	1197.25	12	1194.28
13	1211.21	14	1206.13	15	1201.77	16	1198.80
17	1210.71	18	1206.68	19	1201.61	20	1197.25
21	1194.28	22	1208.73	23	1204.70	24	1199.63
25	1195.27	26	1192.30				

Next the nearest standard pipe diameters are selected to replace the computed diameters, this network is analyzed, and the costs are computed. The cost of the tank will be ignored for now, under the assumption that its size is independent of the pipe sizes and the amount of energy used by the pumps. The summary of these costs is given below.

Standard Pipe Diameters used in Analysis Solution

Pipe	1	2	3	4	5	6	7	8	9	10	11	12	13
D, in	24	15	15	15	10	8	8	12	10	8	8	12	10

Pipe	14	15	16	17	18	19	20	21	22	23	24	25
D, in	8	8	15	12	10	8	8	15	12	10	8	8

SUMMARY OF COSTS

ITEM	TYPE	PRESENT WORTH	SERIES AMOUNT
2	PIPE	832,560.00	75,951.85
8	PUMP	2,734,285.00	299,531.30
	TOTAL	3,566,845.00	375,483.10

From this solution the head at node 1 is 219.2 ft, which is the head that the pump(s) must supply.

A second alternative is to use special input that is employed by NETWK in defining branched networks; the analyst would indicate that the design solution should be followed by an analysis solution based on nearest standard pipe sizes and then obtain an economic analysis of this solution. The input data to request these analyses is given below, in which the HGL-slope has been specified as 0.002.

SPECIAL INPUT TO SOLVE MUNICIPAL DESIGN BRANCHED NETWORK:

```
/*
$SPECIF IHGL=-2,DESIGN=1,NFLOW=1,NPGPM=1,NOMSOL=1,ICOST=1 $END
1214.1 -6000 1000 240 .005
ELEV
1100
1 2 .002 3000/
2 8 .002 1320/
3 12/
2 16/
2 21/
17 26/
END
RUN
8 1192.3
INTEREST=.09
LIFE=20
PIPES
UNIT=8
6 12 8 14.2 10 20.4 12 26.6 15 24 18 54.2 24 69.4 36 109.6
EFFIC
.7
PUMPS
UNIT=.09
CAPI=180000.
END
```

In brief: (1) The option IHGL=-2 tells NETWK that special input will be given that defines a branched pipe system. The first line of this input contains (a) the HGL-elevation at the beginning node, (b) the demand here, (c) the elevation to use until it is changed, (d) the demand to apply at subsequent nodes until it is changed, (e) the pipe roughness to use until it is changed. Subsequent lines define the branched system by giving (a) the initial node, (b) the final node, (c) the HGL-slope, and (d) a list of pipe diameters, ending with / if the last given diameter is to be used for the remaining pipes along this branch. (2) The option NOMSOL=1 requests, after a solution to find precisely the pipe diameters (because DESIGN=1) that conform to the head differences between nodes (the heads will be established from the specified slopes of the HGL's), that a regular analysis be performed, in which the nearest standard pipe sizes are used. (3) The option ICOST=1 requests an engineering economic analysis. The HGL elevation at node 1 has been determined as 1192.3 (the HGL elevation at node 8) plus the sum of products of slope and pipe length between node 8 and node 1, or 1192.3 + 0.002(10920) = 1214.1 ft. NETWK must be given an initial HGL so it can compute HGL elevations, pressures, and heads. Since the above data does not contain any supply sources, a node and HGL at this node must be given immediately after the RUN command for the analysis solution.

To obtain solutions for different HGL-slopes, three values in the foregoing input data must be changed, the two HGL slopes (0.002) and the initial HGL elevation (1214.1 ft). The cost summary from the economic analysis for this solution is as follows:

SUMMARY OF COSTS

ITEM	TYPE	PRESENT WORTH	SERIES AMOUNT
2	PIPE	960,072.00	105,172.50
8	PUMP	2,686,039.00	294,246.10
	TOTAL	3,646,111.00	399,418.60

Similar solutions for HGL slopes of 0.0003, 0.0005, 0.00075, 0.001, 0.003, 0.004, and 0.005 provide the following total series costs: \$430,927 for S = 0.0003, \$405,135 for S = 0.0005, \$396,590 for S = 0.00075, \$393,539 for S = 0.001, \$399,418 for S = 0.002, \$397,424 for S = 0.003, \$410,013 for S = 0.004, and \$410,014 for S = 0.005. The least cost is found for S = 0.001 (but costs vary little between S = 0.00075 and 0.003, inclusive), and the nearest standard pipe sizes for this solution are listed in this table:

Pipe	1	2	3	4	5	6	7	8	9	10	11	12	13
D, in	30	20	15	15	12	10	8	15	12	10	8	15	12

Pipe	14	15	16	17	18	19	20	21	22	23	24	25
D, in	10	8	20	15	12	10	8	15	15	12	10	8

The head that the pump(s) must supply for this latter approach is 203.2 ft, or 16 ft less than the result from the first analysis. Several pipes are now one standard pipe size larger than was obtained when the best HGL slope was obtained for each individual pipe. For the second alternative the total annual cost is \$396,971. The other alternative led to an annual recovery cost of \$375,483, a reduction of \$21,489.

Step 7. Since pipe 1 is important in providing emergency flows, we will choose a 30-inch diameter for it, as indicated by the second analysis. A storage tank will be connected to the system by a 200-ft long 24-in-diameter pipe from node 16, and the pipes that were removed will be given 6-in diameters. For a preliminary choice of pumps we choose two parallel pumps, each with characteristics defined by the three (Q, h_p) pairs in the table:

Pump Operating Characteristics

Q, gal/min.	1500	3000	4500
h_p, ft	234.0	219.2	197.0

When the demands are 0.8 times 240 gal/min, we decide that we want the tank neither to receive nor supply any of the demand, and both pumps are operating then. The following input to NETWK will provide a solution that determines the tank's water surface elevation in this instance:

Analysis based on nearest standard diameters

```
/* 37 20 25
$SPECIF NODESP=1,PEAKF=.8,
  NFLOW=1,NPGPM=1 $END
PIPES
1 1 2 3000 24 .005
2 2 3 1320 15
3 3 4 1320 15
4 4 5 1320 12
5 5 6 1320 10
6 6 7 1320 8
7 7 8 1320 8
8 3 9 1320 12
9 9 10 1320 10
10 10 11 1320 8
11 11 12 1320 8
12 2 13 1320 12
13 13 14 1320 10
14 14 15 1320 8
15 15 16 1320 8
16 2 17 1320 15
17 17 18 1320 12
18 18 19 1320 10
19 19 20 1320 8
20 20 21 1320 8
21 17 22 1320 15
22 22 23 1320 12
23 23 24 1320 10
24 24 25 1320 8
25 25 26 1320 8
26 9 5 1320 6
27 13 9
28 13 18
29 18 23
30 10 6
31 14 10
32 14 19
33 19 24
34 11 7
35 15 11
36 15 20

38 12 8
39 16 12
40 16 21
41 21 26
NODES
1 0 1000
2 240 1100
3 240 1100
4 240 1100
5 240 1100
6 240 1100
7 240 1100
8 240 1100
9 240 1100
10 240 1100
11 240 1100
12 240 1100
13 240 1100
14 240 1100
15 240 1100
16 240 1100
17 240 1100
18 240 1100
19 240 1100
20 240 1100
21 240 1100
22 240 1100
23 240 1100
24 240 1100
25 240 1100
26 240 1100
PUMPS
1 1500 234 3000 219.2 4500 197 4000
PARALLEL
1 2/
RUN
```

The solution indicates that the pumps will supply 0.8(6000) = 4800 gal/min and provide a head of 274.2 ft (which appears to be more than needed), and the head at node 16 is 109.8 ft, so the middle level of the storage tank should be near this elevation. As a preliminary design we shall select the tank so its volume will supply the network for one day. The average total demand is 6000/2.3 = 2610 gal/min = 5.81 ft^3/s. Multiplying this demand by 24x3600 sec/day yields a volume of 502,000 ft^3. If the tank is 10 ft high and circular, then it should have a diameter of 253 ft. Let us specify the diameter as 250 ft (Volume = 490,874 ft^3) with a mid-level water surface elevation of 1200 ft. Then the bottom of the tank will be placed at elevation 1195 ft, and its top will be at 1205 ft.

Now several steady state solutions must be obtained to verify the adequacy of the network under a variety of possible operating conditions. In testing the network for fire flows, extra demands of 2000 gal/min will be located at nodes 6, 8, 9, and 20. The next table lists the smallest pressures and the flows from the pumps and the reservoirs for these analyses under the assumptions that both pumps were operating, that the water surface elevation in the storage tank is at 1200 ft, and that the peak daily demands are occurring at the time of the fires.

Fire Demand Consequences at a Node

Node	Min. Pressure lb/in^2	*Q*, Pumps gal/min	ft^3/s	*Q*, Reservoir gal/min	ft^3/s
6	31.4	6420	14.3	1580	3.52
8	2.5	6290	14.0	1710	3.82
9	39.3	6470	14.4	1530	3.41
20	29.5	6250	13.9	1750	3.91

These fire flow analyses indicate that the network can not supply an additional 2000 gal/min at node 8 (and obviously not at node 26 either) to combat a fire. This inadequate performance is caused by the (initially removed) pipes which run in the North-South direction and were assigned the minimum diameter of 6 inches; the problem can be ameliorated and possibly corrected fully by increasing the diameter of some of these pipes to 8 in. Let us try increasing the size of pipes 38, 39, 40, and 41 to 8 in. The same fire flow analyses now produce the following results, which shows that all pressures are now above the required minimum of 20 lb/in^2.

Fire Demand Consequences at a Node

Node	Min. Pressure lb/in^2	*Q*, Pumps gal/min	ft^3/s	*Q*, Reservoir gal/min	ft^3/s
6	35.0	6070	13.5	1930	4.29
8	22.0	5860	13.1	2140	4.76
9	42.0	6150	13.7	1850	4.13
20	34.5	5870	13.1	2160	4.74

If the water surface elevation in the tank is at its lowest level but still able to supply water when these fire flows occur, and if both pumps are then operating, another computation will produce the following pressures and discharges:

Fire Demand Consequences at a Node
Tank Water Surface at 4195 ft, Two Pumps Operating

Node	Min. Pressure lb/in^2	*Q*, Pumps gal/min	ft^3/s	*Q*, Reservoir gal/min	ft^3/s
6	31.5	6700	14.9	1300	2.89
8	17.9	6460	14.4	1540	3.43
9	38.5	6820	15.2	1180	2.64
20	30.5	6460	14.4	1540	3.42

The pressure was only slightly above the required minimum when the tank water surface was at mid-level, and now with the water surface at the base of the tank the pressure is only 18 lb/in^2 at node 8. The same would apply to node 26. If only one of the two parallel pumps were in operation with the tank nearly empty, then the following results would be found:

Fire Demand Consequences at a Node
Tank Water Surface at 4195 ft, One Pump Operating

Node	Min. Pressure lb/in^2	Q, Pumps gal/min	ft^3/s	Q, Reservoir gal/min	ft^3/s
6	22.8	5130	11.4	2870	6.40
8	11.2	5020	11.2	2990	6.65
9	29.4	5170	11.5	2840	6.32
20	23.5	5030	11.2	2970	6.63

As might have been anticipated, now the pressure at node 8 (and 26) is significantly deficient for a fire demand at these nodes. Some means of correcting the problem should be sought. From the solution for the fire demand at node 8, it is observed that the head losses in pipes 15, 34, and 35 are 15.1 ft, 15.1 ft, and 15.3 ft, respectively. Thus one possible solution might be the use of a 10-in diameter for pipe 15 and an 8-in diameter for pipes 34, 35, 36, and 37. With these additional pipes enlarged, the following pressures and discharges are obtained with the reservoir empty and only one pump in operation:

Fire Demand Consequences at a Node
Tank Water Surface at 4195 ft, One Pump Operating

Node	Min. Pressure lb/in^2	Q, Pumps gal/min	ft^3/s	Q, Reservoir gal/min	ft^3/s
6	25.9	4930	11.0	3070	6.84
8	15.5	4820	10.7	3180	7.09
9	31.5	4970	11.1	3030	6.76
20	28.8	4830	10.8	3170	7.06

Although the 15.5 lb/in^2 pressure at node 8 is deficient, it is notably better than in the previous case and not markedly less than 20 lb/in^2. Since the joint probability of finding the tank empty with only one pump in operation should be very low, we will tentatively accept these pipe sizes, move to step 8 and investigate this network further with an extended time simulation.

Step 8. A primary purpose of the use of an extended time simulation for this network is to examine the adequacy of the storage tank. Will it empty or overflow, and will the water depth be approximately unchanged after a 24-hour simulation? We begin this simulation at a moment when the average demands occur. We assume a demand function that is described by the following table, and that this function applies at all nodes.

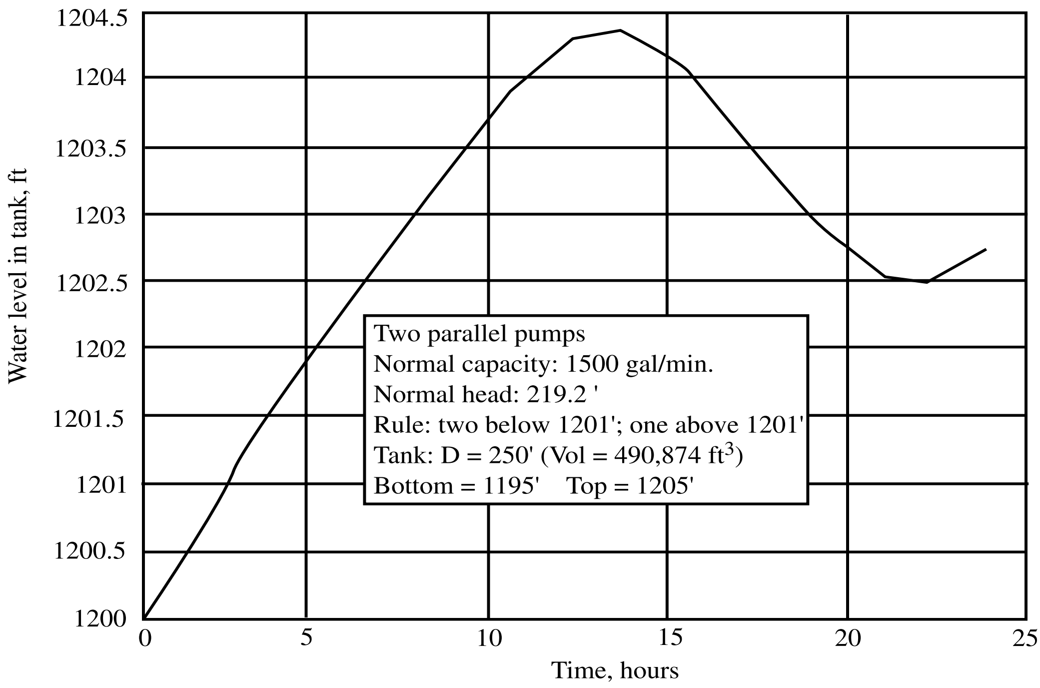
Demand Function

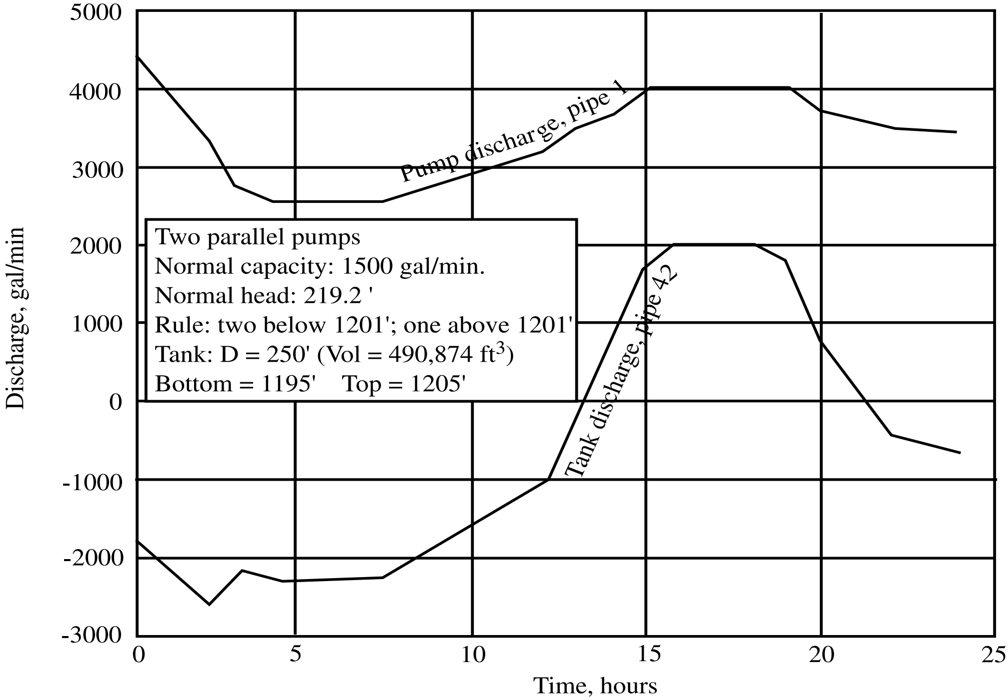
Hour	Peaking Factor
0	1.00
2	1.13
4	1.70
5	2.20
6	2.30
8	2.30
12	0.80
15	0.30
17	0.10
20	0.10
22	0.30
24	1.00

Assume the operation of the pumps will be determined by a level switch in the tank; when the water surface elevation reaches 1201 ft, a pump is turned off so only one pump is in operation, and when the level drops below elevation 1201, the second pump is turned on again. The additional input data for NETWK for the simulation, after the input that defines the network, is then as follows:

```
$TDATA ALTV=1,INCHR=1,ISUNIT=0,LINEAR=1,PRINTT=3,NPUNOD=0,NPNRES=1 $END
PIPE TABLE
ALL
NODE TABLE
ALL
RESER. TABLE
1/
END TABLES
DEMAND FUNCTION
1 0 1 2 1.13 4 1.7 5 2.2 6 2.3 8 2.3 9 2.15 12 .8 15 .3 17 .1 20 .1 22 .3
24 1/
2-26/
STORAGE FUNCTION
1 1195 0 1200 245437 1205 490874/
1/
PUMP RULES
1 2 1 1 1201 1 1199 2 1201 1/
END SIMULATION
```

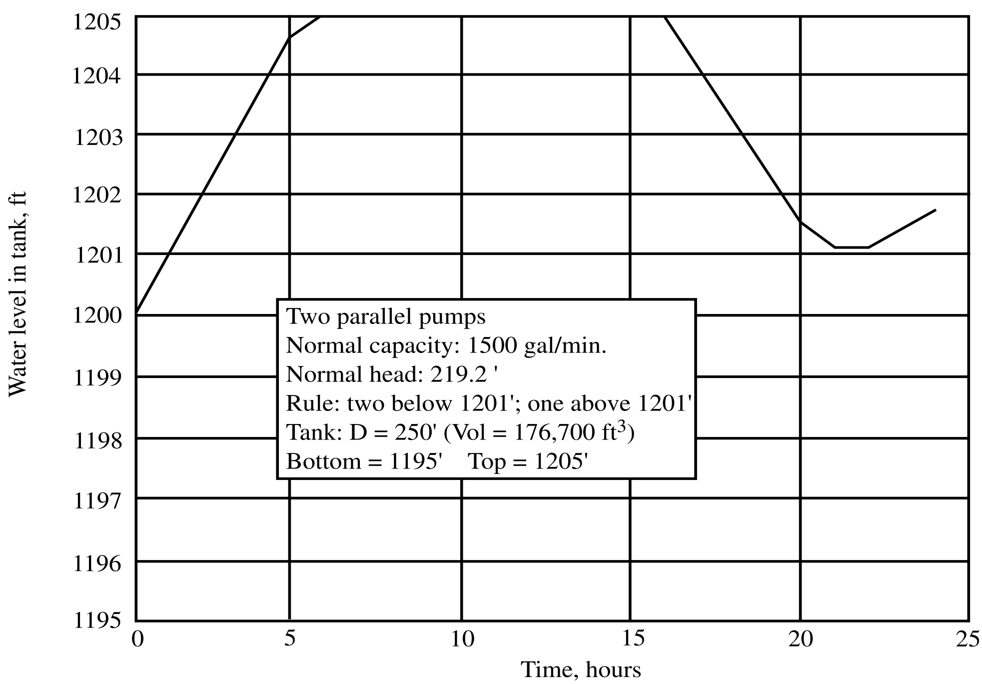
The results of this simulation are summarized in the following two plots. From the first plot we see that the tank's water level rises to within 0.7 ft of the top of the tank at 14 hours, and in satisfying the peak demands the water surface drops to elevation 1202.5, 2.5 ft above its initial mid-level elevation. The results indicate we have more pump capacity than is needed; if this operation were to continue for several days, the tank clearly

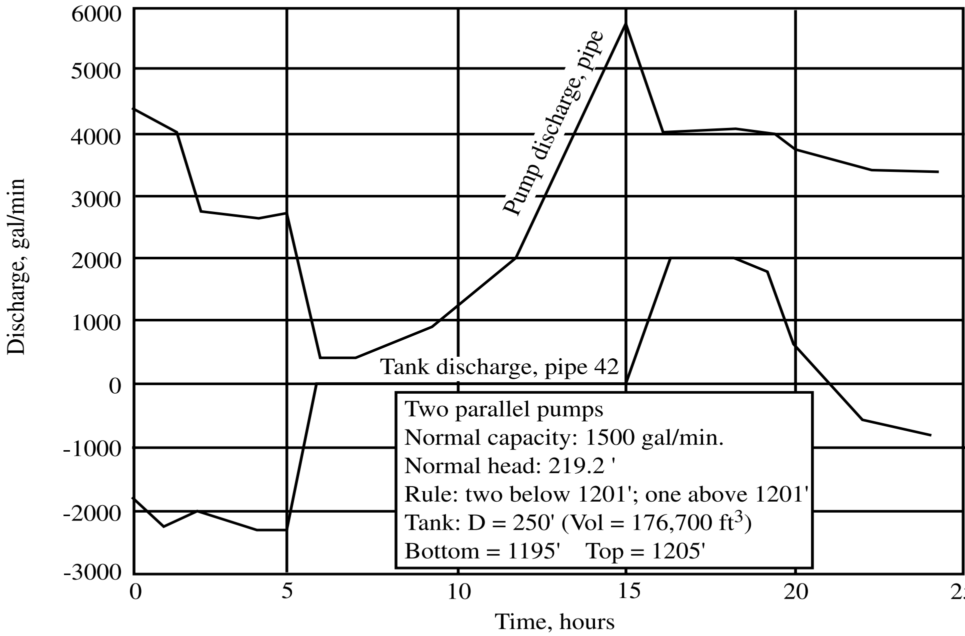




would overflow. We would not be using the tank storage effectively. We should consider reducing the diameter of the tank.

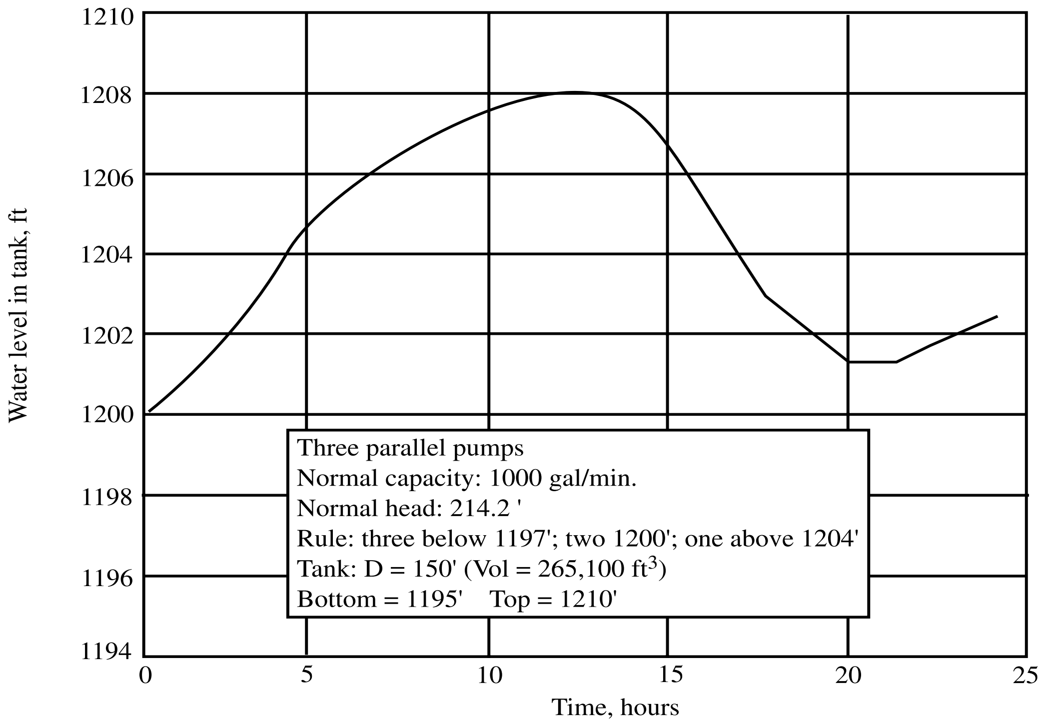
In the next simulation the tank diameter was reduced to 150 ft but the base remained at 1195 ft and the top at 1205 ft, so its new volume was 176,700 ft^3. The results of this simulation are shown in the following two graphs. Now the tank fills after 6 hours of

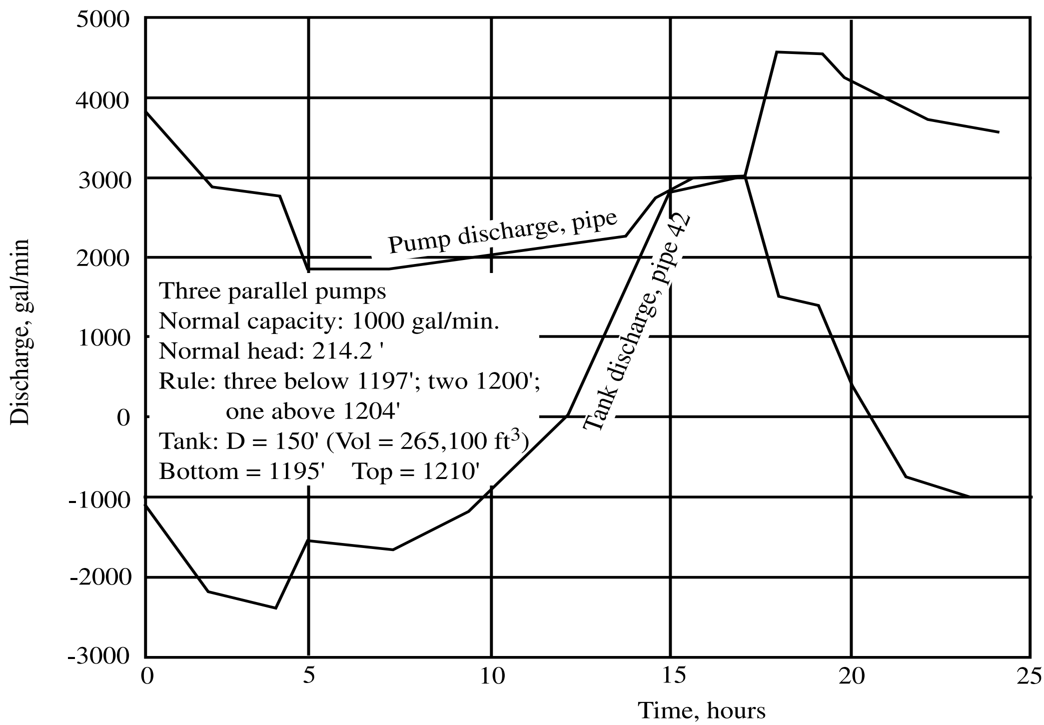




the simulation, and the demands must be met exactly by the pumps until the demand exceeds their capability to do so at 16 hours. Clearly this is not a good tank-pump configuration and operation. The simulation results thus force us to two conclusions: (1) the pumping capacity can be reduced so the water surface elevation in the tank returns nearly to mid-level at the end of the 24 hour simulation, and (2) the elevation of the top of the tank should be increased.

The third simulation will therefore employ three pumps in parallel, each with a normal capacity of 1000 gal/min and a normal head of 214.5 ft (5 ft less than the head of the previous two pumps). The tank will be given a diameter of 150 feet, but we shall raise the top of the tank by 5 ft to elevation 1210, so the tank is now 15 ft high rather than 10 ft, and it has a volume of 265,100 ft^3. The new pump rule is to start the simulation with two pumps operating but to utilize only one pump when the water surface in the tank equals or exceeds 1204 ft. When the water surface drops below 1197 ft, all three pumps will be placed in operation. The results from this simulation are shown below. At the end of the 24-hour simulation the water surface elevation in the tank is just over 2 feet above its initial level, and at 13 hours the water level is within 2 feet of the top. Considering the possibility of occurrence of an emergency demand, this operation is quite satisfactory. The input data to NETWK for this last simulation is given below. What improvements might you suggest? (The final cost analysis is left as an exercise.)


1210
1208
1206
1204
1202
1200
1198
1196
1194
Water level in tank, ft
0
5
10
15
20
25
Time, hours
Three parallel pumps
Normal capacity: 1000 gal/min.
Normal head: 214.2 '
Rule: three below 1197'; two 1200'; one above 1204'
Tank: D = 150' (Vol = 265,100 ft3)
Bottom = 1195' Top = 1210'


5000
4000
3000
2000
1000
0
-1000
-2000
-3000
Discharge, gal/min
0
5
10
15
20
25
Time, hours
Pump discharge, pipe
Tank discharge, pipe 42
Three parallel pumps
Normal capacity: 1000 gal/min.
Normal head: 214.2 '
Rule: three below 1197'; two 1200';
one above 1204'
Tank: D = 150' (Vol = 265,100 ft3)
Bottom = 1195' Top = 1210'

Input to NETWK for the third extended time simulation:

```
Analysis solution based on
nearest standard diameters
/*
$SPECIF ISIML=1,PEAKF=.4347826
,NODESP=1,NFLOW=1,NPGPM=1 $END
PIPES
1 1 2 3000 24 .005
2 2 3 1320 15
3 3 4 1320 15
4 4 5 1320 12
5 5 6 1320 10
6 6 7 1320 8
7 7 8 1320 8
8 3 9 1320 12
9 9 10 1320 10
10 10 11 1320 8
11 11 12 1320 8
12 2 13 1320 12
13 13 14 1320 10
14 14 15 1320 8
15 15 16 1320 10
16 2 17 1320 15
17 17 18 1320 12
18 18 19 1320 10
19 19 20 1320 8
20 20 21 1320 8
21 17 22 1320 15
22 22 23 1320 12
23 23 24 1320 10
24 24 25 1320 8
25 25 26 1320 8
26 9 5 1320 6
27 13 9
28 13 18
29 18 23
30 10 6
31 14 10
32 14 19
33 19 24
34 11 7 1320 8
35 15 11
36 15 20
37 20 25
38 12 8 1320 8
39 16 12
40 16 21
41 21 26
42 27 16 200 24
NODES
1 0 1000
2 240 1100
3 240 1100
4 240 1100
5 240 1100
6 240 1100
7 240 1100
8 240 1100
9 240 1100
10 240 1100
11 240 1100
12 240 1100
13 240 1100
14 240 1100
15 240 1100
16 240 1100
17 240 1100
18 240 1100
19 240 1100
20 240 1100
21 240 1100
22 240 1100
23 240 1100
24 240 1100
25 240 1100
26 240 1100
27 0 1100
PUMPS
1 1000 229 2000
214.2 3000 192 4000
RESER
27 1200.
RUN

$TDATA ALTV=1,INCHR=1,ISUNIT=0,LINEAR=1,PRINTT=3,NPUNOD=0,NPNRES=1 $END
PIPE TABLE
ALL
NODE TABLE
ALL
RESER. TABLE
1/
END TABLES
DEMAND FUNCTION
1 0 1 2 .3 4 .1 7 .1 9 .3 12 .8 15 2.15 16 2.3 18 2.3 19 2.2 20 1.7 22 1.13
 24 1/
2-26/
STORAGE FUNCTION
1 1195 0 1200 88357 1205 176715 1210 265072/
1/
PUMP RULES
1 2 1 2 1197. 3 1200. 2 1204 1/
END SIMULATION
```

* * *

6.4.2. DESIGN GUIDELINES FOR COMPLEX NETWORKS

The design procedure, outlined in eight steps, works well if there is one major supply source so the branched network can be used to start the design process. If several major supply sources exist, the same basic procedure can be followed, with each supply at the head of separate branched systems that are later connected. However, this approach presupposes a knowledge of which portion of the network is supplied by each source. A more general procedure may use the following steps as guideline in the design:

1. Identify the two dominant supply sources for the system. A criterion for selecting these sources is to seek the sources with potentially the largest total heads. Connect these sources by the shortest path of pipes between them. In this methodology this path will be called the dominant path. In the branched system to be defined, all other paths will ultimately terminate at one of the nodes along this dominant path. If only one supply source exists, this dominant path is not defined.
2. Connect each other supply source to one of the nodes on the dominant path via the shortest available path of pipes. These additional paths will be called primary paths. In selecting the node of the dominant path at which a primary path terminates, preference is given to nodes closer to the dominant source with the largest total head. However, all primary paths can be sequenced in descending magnitude of the total head available at the primary path. If only two supply sources exist, they are the dominant sources and this step is omitted.
3. Connect the remaining nodes of the network, ones that are not included in the dominant path or any primary path, by the shortest path of pipes to one of the nodes of the dominant path. Whenever this path intersects a node in a previous path, it is terminated. These paths of pipes will be called secondary paths. The sequence in which these secondary paths are formed is first from nodes of degree one, i.e., dead end pipes, next from nodes of degree two, i.e., that have only two connecting pipes, and so on. The order in which nodes are selected within a given degree is by descending elevation.

Upon completing these three steps, a branched system of pipes has been formed. It includes all nodes of the network and presumably contains the pipes that will convey the majority of the flow from the supply sources to the various demand points throughout the network. The pipes that are not included in this branched network are called additional loop-forming pipes. Their diameters can be arbitrarily specified and, if not based on other criteria, will be the minimum diameter that is permitted.

4. Establish an appropriate head at each node of the network. We do this by working through the paths in the reverse order of their formation. By ignoring the carrying capacity of the additional loop-forming pipes, the discharge in each pipe of the branched system is determined. At the beginning node of each path, the total head is equated to the minimum allowable pressure head plus the elevation of the node. Proceeding from this node to succeeding nodes on the path, the total heads are established by utilizing the optimum S associated with this discharge, as established above. If the head at this node was previously assigned, then the currently computed head is compared with the previous head, and the larger of the two is retained. If any pressure head is computed to be less than the minimum acceptable pressure head, then all previously assigned HGL values along that path are raised. If the pressure head exceeds a maximum specified value that requires the inclusion of a booster pump, then consider putting a booster pipe in this pipe. The total head at nodes that are upstream from the pipe in which a booster pump is placed should then be reduced by the amount of head supplied by the pump.

When this procedure for establishing HGL elevations has progressed to the primary paths, it is necessary to know the discharges that the reservoirs supply, or receive, in order to determine the discharges in the pipes on these primary paths. Rules might be used to assign a fraction of the total demand (positive or negative) for each source to supply.

Aside from this discharge requirement, the total heads are computed at nodes along primary paths in the same manner as along secondary paths.

5. The total heads at nodes on the dominant path, which have not been assigned previously, are determined last by a process that is designed not only to allow an optimal, or near optimal, choice of the size of the pipes, but also to assist the designer in determining the minimum heads that the two dominant sources of supply should have.

To understand how these heads are determined, it will be helpful to assume that N_d nodes exist along the dominant path, excluding the two dominant supply sources themselves. The sketch in Fig. 6.2 has $N_d = 3$. A total of N_d different cases will be examined, which each assume the flow is directed to one of the N_d nodes from both sides.

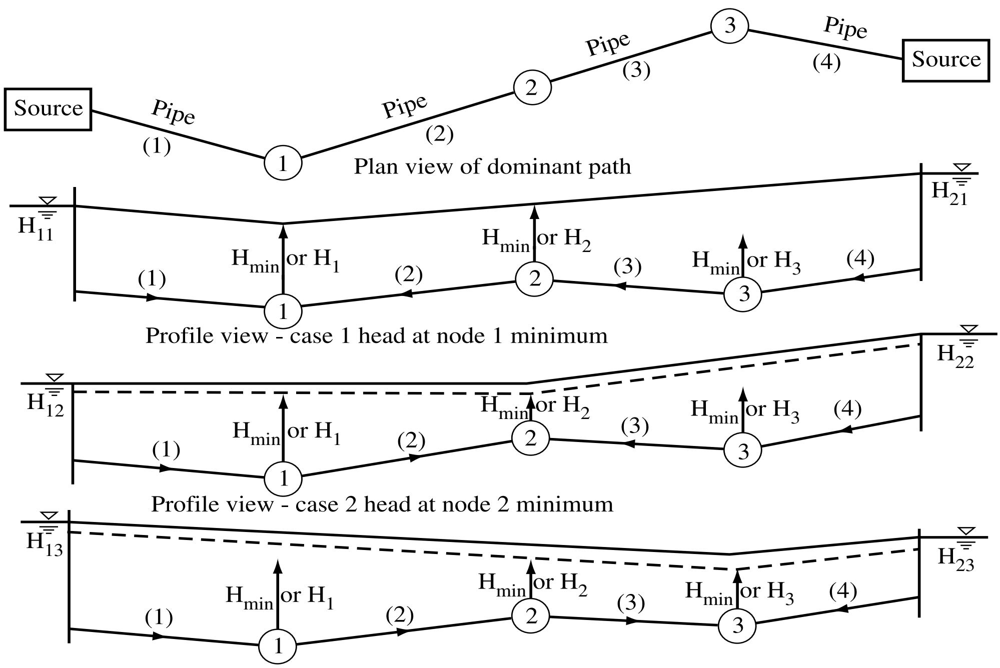


Figure 6.2 Dominant path cases.

For case 1 the HGL must slope from both directions toward node 1, which is nearest to one of the dominant sources; for case 2 the HGL slopes from both directions toward node 2, etc. until for the N_d-th case the HGL slopes toward the node which is nearest to the other dominant source. The elevation of the HGL for each case starts at the node at either the minimum head, H_{min}, above the elevation of the node, or at the head, H, that may have been established during step 4. The slope, or gradient, of the HGL is the optimum slope corresponding to the discharge carried by the pipe. These discharges can be determined since the discharges, or demands, leaving each of the nodes of the dominant path are known when step 4 has been completed. At the starting node for each case, the demand at this node is directed away from the dominant source, from which it receives its supply, and toward node $N_d + 1$. If the HGL should fall below the minimum head, H_{min}, or the total head, H, required from step 4, whichever is larger, then the entire HGL must be raised so it will not fall below any required head. The raising of the HGL is illustrated in cases 2 and 3 in Fig. 6.2. For each case, i, the required total heads, H_{1i} and H_{2i}, for the two dominant supply sources are computed. The case that produces the smallest sum of these source heads is selected, i.e. the case which produces the minimum value of $(H_{1i} + H_{2i})$ is used to establish the total head for all nodes along the dominant path unless judgment suggests that some other heads should be assigned to these supply sources. If the dominant source is a reservoir, then this sets the mid-level water surface elevation, or if this source is a pump, then this determines the head that the pump is to supply.

This procedure not only establishes the heads of the dominant sources, but it also determines the water surface elevations or heads at the other sources at the beginning of the primary paths.

6. With the total heads and discharges now known for each pipe in the branched system from steps 1 through 5, the diameters of all pipes can be computed. These diameters may be computed from the Darcy-Weisbach or Hazen-Williams equation, or even from some other equivalent equation.

6.5 PROBLEMS

6.1 Decide whether (a) an extended time simulation, (b) an unsteady solution that accounts for inertial but not elastic effects, or (c) a full transient analysis might be most appropriate for each of the following situations:

(1) The overall performance of a city water system is to be analyzed to evaluate its ability to accommodate an proposed new subdivision.
(2) A pipe supplies a lumber mill that uses a large jet of water to debark tree stumps, and the valves that control the jet are able to respond quickly so the jet can be shut down rapidly if needed.
(3) A network of pipes is used in a manufacturing plant to supply large amounts of water to numerous locations, and the usage at these locations varies rapidly.
(4) A pipe network in a large building is to be installed for fire protection.
(5) An automatic sprinkler system for a golf course has a timer that controls the irrigation of different portions of the course on a regular schedule.

6.2 Obtain an extended time simulation for the 30-pipe, 16-node network in Example Problem 6.1 if the nodes at which the two demand functions apply are interchanged, that is, the first demand function now applies to nodes 3, 4, 7, 8, 11, 12, 15, and 16, and the second demand function now applies to the other set of nodes.

6.3 Obtain an extended time simulation for the network of Example Problem 6.1 so you can examine the operational consequences of using "pump rules" that you create, based on the water level in the tank that is connected to the network by pipe 30.

6.4 Obtain an extended time simulation of the network in Example Problem 6.1. Start the simulation with two pumps on. Reverse in time the application of the two demand functions so, for example, the demand functions in this problem at 2 hours correspond to those at 23 hours in the original Example Problem 6.1, the demand functions at 3 hours correspond to those at 22 hours etc.

6.5 Obtain an extended time simulation for the network shown below if all of the demands change according to the peaking factor schedule in the table. In the diagram the ground elevation is listed at each node and below the base of the storage tank. The demands on the diagram are 1.5 times the average demands. The peaking factor is expressed as a multiple of the average demands.

Peaking factor PF as a function of time

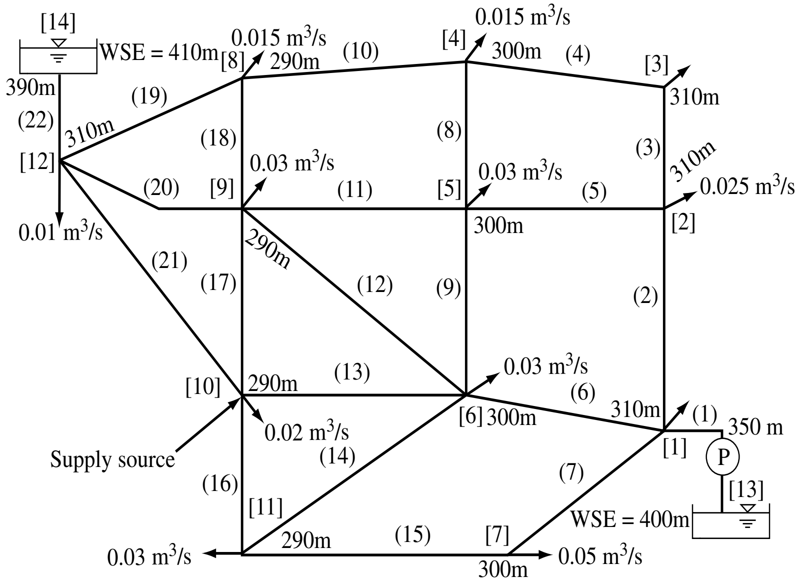
Time, hrs.	0	2	4	6	8	9	10	11	12
PF	1.0	1.2	1.5	2.1	2.5	2.5	2.0	1.4	1.0

Time, hrs.	14	16	18	20	21	22	23	24
PF	0.8	0.4	0.3	0.25	0.25	0.4	0.8	1.0

Pipe No.	Dia. mm	Length m	Pipe No.	Dia. mm	Length m	Pipe No.	Dia. mm	Length m
1	380	1000	9	205	1100	17	205	1000
2	305	1200	10	255	1200	18	150	800
3	305	800	11	205	1100	19	205	1100
4	255	1200	12	205	2000	20	205	1000
5	255	1000	13	255	1200	21	205	2200
6	255	1200	14	255	2000	22	305	800
7	305	1200	15	255	2000			
8	205	1000	16	150	800			

Pump Table, Two Parallel Pumps

Q, m^3/s	0.40	0.45	0.50
h_p, m	48	45	38



There are three parallel pumps that can be operated to increase the head in pipe 1, with the pairs of values given in the pump table actually representing two parallel pumps in operation. The pressure at node 9 is used to control the operation of these pumps as follows: if $p < 1000$ kPa, then 3 pumps are on; if $p = 1120$ kPa, then 2 pumps are on; if $p > 1200$ kPa, then 1 pump is on. The tank [14] at the end of pipe 22 has a diameter of 60 m; its bottom is at elevation 405 m, and it is 15 m high. At node 10 there is a supply source of water that can be purchased (at a relatively high price), and so it is only used when the water level in the storage tank [14] is below 403 m, and then the source will be turned on to supply 0.40 m^3/s.

6.6 From the extended time simulation of Problem 6.5 decide what components of the network should be altered to improve its performance. This may also include rules for the operation of the pumps and/or the purchase of water from the source at node 10.

6.7 Obtain an extended time simulation in hourly increments over a 24 hour period for the operation of the 20-pipe network shown below that is supplied by mountain reservoirs on pipes 1, 2 and 3, and has a storage reservoir on pipe 15. The water surface elevations of the reservoirs are shown on the sketch at time 0, and data in the table below provides the storage vs. water surface elevation relationship for the reservoirs. The demands for the 24-hour period begin with those shown on the diagram, and they then increase to 1.5 times these values in 3 hours and remain constant thereafter according to (t = 1 hr, PF = 1.1), (t =2 hr, PF = 1.2), (t = 3 hr, PF = 1.5). Two butterfly valves are used to control the discharges in pipes 14 and 16, with loss coefficients K given by the equation $K = 244e^{-0.0567x}$, in which x is the number of degrees of opening (0^o is closed and 90^o is open). The amount of opening of the valve in pipe 14 is controlled as a schedule:

Time, hr	0	1	2	3
x, degrees	8.57	15.71	56.29	90.00

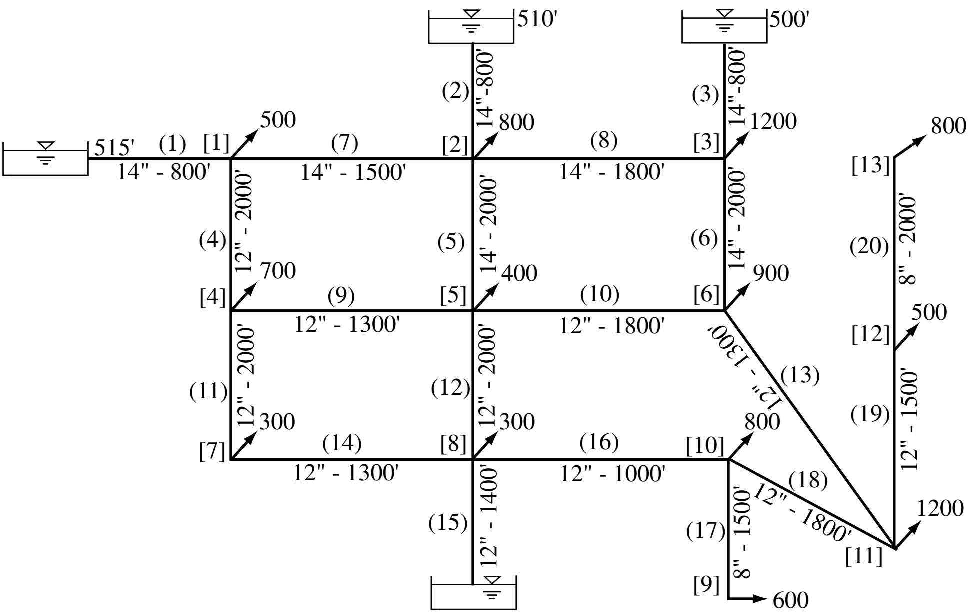
The opening of this valve at $t = 0$ produces a loss coefficient $K_{14} = 150$. The valve in pipe 16 operates on the following rule that depends on the pressure head at node 9:

Head at node 9	70	80	90	100
x, degrees	15.71	40.15	56.29	15.71

Its opening at t = 0 produces a loss coefficient $K_{16} = 24.6$.

Reservoir Storage Function:

For pipes 1, 2, and 3		For pipe 15	
Water surface ft	Volume acre feet	Water surface ft	Volume acre feet
490	0	420	0
510	40	440	15
530	80	460	30



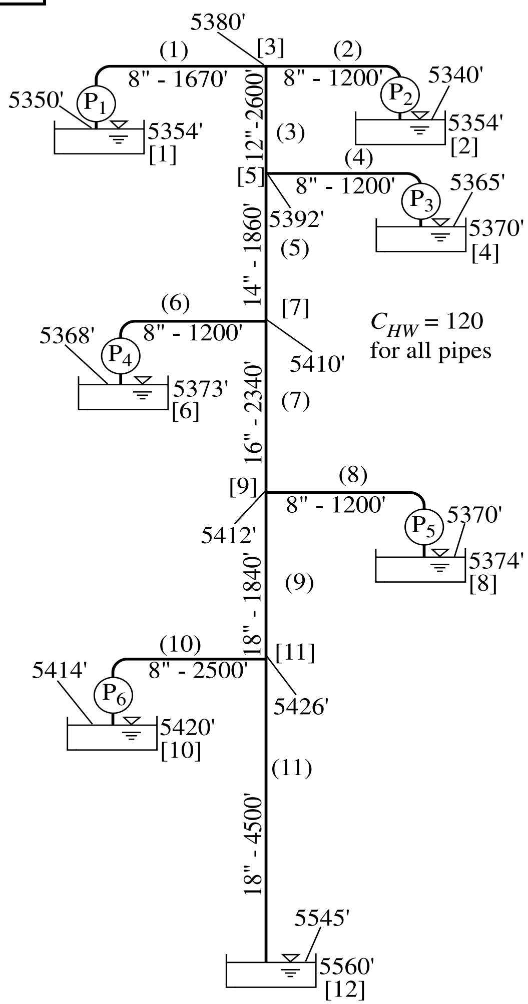
All demands in gal/min

6.8 Water at a rate $Q = 0.045$ m^3/s is to be pumped continuously from a well that has a water table that is 30 feet below the pump to a reservoir that has an average water surface elevation that is 100 ft higher than the pump. The capital investment for the pump and well is \$150,000; electrical energy costs \$0.09/kWh, and the life of the system is estimated to be 50 years. The length of pipe between the well and reservoir is 1200 m, and the cost of pipe for a selection of sizes is as follows: 150 mm pipe costs \$60/m; 205 mm pipe costs \$80/m; 255 mm pipe costs \$110/m; 305 mm pipe costs \$150/m; 375 mm pipe costs \$200/m. The interest rate is 10 percent. Select the most economical pipe size to use. Assume $e = 0.15$ mm.

6.9 The pumps which supply water to the reservoir of the Colorado Springs system are shown below during a period when no demands occur. For this operation determine the amount of energy dissipated by fluid friction, and how much energy is supplied by the pumps per hour of operation. If the motor-pump's combined efficiency, on average, is 70 percent, and energy costs \$0.12/kWh, what is the daily electric bill for each of these units? What is the cost per acre foot of water supplied to the reservoir? If the life of the system is 35 years and the interest rate is 10 percent, what is the equivalent capital recovery cost of each of these items?

All pumps are identical with the following characteristics:

Q gal/min	h_p ft
400	351
600	285
700	234



6.10 A 13,100 ft long 12-inch diameter pipe line is anticipated to have a life of 60 years. If the pipe will cost \$200 per foot to install, what annual benefit must the pipe produce to be economically viable? The interest rate for money is 10 percent.

6.11 Assume that the combined motor-pump efficiencies in Problem 6.9 vary with the discharge through the pumps according to the values in this table:

Q, gal/min	400	600	700
Efficiency	0.68	0.77	0.60

Recompute all the costs that were requested in Problem 6.9 by using linear interpolation and the data in this table.

6.12 Rework Problem 6.9 with (a) all pipes reduced in size by one inch, and (b) all pipes enlarged by one inch. Then compare these costs with the costs that were determined in Problem 6.9.

6.13 In Problem 6.9 all pipes were assumed to be steel with a Hazen-Williams coefficient $C_{HW} = 120$. If all pipes were made of PVC with an equivalent sand grain roughness $e = 0.000008$ in (with unchanged inside diameters that are the standard pipe sizes), again determine all of the quantities that were requested in Problem 6.9.

6.14 For the Colorado Springs network of Problem 6.9 assume there is a demand of 1000 gal/min at each of the tee intersections of the pipes, i.e. nodes 3, 5, 7, 9, and 11. Now what are the quantities requested in Problem 6.9?

6.15 A set of pump curves (not shown) describes the operating characteristics of a pump which is to supply a discharge Q (in gal/min) that varies in time according to

$$Q = 140 + 400\,sin(\pi t/24)$$

in which the time t is in hours. Accounting for the temporal variation in efficiency and head with discharge, determine the energy consumed by the pump during one day's operation. Either a cubic spline function or a second-order polynomial may be used to interpolate the variables. The tables below provide data which has been extracted from the original pump curves. If the efficiency of the electric motor that drives the pump is constant at 85 percent and electrical energy costs \$0.10/kWh, what is the cost per year for electrical power to operate the pump, assuming it operates 365 days per year? The pump's life expectancy is 30 years, and interest is 8 percent. What is the equivalent capital recovery cost for this electrical energy?

Q, gal/min	140	152	188	224	260	280	308
Efficiency	0.58	0.60	0.65	0.70	0.73	0.75	0.78

Q, gal/min	352	400	428	456	480	505	534
Efficiency	0.80	0.78	0.75	0.73	0.70	0.65	0.60

Q, gal/min	140	260	340	420	540
h_p, ft	38.5	35.5	30.5	24.5	12.0

CHAPTER 7

INTRODUCTION TO TRANSIENT FLOW

7.1 CAUSES OF TRANSIENTS

To this point we have emphasized steady flows, flows that do not change with time at any location in the pipeline system. In this brief chapter we will introduce two general categories of unsteady flow that we call *transient flow*. All transient flows are transitions, of long or short duration, from one steady flow state to another. Either of these end states may be the rest state. Each transient flow is a response of the fluid to some change in the hydraulic facilities that control and convey the fluid, or in the surrounding environment, that influences the flow.

The first type of transient, which we will refer to as *quasi-steady flow*, is characterized by the absence of inertial or elastic effects on the flow behavior. In such a flow the variation of discharges and pressures with time is gradual, and over short time intervals the flow appears to be steady. Typical examples are the drawdown of a reservoir, the draining of a large tank, or the variation in demand in a water distribution system over a 24-hour period. This type of transient was considered briefly in Chapter 6 and will be examined in more detail in Section 7.2.

The second kind of transient is known as *true transient flow*, in which the effect of the fluid inertia and/or the elasticity of the fluid and pipe is an essential factor in the flow behavior and must be considered. If inertial effects are significant but pipe and fluid compressibility effects are relatively minor or negligible, then we have a true transient flow which we will refer to as a *rigid-column flow*. If in addition we must retain the elastic effects of the fluid and pipe in order to obtain an accurate characterization of the transient, we will call this a *water hammer* condition. The distinction between rigid-column flow and water hammer is not easily categorized and depends, in a general way, on how rapidly events change in a system. For example, the oscillation of the water level in the surge chamber of a hydroelectric facility can be analyzed accurately as a rigid-column flow. In this case inertial effects must be considered, but elastic or compressibility effects clearly are minor. On the other hand, the sudden closure of a valve in a pipeline is a water hammer situation; to simulate accurately the resulting behavior would require the inclusion of the elasticity of both the pipe and the liquid in the analysis. When the valve is closed more slowly, however, uncertainty arises. If the closure time is sufficiently long, then a rigid-column flow analysis may represent the physics of the problem well and produce good results. If the analyst is in doubt, then a water hammer analysis should be used because it is a more complete and general characterization of the flow. The groundwork for the study of true transients will be laid in Section 7.3 where both rigid-column flow and water hammer will receive attention. The study of water hammer problems will build on this foundation with extensive coverage in Chapters 8 through 13. Chapter 12 will treat both rigid-column flow and water hammer analyses in pipe networks.

7.2 QUASI-STEADY FLOW

We begin by considering a large tank, or even a small reservoir, that is full of water. By large we mean that the depth of the water from the base to the top of the tank is large, and the area of the water surface at any particular level within the tank is also large in comparison with the dimensions of the discharge opening. We wish to drain the water from the tank, a task that can be accomplished in many ways. At one extreme the tank could be emptied by attaching to the base of the tank at point A a long pipe of small but constant cross-sectional area or diameter, with a control valve at the downstream end of the pipe at point B, as shown in Fig. 7.1. Almost irrespective of how fast the valve is opened at B, the fluid will drain relatively slowly from the tank if the tank dimensions are sufficiently larger than the pipe cross-sectional area. At any instant there will be almost no perceptible motion in the tank itself, and there will be only a gradual temporal acceleration (positive or negative) of the fluid in the pipe; thus inertial effects are insignificant. A very small region of local convective acceleration will be found at the pipe entrance. At any instant the flow processes largely appear to be no different than a truly steady flow. After some time has elapsed, it will be found that the water level in the tank is indeed dropping, and the tank will eventually be empty. This flow is a good example of a quasi-steady flow.

Figure 7.1 The draining of a large tank via a quasi-steady flow process.

Turning briefly to Fig. 7.2, which shows the same tank with a drain line of much different dimensions, it is immediately apparent that the tank will now empty very quickly, with all the fluid undergoing a significant acceleration during the process. This particular illustration is a deliberate caricature to emphasize the role that fluid acceleration plays in fluid transients. In true fluid transients at least one of two kinds of fluid acceleration, temporal or convective, will be a significant factor in any energy principle that is utilized in an analysis.

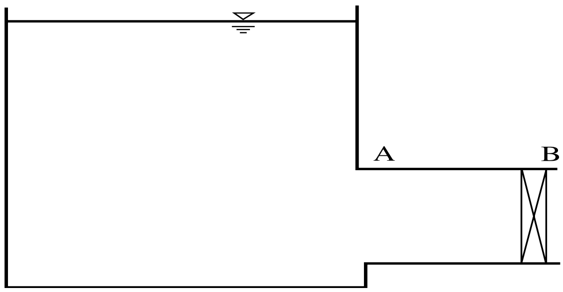


Figure 7.2 A change in dimensions leads to a flow which is not quasi-steady.

We turn now to a quasi-steady analysis of the flow from a generalization of the tank or reservoir in Fig. 7.1 to allow the cross-sectional area A of the tank to vary with elevation, as Fig. 7.3 depicts. A pipe with properties described by the Darcy friction factor f, length L, diameter D, and cross-sectional flow area A_p conveys a discharge Q

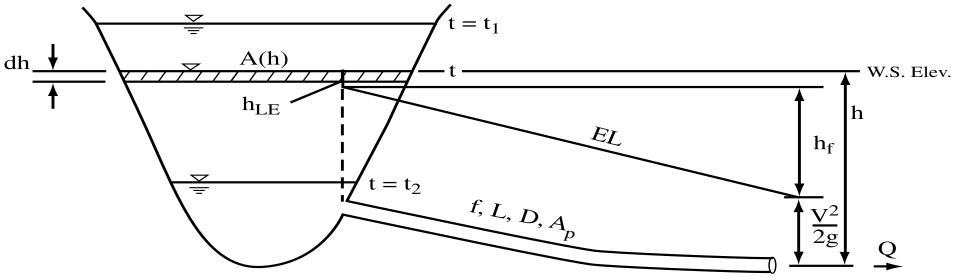


Figure 7.3 Quasi-steady flow from a tank of arbitrary cross-section.

from the tank. In the figure the EL is drawn for the time instant t. In passing from the tank to the atmosphere through the pipe, the fluid undergoes a local entrance loss h_{LE}, a Darcy pipe friction loss h_f, and exits with a mean velocity head $V^2/2g$. The sum of these terms is the instantaneous head h recorded in Eq. 7.1. The loss coefficient K_E for

$$h = \frac{V^2}{2g} + f\frac{L}{D}\frac{V^2}{2g} + K_E\frac{V^2}{2g} = \left[1 + K_E + f\frac{L}{D}\right]\frac{V^2}{2g} \tag{7.1}$$

the entrance loss is representative of any local loss, or sum of local losses, encountered by the flow between the tank and the exit; all such losses are treated identically. From Eq. 7.1 the exit velocity is

$$V = \frac{\sqrt{2gh}}{\left[1 + K_E + f\dfrac{L}{D}\right]^{1/2}} \tag{7.2}$$

and the discharge is

$$Q = VA_p = \frac{A_p\sqrt{2gh}}{\left[1 + K_E + f\dfrac{L}{D}\right]^{1/2}} \tag{7.3}$$

If we apply the continuity principle between the tank water surface and the pipe exit, we equate the fluid volume that exits from the pipe over the small time interval dt to the amount of fluid that is removed from the tank over that interval, and

$$Qdt = A(-dh) \tag{7.4}$$

The minus sign is needed because the quantity on the left side is intrinsically positive, but dh is itself negative as the water surface elevation drops with time. Thus we find the time interval $t_2 - t_1$ for the water surface elevation to change from h_1 to h_2 is

$$t_2 - t_1 = \int_{t_1}^{t_2} dt = -\int_{h_1}^{h_2} \frac{Adh}{Q} \tag{7.5}$$

in which the area $A = A(h)$ is in general a function of the tank configuration, and the discharge $Q = Q(h)$ from the steady-flow work-energy principle, Eqs. 7.1-7.3. One common additional notational simplification is to let

$$\frac{1}{C} = \left[1 + K_E + f\frac{L}{D}\right]^{1/2} \tag{7.6}$$

so the discharge can be written in the form of the standard orifice equation

$$Q = CA_p\sqrt{2gh} \tag{7.7}$$

Now we apply these equations to determine the time that is required to drain partially a tank of constant cross-sectional area $A = A_o$. This situation includes the common case of a cylindrical tank of fixed diameter with a vertical centerline, and it also includes tanks having square and other cross-sectional shapes. In this case the time to drain the tank from level h_1 to level h_2 is

$$\Delta t = t_2 - t_1 = -\int_{h_1}^{h_2} \frac{Adh}{CA_p\sqrt{2gh}} = \frac{-A_o}{CA_p\sqrt{2g}}\int_{h_1}^{h_2} h^{-1/2}dh = \frac{-2A_o}{CA_p\sqrt{2g}}\left[\sqrt{h_2} - \sqrt{h_1}\right] \tag{7.8}$$

in which the removal of C from within the integral is only permissible when f is constant. If the top and bottom of this fraction are multiplied by the common factor $\left[\sqrt{h_2} + \sqrt{h_1}\right]$, an interesting practical interpretation of this result is obtained:

$$\Delta t = \frac{A_o(h_1 - h_2)}{\frac{1}{2}CA_p\sqrt{2g}\left[\sqrt{h_1} + \sqrt{h_2}\right]} = \frac{Volume}{Average\ Q} \tag{7.9}$$

In words, the elapsed time is the ratio of the tank volume that is emptied to the average of the discharges that occur at the beginning and end of the time period, a result that can aid computations and is intuitively appealing. For this result to be valid, however, the cross-sectional area and also the friction factor that is a part of C must remain constant throughout the draining process.

If either of the foregoing restrictions does not hold, the integral in Eqs. 7.5 and 7.8 will not simplify as it did in Eq. 7.8. For example, if the cylindrical tank is laid on its side, then A(h) no longer is constant. It is then possible (but not very practical) to evaluate the resulting expression as an elliptic integral (Byrd and Friedman, 1971), but it is normally more convenient just to evaluate Eq. 7.5 by use of some numerical integration procedure; the Trapezoidal rule or the more accurate Simpson's rule (Press et al., 1992) are just two of many possibilities. Closed-form solutions are also known to exist for certain area variations A(h) with a vertical centerline, specifically the cone, pyramid and paraboloid, but the form of these solutions is algebraically more complex and of limited utility.

The flow defined in Fig. 7.3 can be made more general by allowing a nonzero constant inflow Q_o at the top of the tank. We will again write the outflow from the pipe in the form of Eq. 7.7. At first glance there appear to be two inflow cases, one with $Q_o > Q$ and the water surface in the tank rises, and the other with $Q_o < Q$ and the water surface falls. Such turns out not to be the case, for an individual consideration of each case leads to the restatement of Eq. 7.4 for both possibilities as

$$Adh = (Q_o - Q)dt \tag{7.10}$$

If we again assume that $A = A_o$ and f are constants, then Eqs. 7.7 and 7.10 lead to

$$dt = \frac{A}{Ca\sqrt{2g}} \cdot \frac{dh}{\dfrac{Q_o}{Ca\sqrt{2g}} - \sqrt{h}} \tag{7.11}$$

With integration between the same limits as in Eq. 7.8, we obtain

$$\Delta t = \frac{2A}{\left(Ca\sqrt{2g}\right)^2}\left\{Q_1 - Q_2 - Q_o \, ln\left[\frac{Q_o - Q_2}{Q_o - Q_1}\right]\right\} \tag{7.12}$$

after some care in integration and several lines of algebra. This result, however, is only valid if Q_o is outside the discharge interval (Q_2, Q_1); otherwise Eq. 7.12 will lead to the logarithm of a negative number. The cause of this behavior is not difficult to understand. During the outflow process the discharge Q takes on all values between Q_1 and Q_2. If Q_o were one of these intermediate values, then an equilibrium between inflow and outflow in the tank would occur at that discharge, the unbalanced driving force for the transient would cease, and the process would not continue on to state 2. Moreover, if the inflow were to match either Q_1 or Q_2, then Eq. 7.12 predicts that the time interval that is required to reach the end state is infinite (i.e., a steady equilibrium is never quite reached, according to this representation of the flow).

One additional generalization that can be useful is to allow the inflow to be $Q_o(t)$, a time-varying inflow. A re-organization of Eq. 7.10 yields

$$\frac{dh}{dt} = \frac{Q_o(t) - Q(h)}{A(h)} = F(t, h) \tag{7.13}$$

in which $F(t, h)$ is simply a shorthand, functional representation of the formula that precedes it. Only a little effort is needed to convince oneself that this equation can not be integrated directly as a quadrature. Press et al. (1992) present a chapter on various alternatives in integrating ordinary differential equations, and others have written entire books; Appendix A on numerical methods presents the fourth-order Runge-Kutta formula (Section A.4.2) as one reliable way to solve this kind of problem. The formula in the appendix is described in terms of the variables $y(x)$ which replace the variables $h(t)$ here.

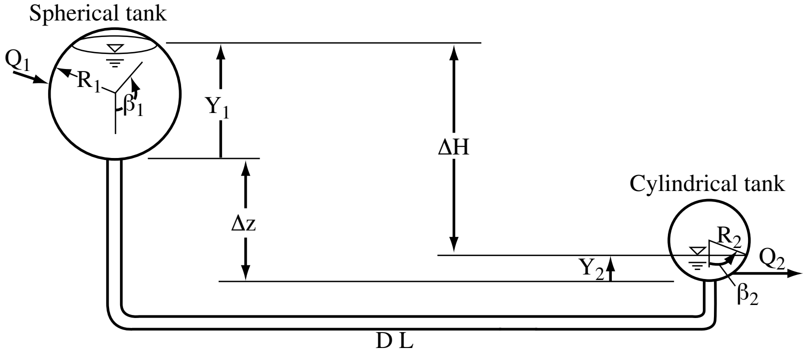
The concept of quasi-steady flow can be applied to a variety of system configurations, including some which are much more extensive than the cases discussed here, so long as it is correct to assume that no large accelerations are present in the transient. In fact, the extended-time simulations in Chapter 6 to determine long-term variations in network demand are quasi-steady flow applications. In such cases the time dependency will only enter the problem through the mass conservation statement.

Example Problem 7.1

A spherical tank of internal radius $R_1 = 20$ ft supplies water to a horizontal cylindrical tank of internal radius $R_2 = 15$ ft and length $L = 20$ ft through an 8-in-diameter pipe that is 500 ft long ($e = 0.004$ in and $\nu = 1.217\text{x}10^{-5}$ ft^2/s). The base of the cylindrical tank is 25 ft lower than the bottom of the spherical tank. A constant inflow $Q_1 = 0.5$ ft^3/s enters the spherical tank, which has a small opening to admit atmospheric pressure (14.7 lb/in^2 absolute) to the top of the tank. The cylindrical tank is closed; at time $t = 0$ the pressure of the air over the water is 13 lb/in^2. In this problem assume air behaves isothermally with a temperature of 60^{o}F. The external discharge Q_2 leaving the cylindrical tank is described by data in the following table, which should be converted into a continuous function by using a cubic spline function.

Time, s	0	500	1000	1600	2000	2400	3000	3600
Q_2, ft^3/s	0.00	3.27	3.85	4.68	5.00	4.80	4.00	0.00

If at time $t = 0$ the water depth in the spherical tank is $Y_{10} = 30$ ft and in the cylindrical tank is $Y_{20} = 8$ ft, determine the discharge between the tanks and the water depths and volumes in each tank over one hour (3600 s) in increments of 30 seconds.



First we must establish some relationships between the volume and depth in each tank. In the spherical tank the differential volume element $d\forall$ is a thin circular slice which can be written as $d\forall = \pi r^2 dY = \pi R^2 \sin^2 \beta_1 \, dY$ with $\cos\beta_1 = 1 - Y/R$, from which we find $dY = R \sin\beta_1 \, d\beta_1$, and the differential volume becomes

$$d\forall = \pi R^3 \sin^3 \beta_1 \, d\beta_1$$

If this expression is integrated over the range of β_1 from 0 to π, we obtain the entire spherical volume $\forall = 4\pi R^3/3$. Partly full volumes can than be expressed as a function of β_1 by integrating from 0 to β_1 to obtain

$$\forall = (\pi R^3/3)[2 - \cos\beta_1 (\sin^2\beta_1 + 2)]$$

With the aid of the identity $\sin^2\beta + \cos^2\beta = 1$ we find the relation $\beta_1(Y)$ between angle and depth is $\sin^2\beta_1 = 2(Y/R) - (Y/R)^2$, which allows us to write the volume of water in the spherical tank directly as a function of depth:

$$\forall = (\pi R^3/3)[3(Y/R)^2 - (Y/R)^3] = \pi Y^2 (R - Y/3)$$

In a similar way the volume as a function of the angle β_2 can be shown to be

$$\forall = LR^2 (\beta_2 - \cos\beta_2 \sin\beta_2)$$

with L being the length of the tank and $Y_2 = R_2(1 - \cos \beta_2)$.

Since $d\forall/dt$ represents the net discharge from a tank, the following two ODEs each describe the rate of change of water surface elevation in a tank:

$$\frac{dY_1}{dt} = \frac{Q_1 - Q}{\pi Y_1 \left(2R_1 - Y_1\right)} \qquad \frac{dY_2}{dt} = \frac{Q - Q_2}{2LR_2^2\left(1 - \cos^2 \beta_2\right)}$$

Here Q_1 and Q_2 are respectively the prescribed inflow and outflow from tanks 1 and 2. The discharge Q in the pipe must also satisfy the hydraulic equation

$$F = \left(\frac{fL}{D} + \sum K_L\right)\frac{Q|Q|}{2gA^2} - Y_1 + \Delta z + Y_2 + \frac{M p_o}{\gamma \rho_o V_{air}} = 0$$

The last term is the pressure head created by the air pressure above the water surface in the tank (more on this topic can be found in Secs. 12.5 and 13.2), in which M is the air mass in the tank, γ is the specific weight of water, p_o is the initial absolute air pressure, and ρ_o is the corresponding air density, found from the perfect gas law.

Program SHPTANK solves this problem. To review the details of its structure, the reader should obtain a listing of it from the CD. Principally it calls two subroutines, SPLINESU to accomplish the cubic spline interpolation, and RUKUST to solve the two ODEs simultaneously. Within subroutine SLOPE that supplies the two derivatives dY_1/dt and dY_2/dt to RUKUST we will find that the hydraulic equation is solved there for the values of Y_1 and Y_2 by use of the Newton method. The input file to solve this problem is

```
20 15 20 30 8 500 0.6666667 0.00033333 1.217E-5 1.5 30 120 32.2 25 420
3556 0.5 8 0 0 500 3.27 1000 3.85 1600 4.68 2000 5 2400 4.8 3000 4 3600 0
```

A portion of the solution is listed in the following table:

Time sec	Y_1 ft	β_2 radians	Y_2 ft	$\not{V}_1$ ft^3	$\not{V}_2$ ft^3	Q ft^3/s
30	29.91	1.086	8.01	28186.7	3032.6	3.414
60	29.82	1.088	8.03	28099.5	3042.2	3.401
90	29.73	1.089	8.06	28012.7	3055.0	3.386
.						
.						
.						
1530	26.18	1.048	7.51	24274.7	2771.1	3.261
1560	26.11	1.045	7.48	24191.8	2752.3	3.266
1590	26.04	1.043	7.44	24108.7	2734.4	3.271
1620	25.96	1.037	7.37	24025.5	2697.0	3.286
1650	25.89	1.030	7.27	23941.5	2646.0	3.309
.						
.						
.						
3480	21.23	0.908	5.77	18294.4	1902.3	3.334
3510	21.16	0.907	5.76	18209.5	1896.6	3.332
3540	21.09	0.905	5.73	18124.5	1886.4	3.332
3570	21.02	0.902	5.70	18039.5	1872.0	3.334
3600	20.96	0.899	5.67	17954.5	1853.9	3.338

* * *

7.3 TRUE TRANSIENTS

The study of true transient flows must include fluid inertia and may also include, in addition, the elasticity or compressibility of the fluid and the conduit. The analysis of transient flows in either case requires the application of Newton's second law which leads to the Euler equation. In Section 7.3.1 the Euler equation is developed; in Section 7.3.2 it is employed to study several rigid-column flow problems where the fluid inertia must be considered but where elasticity is unimportant and can be omitted. In Section 7.3.3 we briefly investigate the effects of elasticity to demonstrate this additional factor requires a different approach to the solution of such problems.

7.3.1. THE EULER EQUATION

The Euler equation is derived by applying Newton's second law to a small cylindrical control volume of fluid at the pipe centerline, as shown in Fig. 7.4. The resulting equation will apply to one-dimensional flow along the pipeline when we disregard variations in fluid or flow properties across the cross section. Further, the equation will apply to flows of both constant and variable density, so it is valid for both rigid-column and water hammer flows.

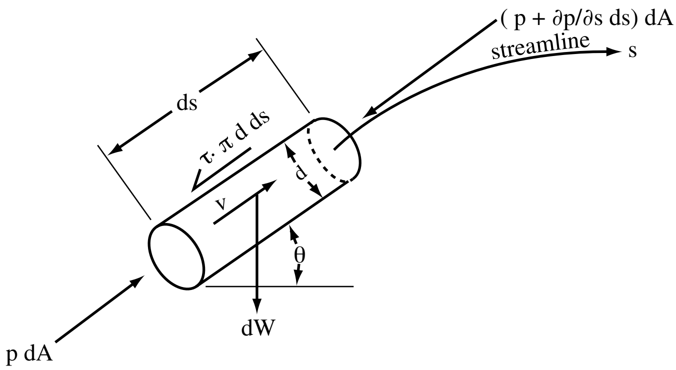


Figure 7.4 A cylindrical fluid element with all forces shown.

Along the streamline direction s, Newton's second law gives

$$\sum F_s = ma_s = m\frac{dv}{dt} \tag{7.14}$$

where m is the fluid mass in the cylindrical fluid parcel. The term dv/dt is in general the total or substantial derivative of the fluid velocity. Substituting the applied forces into this equation and writing the mass in terms of density and volume results in

$$pdA - \left(p + \frac{\partial p}{\partial s}ds\right)dA - dW\sin\theta - \tau\,\pi d(ds) = \frac{dW}{g}\frac{dv}{dt} \tag{7.15}$$

If we divide by dW to produce a non-dimensional equation that is written per unit weight, and the local pipe slope is expressed in terms of distance and elevation along the pipe, we arrive at the one-dimensional Euler equation

$$-\frac{1}{\gamma}\frac{\partial p}{\partial s} - \frac{\partial z}{\partial s} - \frac{4\tau}{\gamma d} = \frac{1}{g}\frac{dv}{dt} \tag{7.16}$$

If we now expand the cross-sectional area of the parcel to fill the pipe cross section and introduce the average velocity V, we obtain a more useful equation:

$$-\frac{1}{\gamma}\frac{\partial p}{\partial s} - \frac{\partial z}{\partial s} - \frac{4\tau_o}{\gamma D} = \frac{1}{g}\frac{dV}{dt} \tag{7.17}$$

Here τ_o is the shear stress at the wall. Because the wall shear stress is usually not of primary interest and because we will be working almost entirely with cylindrical pipes, we prefer to express the shear stress in terms of the Darcy-Weisbach friction factor f as

$$\tau_o = \frac{1}{8} f \rho V|V| \tag{7.18}$$

The form of the velocity representation in this equation is desirable because it preserves the proper direction of the shear force whenever the flow reverses direction.

With the substitution of Eq. 7.18 into Eq. 7.17 and the assumption that the local elevation of the pipe can be described solely as a function of location s, we obtain the Euler equation of motion

$$\frac{1}{g}\frac{dV}{dt} + \frac{1}{\gamma}\frac{\partial p}{\partial s} + \frac{dz}{ds} + \frac{f}{D}\frac{V|V|}{2g} = 0 \tag{7.19}$$

If we also introduce the piezometric head H via $p = \rho g(H - z)$ and expand the total derivative in the form

$$\frac{dV}{dt} = \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial s} \tag{7.20}$$

then the Euler equation can be written in the alternative form

$$\frac{1}{g}\frac{\partial V}{\partial t} + \frac{\partial}{\partial s}\left(H + \frac{V^2}{2g}\right) + \frac{f}{D}\frac{V|V|}{2g} = 0 \tag{7.21}$$

since $V\partial V/\partial s = \partial(V^2/2)/\partial s$. The sum of piezometric head and velocity head that appears in the middle term is the sum that is usually displayed in diagrams as the Energy Line.

7.3.2. RIGID-COLUMN FLOW IN CONSTANT-DIAMETER PIPES

The neglect of the elasticity of the pipe and fluid in a pipe of constant diameter forces any change in velocity, in theory, to occur instantaneously throughout the entire pipe. In addition, the steady form of the mass conservation equation applies throughout the pipe so that the velocity everywhere in the pipe is the same at any given time. This section will only examine such flow in single pipes in order to emphasize basic principles; similar transient flows in pipe networks are studied in Chapter 12.

There are relatively few closed-form solutions of Eqs. 7.19 or 7.21, even with these restrictions. One of these solutions describes the development or establishment of flow from rest through a horizontal pipe from a constant-head reservoir, as shown in Fig. 7.5.

In the simpler version of the flow establishment problem we assume that the fluid is inviscid, i.e. without friction, so the last term in Eq. 7.21 is dropped and only the inertia of the fluid is important. First we must choose the limits of integration, with respect to the distance s, to begin the solution of Eq. 7.21; we select section 1 at the upstream end of the pipe (even though the pressure is changing rapidly and the velocity is indeed nonuniform over this section; see Street et al., 1996, pp. 362, 367) and section 2 to be

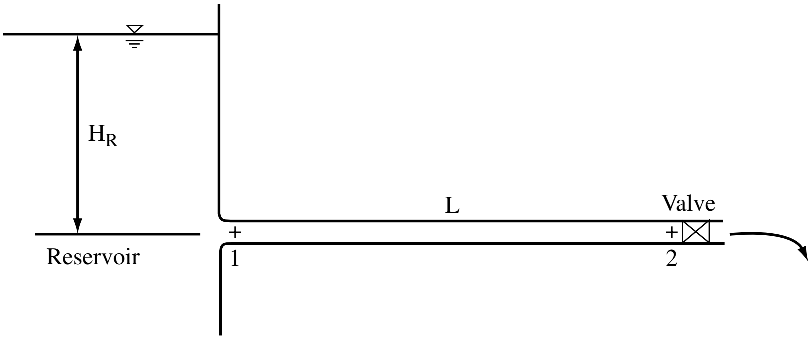


Figure 7.5 Constant-head reservoir with valve at downstream end of horizontal pipe.

just upstream from the valve. Now we formally integrate with respect to the distance s from the reservoir, point 1, to the valve, point 2:

$$\frac{1}{g}\int_{1-2}\frac{\partial V}{\partial t}\,ds = -\int_{1-2}\frac{\partial}{\partial s}\left(H+\frac{V^2}{2g}\right)ds \tag{7.22}$$

Since continuity assures us that all of the fluid in the pipe must undergo the same acceleration, this leads to

$$\frac{L}{g}\frac{\partial V}{\partial t} = -\left(H+\frac{V^2}{2g}\right)_2 + \left(H+\frac{V^2}{2g}\right)_1 \tag{7.23}$$

From Eq. 7.23 we see that the acceleration term is the difference in fluid energy per unit weight, which is also the difference in energy grade line values, between the two end points of the integration. The valve is instantaneously opened fully at $t = 0$, and flow develops thereafter. When $t = 0^+$ at section 2, H_2 drops to zero and remains so, and $V_2 = V$ will grow with time from 0 to the steady state velocity V_0. If a separate Bernoulli equation is now written between the reservoir and section 1, assuming no energy loss, then the sum of the two terms at section 1 is simply the reservoir energy per unit weight or head H_R, a constant. Thus

$$\frac{L}{g}\frac{dV}{dt} = H_R - \frac{V^2}{2g} \tag{7.24}$$

in which the remaining derivative is a function of time only, an ordinary derivative. After the flow has become established, i.e. steady, the left term becomes zero, and the steady velocity V_0 can be found from the remainder of Eq. 7.24 to be

$$V_0 = \left(2gH_R\right)^{1/2} \tag{7.25}$$

To determine the discharge behavior as a function of time during the establishment time interval, we solve Eq. 7.24 for dt and integrate the resulting expression as

$$\int_0^t dt = \frac{L}{g}\int_0^V \frac{dV}{H_R - \frac{V^2}{2g}} \tag{7.26}$$

or

$$t = 2L\int_0^V \frac{dV}{V_0^2 - V^2} \tag{7.27}$$

We can either use partial fractions or a table of integrals to evaluate this expression. Upon some additional algebra, we find the final result as

$$t = \frac{L}{V_0} ln\left[\frac{V_0 + V}{V_0 - V}\right] \tag{7.28}$$

From Eq. 7.28 we learn that, strictly speaking, the flow-establishment time is infinite, since the logarithm does not remain bounded as V approaches V_0. Since this result is not a practical one, it is usual to declare the flow to be steady when $V = 0.99V_0$.

It is more realistic to consider the establishment of flow in the presence of fluid friction and local losses, so we now re-examine this problem. We assume for simplicity that the friction factor remains constant. In this case the integration of Eq. 7.21 with the inclusion of pipe friction yields

$$\frac{L}{g}\frac{\partial V}{\partial t} = -\left(H + \frac{V^2}{2g}\right)_2 + \left(H + \frac{V^2}{2g}\right)_1 - \frac{fL}{D}\frac{V^2}{2g} \tag{7.29}$$

so long as V is always positive. The result is similar to Eq. 7.24, but the evaluation of the energy at sections 1 and 2 now changes to account for the local losses at the entrance and through the valve, respectively. Writing an energy equation between section 1 and the reservoir now produces H_R - $K_E V^2/2g$ as the sum of terms at section 1; in other words, the energy level in the reservoir is now reduced by the local head loss of the entrance as the flow moves to section 1. Between the downstream exit and section 2 a local energy loss occurs at the valve, causing the head to be $H_2 = K_V V^2/2g$ above the datum. With these substitutions and some algebraic rearrangement, we find

$$\frac{L}{g}\frac{dV}{dt} = H_R - \left(1 + K_E + K_V + \frac{fL}{D}\right)\frac{V^2}{2g} = H_R - C_1\frac{V^2}{2g} \tag{7.30}$$

defining C_1 in Eq. 7.30 to shorten subsequent algebra. The steady-state velocity V_0 is in this case found to be

$$V_0 = \left(2gH_R / C_1\right)^{1/2} \tag{7.31}$$

The integration of Eq. 7.30 closely follows the procedure for integrating Eqs. 7.26 and 7.27, but with $2L/C_1$ replacing $2L$ in Eq. 7.27, assuming C_l is constant. The solution is

$$t = \frac{L}{V_0 C_1} ln\left[\frac{V_0 + V}{V_0 - V}\right] \tag{7.32}$$

We see that the time to reach steady flow remains infinite, but the steady-state velocity itself has been reduced by the effects of pipe friction and the local losses. We also see that the deletion of these real-fluid effects leads to $C_1 = 1$ and the previous solution.

Example Problem 7.2

A horizontal pipe 24 inches in diameter and 10,000 ft long leaves a reservoir 100 ft below its surface and ends at a valve. The steady-state friction factor is 0.018 and is assumed to remain constant during the acceleration process.

(a) If the valve is suddenly opened completely, what is the time that is required to attain 99% of the steady-state velocity? Neglect the frictional loss and local losses in this part.
(b) Solve the problem again, including pipe friction but omitting local losses.
(c) Solve the problem again, including pipe friction and using loss coefficients of 0.5 and 5.0 for the entrance and valve, respectively.
(d) Plot the results of (c) to show how the velocity approaches the steady state.
(e) Repeat (c) but allow f to vary, selecting e to produce $f = 0.018$ at steady state.

For part (a) we first use Eq. 7.25 to find the steady-state velocity:

$$V_0 = \left(2gH_R\right)^{1/2} = [2(32.2)(100)]^{1/2} = 80.2 \ ft/s$$

From Eq. 7.28 the time to reach 99% of this velocity is

$$t = \frac{L}{V_0} ln\left[\frac{V_0 + V}{V_0 - V}\right] = \frac{10,000}{80.2} ln\left[\frac{V_0 + 0.99V_0}{V_0 - 0.99V_0}\right] = \frac{10,000}{80.2} ln\left[199\right] = 660 \ s$$

For part (b) we begin by computing V_0 from Eqs. 7.30 and 7.31 with $K_E = K_V = 0$:

$$C_1 = 1 + K_E + K_V + \frac{fL}{D} = 1 + 0 + 0 + \frac{0.018(10,000)}{24/12} = 91.0$$

$$V_0 = \left[2(32.2)(100.0)/91.0\right]^{1/2} = 8.41 \ ft/s$$

From Eq. 7.32 we find

$$t = \frac{L}{V_0 C_1} ln\left[\frac{V_0 + V}{V_0 - V}\right] = \frac{10,000}{(8.41)(91.0)} ln(199) = 69.2 \ s$$

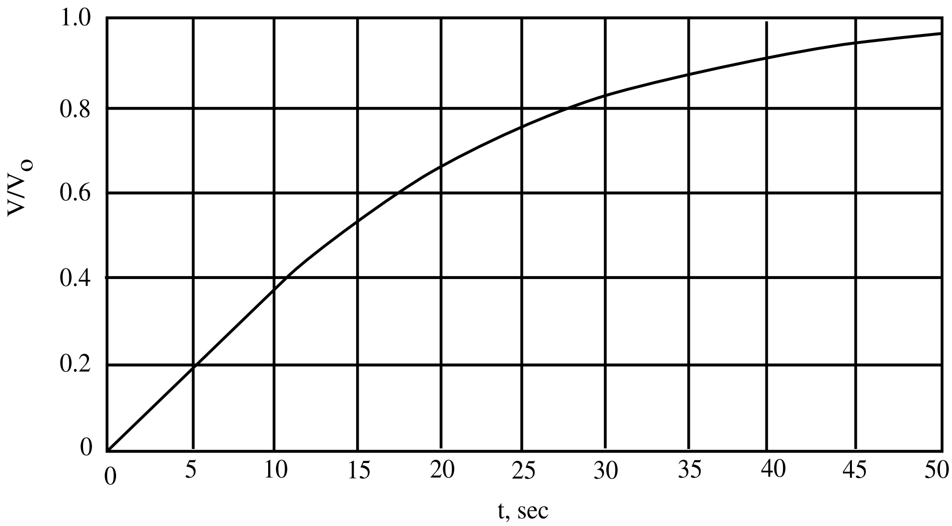
In part (c) we repeat the part (b) calculation with $K_E = 0.5$ and $K_V = 5.0$, leading to $C_1 = 96.5$, $V_0 = 8.17$ ft/s, and a time $t = 67.2$ s for the flow to become established.

A short computer program should be written to solve part (e) with f varying. To determine the correct pipe roughness e for the simulation, Eq. 7.30 with $dV/dt = 0$ is solved simultaneously with the Colebrook-White equation; the results are $e = 0.00129$ ft and $V_0 = 8.169$ ft/s. Then Eq. 7.30 is re-arranged to separate the variables, and a program is written to perform the numerical integration. A program to accomplish the integration is listed below; for the integration the program calls SIMPR, a subroutine that uses Simpson's rule (see Appendix A). The friction factor will be determined by equations in Table 2.2 with a few small changes: if $Re < 100$, then $f = 0.64$; the transitional Colebrook-White formula will be used whenever $Re > 2100$ rather than 4000. Some care must be taken to assure that the right side of Eq. 7.30 never becomes zero or negative;

hence the range of integration for V is from 0 to $0.99V_0 = 8.087$ ft/s. In this instance the time to steady flow is 66.4 s.

```
      EXTERNAL EQUAT
      COMMON SF
      SF=7.5
      CALL SIMPR(EQUAT, 0.0, 8.09, TIME, 1.0E-04, 30)
      WRITE(*,*)  ' Time = ', TIME
      END
C
      FUNCTION EQUAT(V)
      COMMON SF
      RE = 1.6433854E5*V
      IF (RE.LT.100.) THEN
      FR = 0.64
      ELSEIF (RE.LT.2100) THEN
      FR = 64.0/RE
      ELSE
    1 SF1 = SF
      SF = 1.14-2.0*ALOG10(6.3625E-4 + 9.35*SF1/RE)
      IF(ABS(SF-SF1) .GT. 1.0E-06) GO TO 1
      FR = 1.0/SF/SF
      ENDIF
      DEM = 100.0 - (6.5 + 5000.0*FR)*V**2/64.4
      IF(DEM .LT. 0.01) THEN
      EQUAT = 31055.9
      ELSE
      EQUAT = 310.559/DEM
      ENDIF
      RETURN
      END
```

The plot of the results from part (c), requested as part (d), shows the velocity increases rather rapidly until it reaches 80 to 90% of the steady-state value. By that time the acceleration has decreased noticeably, and steady state is approached asymptotically. As part (e) shows, only a short program is needed to add more accuracy to the computation, but in this particular example the difference in time to steady state is only four percent.



From these computations we see that pipe friction is the dominant factor in the flow establishment process when the pipe is sufficiently long. Overlooking this factor would be a severe error, part (a), but for very long pipes the effect of local losses is truly a minor effect, with the the steady-state velocities and flow-establishment times only differing by a few percent in this problem.

* * *

The simulation of flow shutdown by use of Eq. 7.30 for the physical problem depicted in Fig. 7.5 is actually a more difficult problem than the startup problem. The principal difficulty is in representing correctly the loss coefficient K_V for the valve, because it is incorrect to model this coefficient as a constant in this problem. Instead we now have a continually increasing head loss, and loss coefficient, across the valve which with pipe friction (and, to a minor extent, local losses) causes the flow to decelerate and eventually stop. We assume that all loss coefficients under unsteady-flow conditions are unchanged from steady-flow conditions at the same velocity. The head loss in the system will be the pipe friction loss described by the Darcy-Weisbach equation, the local entrance loss, and the valve head loss, as the governing equation, Eq. 7.30, shows. Since K_V varies with the valve setting, which in turn changes in some predetermined manner with time, a closed-form solution of this ODE is not possible. Thus we must solve this nonlinear equation by numerical methods.

Here we choose the fourth-order Runge-Kutta method, described in Appendix A.4.2, as the numerical solution technique for this problem. Equation 7.30 is rewritten in the form

$$\frac{L}{g}\frac{dV}{dt} = H_R - \left(1 + K_E + K_V + \frac{fL}{D}\right)\frac{V^2}{2g} = F(t, V) \tag{7.33}$$

Now the Runge-Kutta method can be applied directly, once the details of computing K_V and $F(t, V)$ are set. To complete the setup, we must know the valve operating schedule (percent open P vs. time) and the relation between K_V and percent open P.

If in addition we wish to know the maximum pressure head to occur (probably at the valve) as time progresses, we can insert the computed velocity in $h_L = K_V V^2/2g$ to find this head. However, one complicating factor occurs at the instant of closure; this loss coefficient becomes infinite as the velocity approaches zero, creating an indeterminate pressure head. Even under the best of circumstances, any numerical procedure will produce unreliable results at this point. Fortunately, the maximum pressure usually occurs somewhat before complete valve closure, so the numerical analysis will be terminated a fraction of a second before complete closure. Example Problem 7.3 presents the solution process for the reservoir-pipe system of Fig. 7.5.

Example Problem 7.3

The reservoir head on the pipeline in Fig. 7.5 is 60 ft. The 12 in-diameter line is 3000 ft long with an equivalent roughness $e = 0.012$ in. Since the valve has been fully open for a long time, the flow of water is steady.

(a) Calculate the steady-state velocity in the line assuming there is no loss at the valve. Then compute the maximum pressure in the line if the valve closes so that the rate of decrease in velocity is linear in time from its steady-state value to zero in 20 sec.

(b) Now assume the valve at the downstream end is a GA Industries 12-in globe valve whose loss characteristics are given in Appendix C.1 as a function of valve opening. Compute again the steady-state velocity with the valve fully open. Assuming the valve closes in 20 sec at a rate that is linear in time, find the maximum pressure in the line.

(c) Repeat part (b) but employ a cubic spline interpolation to represent the valve data.

To begin part (a) and find the steady-state velocity, we can apply Eq. 7.30 directly with $dV/dt = 0$ and $K_V = 0$:

$$H_R = \left(1 + K_E + K_V + \frac{fL}{D}\right)\frac{V_0^2}{2g}$$

$$60 = \left(1 + 0.5 + 0 + \frac{f(3000)}{1.0}\right)\frac{V_0^2}{64.4}$$

If we solve this equation with the Colebrook-White equation, we obtain $V_0 = 7.91$ ft/s and $f = 0.0201$. The linearly decreasing velocity creates a constant deceleration so that

$$\frac{L}{g}\frac{dV}{dt} = \frac{3000}{32.2}\frac{(-7.91)}{20} = -36.8\ ft$$

Now we can apply Eq. 7.29 from section 1 to 2. Since $(H + V^2/2g)_1 = H_R - K_E V^2/2g$ and $H_2 = z_2 + p_2/\gamma = 0.0 + p_2/\gamma$, we have

$$\frac{L}{g}\frac{dV}{dt} = -\left(\frac{p_2}{\gamma} + \frac{V^2}{2g}\right) + \left(H_R - K_E\frac{V^2}{2g}\right) - \frac{fL}{D}\frac{V^2}{2g}$$

$$\frac{L}{g}\frac{dV}{dt} = -\left(\frac{p_2}{\gamma}\right) + H_R - \left(1 + K_E + \frac{fL}{D}\right)\frac{V^2}{2g}$$

$$-36.8 = -\left(\frac{p_2}{\gamma}\right) + 60 - \left(1 + 0.5 + \frac{f(3000)}{1.0}\right)\frac{V^2}{64.4}$$

or

$$\frac{p_2}{\gamma} = 96.8 - (1.5 + 3000f)\frac{V^2}{64.4}$$

From this equation we see clearly that the pressure head at the valve increases as the velocity decreases, reaching a maximum at the instant of closure. We conclude that

$$\left(\frac{p_2}{\gamma}\right)_{max} = 96.8\ ft \qquad or \qquad (p_2)_{max} = 96.8\left(\frac{62.4}{144}\right) = 42.0\ lb/in^2$$

In part (b) we repeat the sequence of computations in part (a) but represent the hydraulic behavior of the valve more accurately. We start by again computing the steady-state velocity, but this time we include the actual valve loss in the computation. Following Appendix C.1, we write

$$K_L = K_V = 890\frac{D^4}{C_V^2}$$

and for the 12-inch globe valve we find in the second table of this appendix $C_V = 1750$, leading to

$$K_V = 890\frac{12^4}{1750^2} = 6.0$$

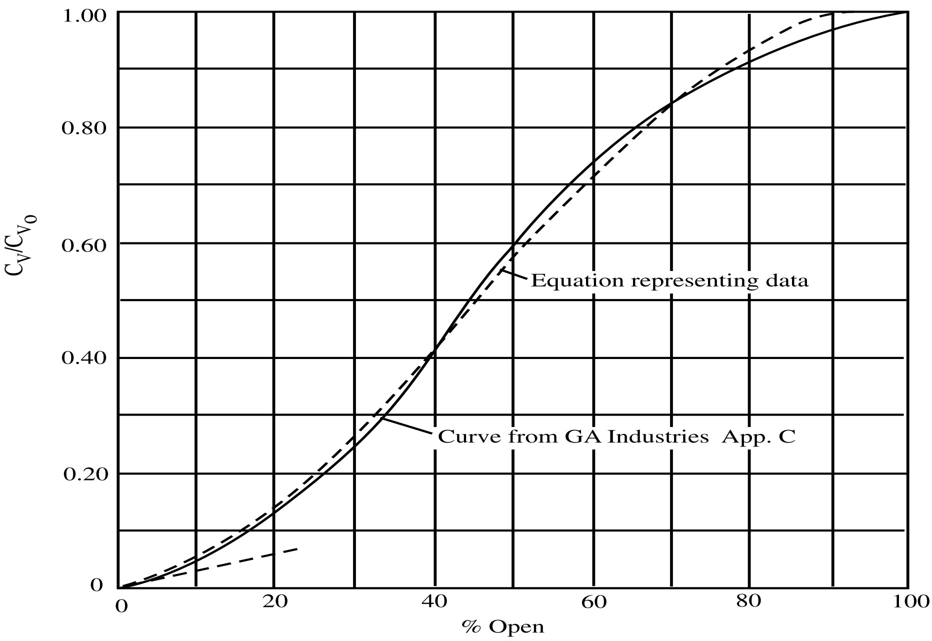
We return to the first two equations of the solution for part (a) and replace $K_V = 0$ with $K_V = 6.0$ there to obtain the steady velocity as $V_0 = 7.55$ ft/s and $f = 0.0201$.

Before we can apply the Runge-Kutta method, we must devise a way to determine the value of K_V as a function of time. The valve opening will be prescribed at any time by the closure schedule; in this case we assume a linear behavior. It only remains to find K_V for any given opening. Because the data for the GA Industries valve are given in terms of C_V, we will determine C_V at a given opening and convert to the corresponding K_V by using the formula from Appendix C.1. The data pairs of P = percent open vs. C_V/C_{V0} in the following table (C_{V0} is the fully open value) are read from Appendix Fig. C.1:

P	0	10	20	30	40	50	60	70	80	90	100
C_V/C_{V0}	0.00	0.055	0.13	0.26	0.42	0.60	0.74	0.83	0.92	0.97	1.00

From line two in this table we clearly see at the instant of valve closure that $C_V = 0.0$, with $K_V \to \infty$. We will avoid this problem by halting the numerical analysis two time increments before the valve closure is complete. If the maximum pressure head occurs before complete closure, little will be lost when the analysis is terminated then.

To develop an equation to fit the tabular data, we first replot the data. The fitting equation must capture the point of inflection that is seen in the plot. Hence we select a



cubic polynomial as the simplest function which will satisfy this requirement. The equation is

$$C_V = C_{V0}\left[aP^3 + bP^2 + cP + d\right]$$

where P is the measure of the valve position and a, b, c, d are the fitting coefficients.

The conditions we will impose on the polynomial to find the four coefficients are

$$\begin{array}{lll} C_V/C_{V0} & = 0 & \text{when } P = 0 \\ C_V/C_{V0} & = 1.0 & \text{when } P = 100 \\ \text{Slope} & = 0.003 & \text{when } P = 0 \\ C_V/C_{V0} & = 0.42 & \text{when } P = 40 \end{array}$$

The third condition seems at first glance to be an odd requirement. However, a valve only generates enough head loss to decelerate the flow significantly when it is very close to complete closure. Hence, it is important to represent the empirical curve as accurately as possible near the closure point. One good way to assure that the shape of the curve near closure is accurate is to measure the slope of the plot at closure and then specify this slope.

Application of these four conditions results in the following equation, which is also plotted on the graph as a dashed curve:

$$C_V = C_{V0}\left[-1.96\times10^{-6}P^3 + 2.66\times10^{-4}P^2 + 3.00\times10^{-3}P\right]$$

The creation of this equation to represent the flow coefficient over the full range of valve motion suggests that Eq. 7.33 could be integrated to yield a closed form solution. However, the variables in the equation do not separate. Further, if the valve closure schedule is more complex than the motion of this example, the relation between C_V and time will become progressively more complex. Hence we will proceed with a numerical solution as the most broadly applicable approach.

Program VALCLO1, to be found on the CD, generates a solution for this example problem for the specified conditions; in it the valve coefficient is modeled by the third-order polynomial. This data file will generate the solution:

```
EXAMPLE PROBLEM 7.3 RIGID COLUMN THEORY VALVE CLOSURE
&SPECS HR=60.,VZERO=7.57,D=12.,L=3000.,E=0.012,KE=0.5,TCLOSE=20.,
       DELT=1.0,A=-0.00000196,B=0.000266,C=0.00300,CVZERO=1750./
```

An increment $\Delta t = 0.10$ sec ought to produce accurate numerical results. To check this assumption, additional runs with $\Delta t =$ 0.50, 0.25, and 0.05 sec. were made, and $\Delta t =$ 0.10 sec does produce a solution that is independent of Δt. The solutions confirmed a maximum pressure head of 228 ft occurring about 18 sec after beginning the 20-sec valve closure. Some output from the computer analysis follows, using a print interval of 1.00 sec to conserve space:

```
              ***************
              * INPUT DATA  *
              ***************

          EXAMPLE PROBLEM 7.3 - RIGID COLUMN THEORY VALVE CLOSURE
                HR =    60.0 FT
             VZERO =    7.57 FT/S
                 D =   12.00 IN
                 L =  3000.0 FT
                 e =   0.012 IN
```

```
        KE =  0.50

    TCLOSE =   20.0 SEC
      DELT =  1.000 SEC

         A = - 0.196E-05
         B =   0.266E-03
         C =   0.300E-02
    CVZERO =  1750.0

        ***********
        * RESULTS *
        ***********

  TIME,SEC  V,FT/S  PRESSH,FT
  --------  ------  ---------
     0.00     7.57        5.4
     1.00     7.57        5.3
     2.00     7.56        5.4
     3.00     7.56        5.6
     4.00     7.55        6.1
     5.00     7.54        6.6
     6.00     7.52        7.5
     7.00     7.50        8.6
     8.00     7.46       10.2
     9.00     7.42       12.4
    10.00     7.35       15.6
    11.00     7.25       20.1
    12.00     7.11       26.8
    13.00     6.91       37.1
    14.00     6.60       53.3
    15.00     6.12       79.1
    16.00     5.37      118.9
    17.00     4.22      172.3
    18.00     2.62      215.0
    19.00     0.98      194.6
```

An alternate approach to the construction of the cubic-polynomial to represent the valve coefficient is to apply a spline fit to the coefficient data and otherwise proceed as before. Program VALCLO, also on the CD, implements this approach. The input and output data files for this alternative follow. The input file is

```
EXAMPLE PROBLEM 7.3 RIGID COLUMN THEORY VALVE CLOSURE
&SPECS HR=60.,VZERO=7.57,D=12.,L=3000.,E=0.012,KE=0.5,TCLOSE=20.,
       DELT=1.,CVZERO=1750./
0 0 10 0.055 20 0.13 30 0.26 40 0.42 50 0.6 60 0.74 70 0.83 80 0.92 90
0.97 100 1.0
```

The output file is

```
      **************
      * INPUT DATA  *
      **************

  EXAMPLE PROBLEM 7.3 - RIGID COLUMN THEORY VALVE CLOSURE

        HR =   60.0 FT
     VZERO =   7.57 FT/S
         D =  12.00 IN
```

```
         L = 3000.0 FT
         e =  0.012 IN
        KE =  0.50

    TCLOSE =   20.0 SEC
      DELT =  1.000 SEC

    CVZERO =  1750.0

        ***********
        * RESULTS *
        ***********

  TIME,SEC  V,FT/S  PRESSH,FT
  --------  ------  ---------
       .00    7.57        5.4
      1.00    7.57        5.5
      2.00    7.56        5.7
      3.00    7.55        5.9
      4.00    7.54        6.3
      5.00    7.53        6.9
      6.00    7.51        7.7
      7.00    7.49        8.5
      8.00    7.46        9.5
      9.00    7.42       11.2
     10.00    7.37       14.1
     11.00    7.28       19.0
     12.00    7.14       27.1
     13.00    6.92       39.6
     14.00    6.57       59.8
     15.00    5.99       93.8
     16.00    5.06      141.6
     17.00    3.77      175.0
     18.00    2.40      177.4
     19.00    1.14      169.8
```

The differences in pressures between the two solutions are clearly a consequence of the differences in the numerical representation of the valve coefficient toward the end of the closure schedule. As the valve approaches its closed position, the loss coefficient K_V, which varies inversely with C_V, increases very rapidly. The functional relationship chosen to represent C_V can have a significant effect on the value of K_V and thus on the pressure. Since the maximum value of the pressure is usually an important part of an analysis, it is imperative to try to define C_V as accurately as possible near closure. Valve closure data from manufacturers is generally provided in a table or graph, which works very well for steady-state analyses. But the precise nature of the change in C_V near closure, which is needed for an accurate analysis of unsteady flow behavior, is rarely available. As a consequence, practitioners must use their judgment and experience in modeling the valve closure and appraising in a conservative way the results of the analysis.

7.3.3. WATER HAMMER*

When velocities in a pipe system change so rapidly that the elastic properties of the pipe and liquid must be considered in an analysis, we have a hydraulic transient commonly known as water hammer. While this type of analysis is more complex than a rigid-column

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* Material in this section on pp. 295-299 is adapted from Elementary Fluid Mechanics, by R. L. Street, G. Z. Watters, and J. K. Vennard, Ed. 7, Copyright 1996 by John Wiley & Sons, Inc. Reprinted by permission.

analysis, it more accurately represents the actual behavior of the flow. Before we embark on an extensive investigation of this type of phenomenon in the next chapter, we will look at a simple water hammer situation. The problem will help us to see what happens in a pipe when velocities change rapidly, and it will introduce fundamental concepts that are important in understanding the phenomenon.

With the aid of the simple pipeline and valve that is attached to the reservoir in Fig. 7.6, we can now observe how water hammer waves evolve in time, according to our simplified equation set. We assume that steady flow occurs in the pipe at velocity V. The piezometric head everywhere in the pipe is H in the absence of friction. If the valve setting is changed in any way, a transient will be caused in the pipe, both upstream and downstream of the valve; we will concentrate only on the pipe section that is upstream.

Now assume we can completely close the valve rapidly, indeed instantaneously. At the valve the water velocity is suddenly forced to zero. As a consequence the head at the valve abruptly increases by an amount $\Delta H = aV/g$, as Chapter 8 will show. The amount of this increase is just sufficient to reduce the momentum of the moving water to zero.

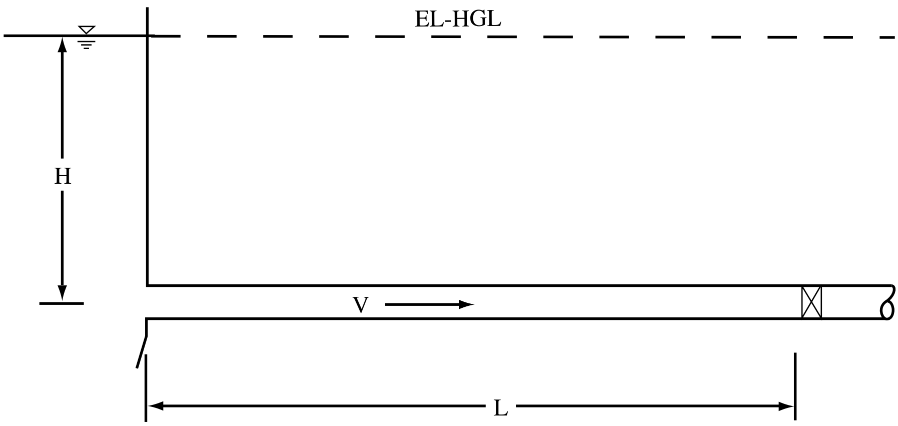


Figure 7.6 Steady flow from a reservoir in the absence of friction.

The increased head immediately creates two other changes at the valve; the pressure increase slightly enlarges the pipe and also increases the density of the fluid. The amount of the stretching of the pipe depends on the diameter and thickness of the pipe and on the compressibility of the pipe material and the liquid, but it normally changes by less than one-half percent. In Fig. 7.7 the amount of the deformation is exaggerated.

The rise in pressure head causes a sharp-fronted pressure wave to propagate upstream at speed a, the magnitude of which is a function of properties of the conduit and the fluid. This wave speed remains constant until the conduit properties or the fluid properties change. The wave front reaches the reservoir L/a seconds after valve closure. At that instant the velocity is zero throughout the pipe, the pressure head is everywhere $H + \Delta H$, the pipe is enlarged and the fluid is compressed.

Under these conditions the fluid in the pipe near the reservoir connection is locally not in equilibrium since the reservoir pressure head is only H. Hence fluid begins to flow toward the region of lower head (the reservoir) as the distended pipeline forces flow in that direction. In the absence of friction this left-ward velocity is equal in magnitude to the original steady velocity as it is driven by the same head increment ΔH; and the source of the liquid for this flow is the compressed liquid that is stored in the enlarged pipe cross section under the increased pressure head.

The process continues to evolve with time. At time $2L/a$ after the beginning, the pressure throughout the pipe has returned to its original value, but with the velocity reversed from its original direction. At this instant the store of compressed liquid is

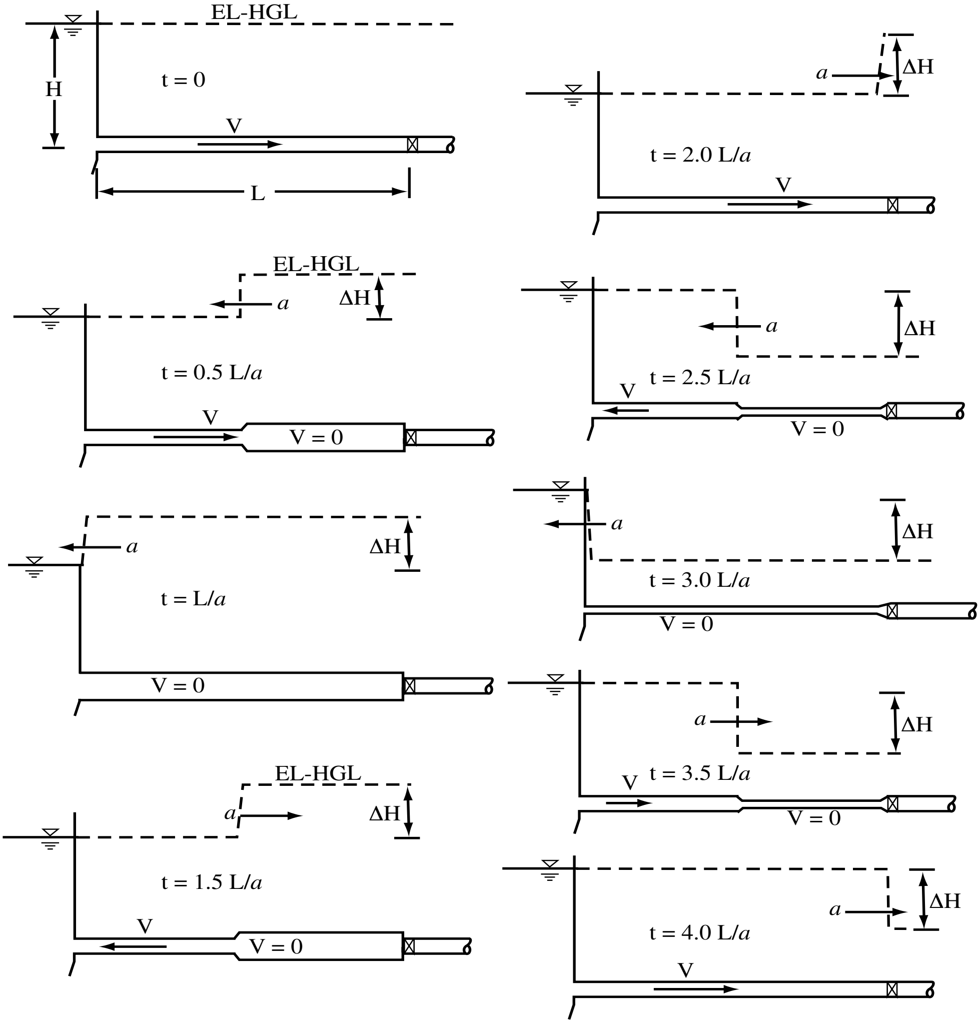


Figure 7.7 Evolution of a transient pressure wave in the pipe in Fig. 7.6.

exhausted, and the pressure wave appears to undergo a reflection. That is, the pressure head drops an amount ΔH below the original steady head, and this pressure drop and the closed valve cause the velocity behind the wave front to return to zero. Behind this negative wave the pipe cross section shrinks and the liquid expands.

By time $3L/a$ this negative wave has reached the reservoir, and the velocity is everywhere zero. However, the pressure head at the reservoir is again not in equilibrium with the reservoir head, so fluid is drawn from the reservoir into the pipe at velocity V. Behind the new, advancing wave the head is in equilibrium with the reservoir head.

At time $4L/a$ the wave has reached the valve; at this instant all variables have returned to the original steady state that existed before the valve was closed. This time interval that has just been described is one full cycle in a hydraulic transient that would, in the absence of friction, continue without abating. The simple fact that this sequence of events is unending, unless friction is present, points out the necessity of retaining the otherwise

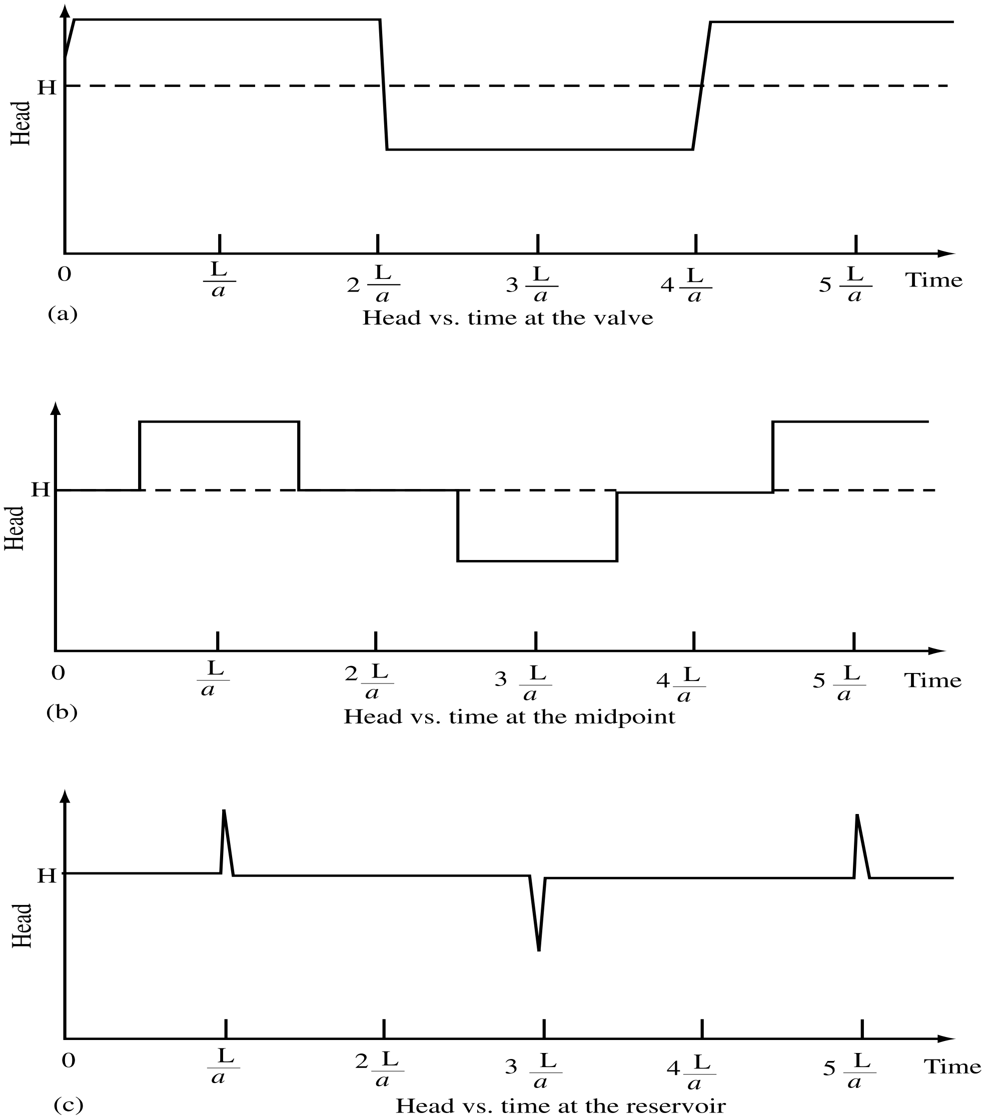


Figure 7.8 Head vs. time at three locations.

small friction term if we are to achieve realistic, practical results in our simulations of such events.

If the variation of piezometric head is plotted as a function of time for selected locations along the pipe, as is done in Fig. 7.8, we can use these results to infer several additional fundamental features. For example, these plots make clear that it is not time alone, but time in units of L/a, that describes the head variation at a point in the most meaningful way. In Fig. 7.8a we see that the head takes on only the values $H + \Delta H$ and $H - \Delta H$

with abrupt transitions, but intermediate points such as the midpoint of the line (see Fig. 7.8b) also have intervals when the head is H itself.

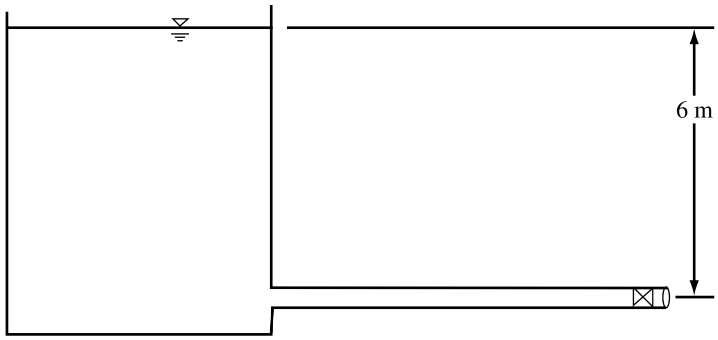
From Figs. 7.8b and 7.8c we see that the disturbance is not initially noticed everywhere; instead the fact that the valve is suddenly closed travels at the finite wave propagation speed a to the other locations and arrives at the midpoint in the pipe in half the time that is needed to traverse the entire pipe. The sudden increase in head at the valve, shown in Fig. 7.8a, remains in place at the valve until one round-trip wave travel time has elapsed, and only then is the returning wave from the reservoir able to reduce the head there. In fact the head increment ΔH need not be created instantaneously for the full incremental head to be present at the valve; it is only necessary for a set of incremental increases in head, which sum to ΔH, to be developed at the valve in a time interval that is less than this round-trip travel time $2L/a$ for the full increment in head ΔH to be present at the valve for a while. We shall later see that, owing to the manner in which a valve decreases the discharge in a pipeline by creating large head losses, it may be necessary to close a valve in a time that is much greater that $2L/a$ if we are to avoid the creation of large transient pressures.

7.4 PROBLEMS

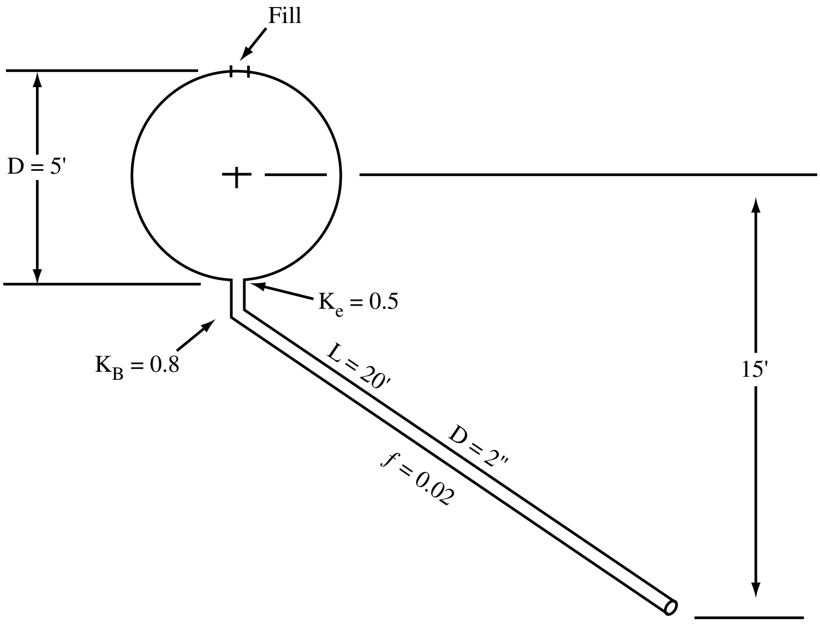
7.1 The water storage tank shown below is square with a side length of 4 m. Initially it is filled to a depth of 6 m. The exit pipe is 20 m long; its diameter is 5 cm, and the pipe entrance is sharp-edged. Assume quasi-steady flow.

(a) If the friction factor is 0.02, how much time is required for the water surface to fall 2.0 m? Plot the water surface elevation as a function of time during this fall.

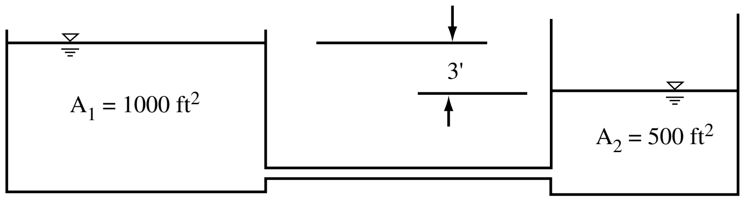
(b) Repeat part (a), but now assume that the pipe is smooth, new PCV pipe. How much do the results change?



7.2 A cylindrical tank 10 ft. long with an outlet pipe is shown below. The tank is filled with water through an opening at the top. Find the time Δt, to within approximately 15 sec, to empty the tank completely. Also plot the water surface elevation vs. time to a reasonable scale.

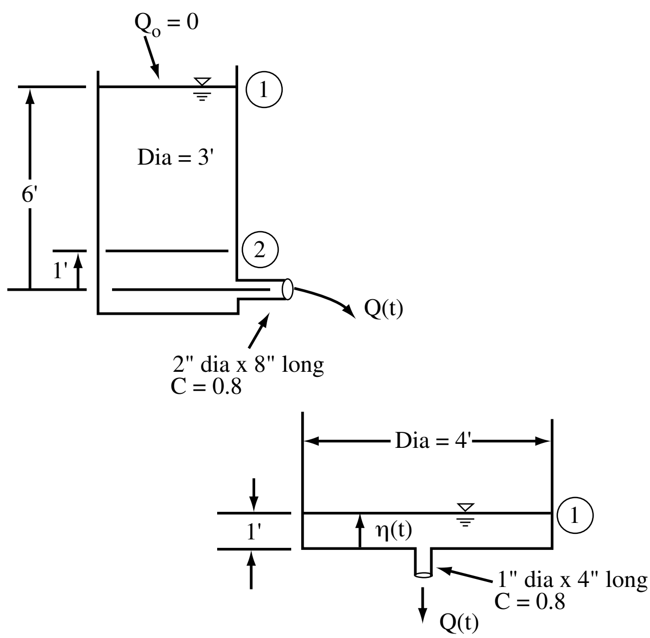


7.3 The two tanks shown atop the next page in cross section are connected by 50 feet of 18-inch-diameter cast iron pipe with a sharp-edged entrance and exit. The initial difference in water surface elevation is 3 ft. Assuming quasi-steady flow and a friction factor of 0.017, find the time for the water surfaces to reach equilibrium.



7.4 You are asked to assist in the design of a tank for a client; when this tank is drained (through a bottom orifice of area a and discharge coefficient C), the fall of the water surface is to be linearly proportional to the elapsed time. (That is, if the surface drops one foot in 30 minutes, then it drops another one foot in another 30 minutes.) What should be the shape of this tank?

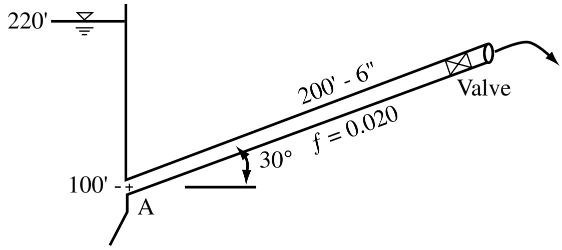
7.5 A two-tank cascade is shown below. Both tanks initially have their outlets closed, and no flow occurs; each water surface elevation is at level 1. At $t = 0^+$ both outlets are opened. When the water level in the upper tank has fallen 5 ft to level 2, the upper outlet is closed. Determine, and plot to a reasonable scale, the water surface elevation $\eta(t)$ so long as η is 1 ft or more.



7.6 Repeat the computations in Example Problem 7.1, assuming now that $R_1 = 15$ ft and $R_2 = 10$ ft.

7.7 A horizontal pipe that is 12,000 ft long and has a 3 ft diameter leaves a reservoir under a head of 125 ft and ends with a valve. Assume a Darcy-Weisbach friction factor of 0.022. If the valve is completely opened and causes no loss, what is the steady state velocity? After sudden opening of the valve, what time interval is required for the fluid velocity to attain (a) 50 %, (b) 99 %, of the steady state velocity?

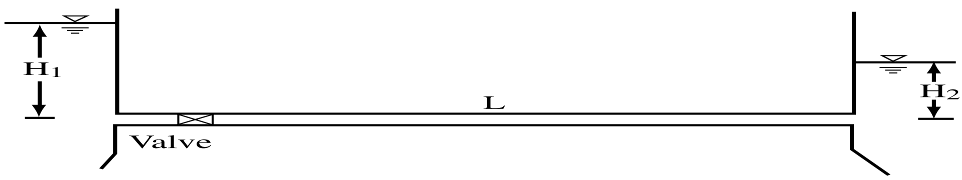
7.8* The pipe shown is initially full of water with the valve completely closed. Compute the time for the velocity to reach 99% of its steady-flow value after the valve is opened suddenly. When the valve is fully open, its head loss is negligible. Neglect the entrance loss.



7.9 For the physical system shown with Problem 7.8, assume an equivalent sand grain roughness for the pipe of e = 0.006 in., allow f to vary and repeat the computation requested in that problem.

7.10 For the same conditions, solve Problem 7.8 with the valve located at section A.

7.11 The pressure head in this horizontal pipeline is H_2 before the valve is opened.



(a) If the valve is opened suddenly, find an equation for the time that is needed for the velocity to reach 99% of its final value. Neglect the head loss across the valve and the entrance loss.

(b) In shutting off flow in the pipeline, the valve is operated so that the fluid velocity decreases linearly with time. The steady-state velocity is 9.92 ft/s, and the time of closure is 100 sec. Find the minimum pressure head in the system, where it occurs, and the time it occurs. Use the following values for this calculation:

L = 3220 ft, H_2 = 100 ft, H_1 = 200 ft, D = 1.0 ft, f = 0.020.

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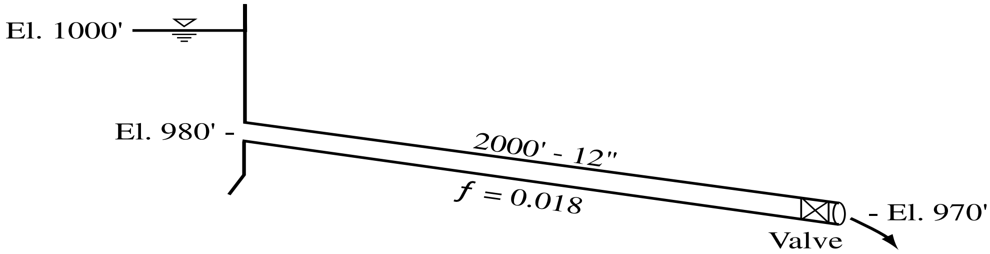
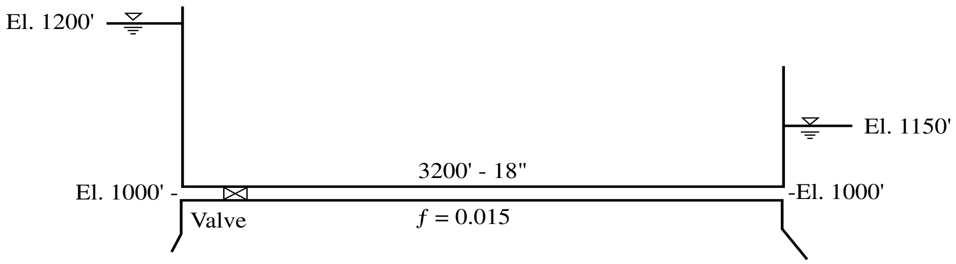
* Material in Problems 7.8 and 7.10-7.16 is adapted from Elementary Fluid Mechanics, by R. L. Street, G. Z. Watters, and J. K. Vennard, Ed. 7, Copyright 1996 by John Wiley & Sons, Inc. Reprinted by permission.

7.12 The globe valve in the pipeline shown is opened instantaneously. If the loss coefficient for the wide-open valve is 3.0, how many seconds are required for the velocity to reach 99% of its final value? Neglect the entrance loss.



7.13 The globe valve in Problem 7.12 is opened instantaneously to establish flow in the pipeline. If the valve's fully-open loss coefficient is 6.3, what is the elapsed time for the velocity to reach 99.9% of its final value? Neglect the entrance loss.

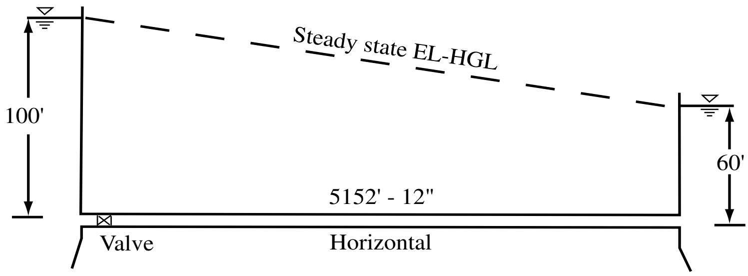
7.14 When the valve in the pipeline shown below is fully open, the steady-state velocity is 9.88 ft/s. Under these conditions the local losses and the valve loss are negligible. The valve is closed in a manner which causes the velocity to decrease linearly with time to 5 ft/s in 10 sec. Find the maximum and minimum pressure heads in the system and where and when they occur. Finally, determine the loss coefficient for the partially-open valve after valve movement has ceased and a steady flow has been re-established.



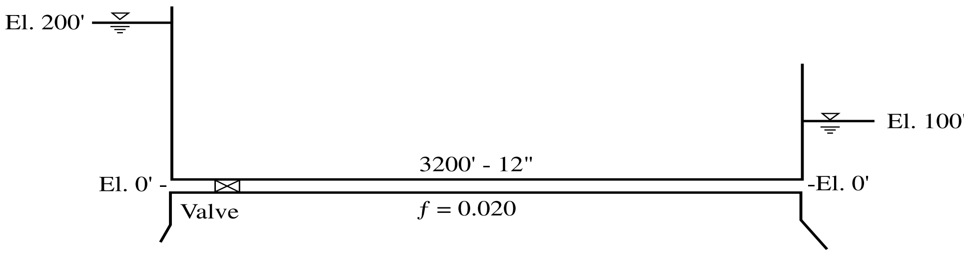
7.15 The valve in the pipeline shown below is closed in such a manner that the velocity follows the quadratic relation

$$V = V_0\left(1 - \frac{t}{T}\right)^2$$

in which the initial velocity is $V_0 = 5$ ft/s, and the time of valve closure is $T = 30$ sec. Find the minimum pressure head in the system and when and where it occurs. Neglect local losses.



7.16 At time zero the valve in the pipeline below is in the closed position. It is proposed to open the valve in such a manner that the velocity will increase linearly with time to its steady-state value of 10 ft/s in 100 sec. Find the maximum and minimum pressure heads occurring in the pipeline for the proposed program of valve movement. Can this operating program work? Explain.



7.17 In the pipeline of Problem 7.8 the valve at the downstream end, which is a GA Industries 6-inch globe valve (see App. C), will be closed linearly in time over 10 sec. Using a time increment of 0.05 sec, compute the maximum pressure which will occur at the valve. Refer to Example Problem 7.3 for numerical procedures and computer programs.

7.18 The valve in the pipeline in Problem 7.12 is a 12-inch Pratt butterfly valve (see App. C). If the valve is closed in 30 sec at an angular rate that is linear in time, calculate the maximum pressure head at the valve. The entrance loss coefficient is 0.50. Use a time increment of 0.10 sec, and refer to Example Problem 7.3 regarding numerical procedures and computer programs.

7.19 Solve Problem 7.18 if the valve in the pipeline is a Pratt 12-inch ball valve (see App. C).

APPENDIX C

VALVE LOSS COEFFICIENTS

C.1 GLOBE AND ANGLE VALVES

Flow coefficients C_v and loss coefficients K_L for fully open Cla-Val globe and angle valves are presented in the following table:

	Size, in	4	6	8	10	12	14	16	24
C_v	Globe	200	460	770	1245	1725	2300	2940	7655
	Angle	240	541	990	1575	2500*	3060*	4200*	---
K_L	Globe	5.8	5.7	6.1	5.8	6.1	5.0	5.2	4.0
	Angle	4.1	4.1	3.7	3.6	2.9	2.8	2.6	---

*Estimated

The factor C_v is the discharge, in U.S. gallons per minute, across a pressure differential of 1 lb/in^2 when water at 60 ^oF is flowing. The formula that relates C_v, the discharge Q, and the pressure drop Δp, may be written in three forms, which are

$$C_v = \frac{Q}{\sqrt{\Delta p}} \qquad Q = C_v\sqrt{\Delta p} \qquad \Delta p = \left(\frac{Q}{C_v}\right)^2 \tag{C.1}$$

in which C_v is the fresh water discharge rate in gal/min for a 1 lb/in^2 pressure difference, Q is the fresh water discharge in gal/min, and Δp is the pressure drop in lb/in^2.

Values of the resistance coefficient K_L for the valve are calculated from

$$h = K_L \frac{V^2}{2g} \tag{C.2}$$

in which h = frictional resistance, V = average velocity in ft/s, and g = 32.2 ft/s^2. The relation between C_v and K_L is

$$K_L = 890\frac{D^4}{C_v^{@}}$$

in which D = pipe diameter in inches.

Condensed from Cla-Val Automatic Control Valve Product Data Catalog, courtesy of CLA-*VAL, Newport Beach, CA*

The next table provides data on flow coefficients C_v for fully open valves for GA Industries globe and angle valves. The coefficient C_v is defined in Eq. C.1.

Size, in	6	8	10	12	14	16	18	20
Globe	447	831	1175	1750	2500	3260	4130	5100
Angle	600	1060	1800	2385	3245	4240	5365	6620

Figure C.1 illustrates how the flow coefficient varies with valve stroke. The graph applies to both globe and angle valves.

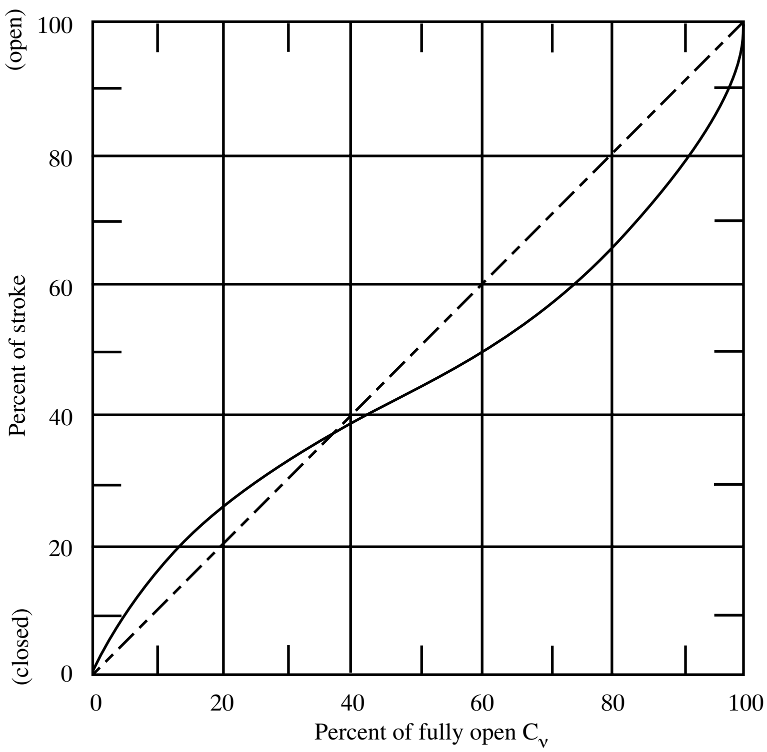


Figure C.1 The flow coefficient C_v as a function of percent of stroke.

Table and graph courtesy of GA Industries, Inc., Cranberry township, PA.

C.2 BUTTERFLY VALVES

This table presents the loss coefficient K_L, as defined in Eq. C.2, as a function of percent open for Pratt butterfly valves.

Degrees Open	3-in to 8-in Valves K_L	10-in to 20-in Valves K_L
5	15625	15625
10	3860	3860
15	935	935
20	337	337
25	145	145
30	71.8	71.8
35	39.6	39.6
40	21.6	21.6
45	12.7	12.7
50	6.61	7.42
55	4.00	4.41
60	2.62	2.64
65	1.79	1.59
70	1.25	0.952
75	0.948	0.620
80	0.800	0.496
85	0.718	0.460
90	0.689	0.447

C.3 BALL VALVES

For Pratt ball valves the following two tables provide (1) values of C_V for fully open valves and (2) percent of fully open C_V as a function of number of degrees that the valve is open. The coefficient C_V is defined in Eq. C.1.

Valve Size, in.	C_v
6	5250
8	9330
10	14600
12	21000
14	28600
16	37300
18	47300
20	58300
24	84000
30	131300
36	189000
42	257300
48	336000
54	425300
60	525100

Degrees Open	Percentage of fully open C_v
5	0.16
10	0.88
15	1.4
20	1.8
25	2.4
30	3.1
35	3.7
40	4.7
45	5.9
50	7.2
55	9.0
60	11.2
65	14.1
70	18.0
75	24.5
80	41.5
85	73.0
90	100.0

The tables in sections C.2 and C.3 are courtesy of Henry Pratt Co., Aurora, IL.

CHAPTER 8

ELASTIC THEORY OF HYDRAULIC TRANSIENTS (WATER HAMMER)*

In situations where the velocity can change suddenly and the pipeline is relatively long, the elastic properties of the pipe and liquid become significant factors. In Chapter 7 we saw how a pipeline responded to the sudden closure of a valve. This valve motion caused an increase in pressure head ΔH to occur, which propagated at a speed a. In this chapter governing relations for ΔH and a will be developed, thereby broadening the range of applications from that of the simple example in Chapter 7.

8.1 THE EQUATION FOR PRESSURE HEAD CHANGE ΔH

The linear momentum equation will be used to develop an equation for ΔH. We know that a change in velocity ΔV will cause a pressure head change ΔH to propagate at some speed a. In Fig. 8.1a ΔV is actually negative, leading to an increase in head ΔH. We begin with a section of pipe of incremental length δL, where δL is arbitrarily small but not differentially small as would be dL. The pressure wave and the pipe bulge (which is caused by the pressure head change ΔH) propagate at speed a. The wave speed is the speed relative to an observer at rest with respect to the pipe rather than the speed relative to the water velocity. For relatively rigid pipes either choice for a reference would give essential-ly the same results. Because this is an unsteady flow situation, the linear momentum

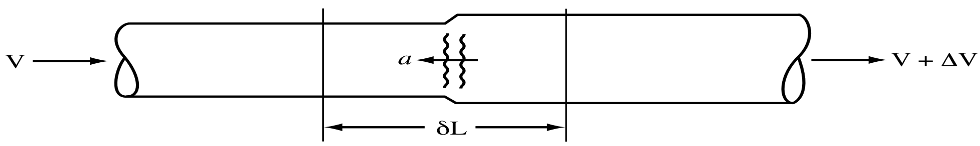


Figure 8.1a The unsteady flow control volume for a momentum analysis.

equation for steady flow does not apply. However, here it is possible to use a translating coordinate system so that the unsteady flow appears to be steady, as shown in Fig. 8.1b:

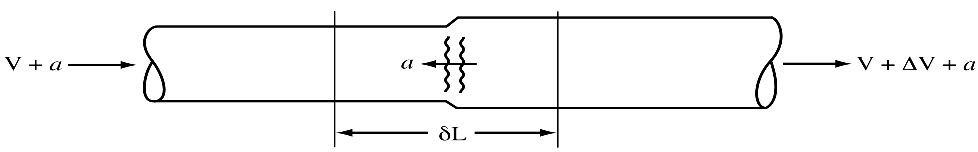


Figure 8.1b The steady flow control volume for a momentum analysis.

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* Pages 305-315 are adapted from Elementary Fluid Mechanics, by R. L. Street, G. Z. Watters, and J. K. Vennard, Ed. 7, Copyright 1996 by John Wiley & Sons, Inc. Reprinted by permission.

If we move our reference system to the left at a speed a, we have for all appearances a steady flow. From basic fluid mechanics we may then apply the steady one-dimensional linear momentum equation

$$\sum \mathbf{F}_{ext} = \left(\sum Q\rho\mathbf{V}\right)_{out} - \left(\sum Q\rho\mathbf{V}\right)_{in} \tag{8.1}$$

where Q is the discharge, ρ is the fluid density, and $\Sigma\mathbf{F}_{ext}$ is the sum of the external forces acting. The momentum correction factor for nonuniform velocity profiles is assumed to be 1.00 in this case.

Considering only the component of this vector equation parallel to the pipe and noting that the momentum flux enters and leaves the pipe section of length δL at only one cross section each, we can write

$$\left(\sum F_{ext}\right)_x = Q\rho\left(V_{out} - V_{in}\right) \tag{8.2}$$

To apply the momentum equation, we must specify a control volume and account for all forces acting on the fluid in the control volume at a particular instant and at that same instant evaluate the momentum fluxes into and out of the control volume. We choose the control volume to coincide with the inside of the pipe walls over the length δL and include the flow cross section at each end of this pipe section. This control volume, the fluid in it, and the external forces acting are shown in Fig. 8.2:

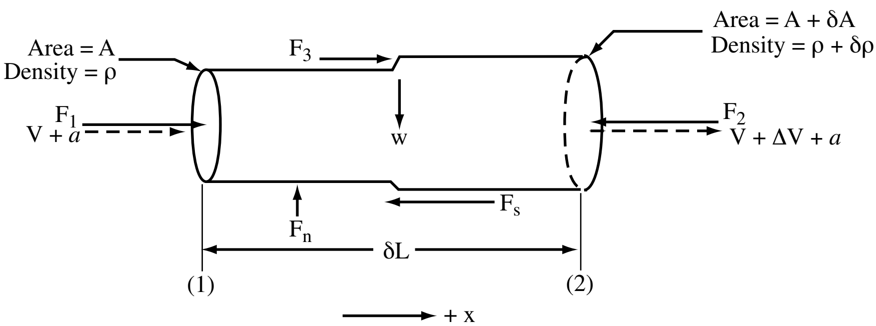


Figure 8.2 The steady-flow control volume with all forces shown.

The side shear force F_s caused by friction will be neglected because its size is proportional to δL. Also, because we consider only relatively strong pipe materials (steel, concrete, etc.), the pipe bulge is very small and so F_3 is also negligible.

Application of Eq. 8.2 gives

$$F_1 - F_2 = Q\rho[(V + \Delta V + a) - (V + a)] = Q\rho[\Delta V] \tag{8.3}$$

in which $Q\rho = (V + a)A\rho$.

If the pressure at (1) were p_o, then the pressure at (2) would be $p_o + \Delta p$, and

$$p_o A - \left(p_o + \Delta p\right)\left(A + \delta A\right) = \left(V + a\right)A\rho\left(\Delta V\right) \tag{8.4}$$

Expanding this equation and recognizing that $\Delta p = \gamma\Delta H$ and δA is very small compared to ΔH, A, and γ, we can neglect the small terms with the result

$$-\Delta H\gamma A = \left(V + a\right)A\rho\left(\Delta V\right) \tag{8.5}$$

This equation can also be written as

$$\Delta H = -\frac{\rho}{\gamma}\Delta V(V + a) \tag{8.6}$$

or

$$\Delta H = -\frac{a\Delta V}{g}\left(1 + \frac{V}{a}\right) \tag{8.7}$$

In most rigid pipe situations (even PVC with a wave speed of only 1200 ft/s), the value of V/a is less than 0.01. Accordingly, Eq. 8.7 is generally (and always in this work) applied as

$$\Delta H = -\frac{a}{g}\Delta V \tag{8.8}$$

From Eq. 8.8 we see that a decrease in velocity ΔV causes an increase in head ΔH. Further, ΔH depends on the wave speed a and cannot be determined until a value of a is established.

8.2 WAVE SPEED FOR THIN-WALLED PIPES

To develop an equation for the wave speed, we will consider conservation of mass in the pipe section δL long that was used to develop Eq. 8.4. We examine the mass flow into and out of the pipe section over the time period required for the wave to pass through that portion of the pipe. The net inflow of mass will be equated to the increase in mass storage in δL to produce an equation for wave speed a. Again we assume that a decrease in velocity occurs, hence mass accumulates.

To begin we note the situation when the wave has first reached the control volume and then at the time the wave has just passed through the section at a time δt later.

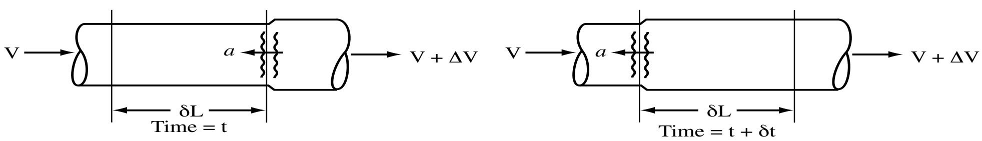


Figure 8.3 Propagation of the pressure wave at two instants.

It is clear that δL and δt are related via the wave speed as $\delta L = a\delta t$.

8.2.1. NET MASS INFLOW

During the time interval required for the wave to pass through the control volume, mass has accumulated in the section in the amount

$$\delta M = VA\rho\delta t - (V + \Delta V)(\rho + \delta\rho)(A + \delta A)\delta t \tag{8.9}$$

Expanding the parentheses and neglecting small terms gives

$$\delta M = -A\rho\Delta V\delta t \tag{8.10}$$

or, in terms of wave speed and δL,

$$\delta M = -A\rho\Delta V\frac{\delta L}{a} \tag{8.11}$$

This extra liquid is stored in the control volume partly by being compressed slightly to a larger density and partly by occupying additional space provided by stretching the pipe cross section a small amount.

We now proceed to quantify the volume changes for the liquid and the pipe.

8.2.2. CHANGE IN LIQUID VOLUME DUE TO COMPRESSIBILITY

Because the pressure has increased during the passage of a positive pressure wave caused by a decrease in velocity, the volume of liquid in the section is compressed to a slightly higher density. The equation relating the increase in pressure and decrease in volume is the equation defining the bulk modulus of elasticity for a liquid, as can be found in any elementary fluid mechanics text:

$$K = -\frac{dp}{d\forall/\forall} \tag{8.12}$$

Here K is the bulk modulus of elasticity of the liquid and p and $\forall$ are the pressure and volume of the liquid, respectively. Since K is relatively constant over a wide pressure range (assuming no entrained gases in the liquid), we can let $dp = \Delta p$ and write Eq. 8.6 as

$$\delta\forall = -\Delta p\frac{\forall}{K} \tag{8.13}$$

where $\delta\forall$ is the change in liquid volume in the control volume resulting from the pressure change Δp.

8.2.3. CHANGE IN PIPE VOLUME DUE TO ELASTICITY

When the increased pressure stretches the pipe, more space is available to store the accumulated net inflow of liquid. The pipe may stretch both circumferentially and longitudinally, so we must consider both contributions to the change in pipe volume.

Developments that are basic to the mechanics of solid materials show the relation between the pipe wall strains in the two perpendicular directions. If a material is strained in one direction by an amount ε_1, then a strain ε_2 will occur in the perpendicular direction (provided the material is free to strain without developing a stress in that direction) according to $\varepsilon_2 = \mu\varepsilon_1$, where μ is Poisson's ratio. If there is a restriction to free strain in either direction caused either by restraint or applied stress, the relation is more complicated. A text on the mechanics of materials will provide the following equations for two-dimensional stress which can be applied to thin-walled pipes:

$$\sigma_1 = \frac{\varepsilon_1 + \mu\varepsilon_2}{1-\mu^2}E \qquad \text{or} \qquad \varepsilon_1 = \frac{\sigma_1 - \mu\sigma_2}{E} \tag{8.14a}$$

$$\sigma_2 = \frac{\varepsilon_2 + \mu\varepsilon_1}{1-\mu^2}E \qquad \text{or} \qquad \varepsilon_2 = \frac{\sigma_2 - \mu\sigma_1}{E} \tag{8.14b}$$

Here σ_1 and ε_1 are the stress and strain, respectively, in the direction along the pipe axis, σ_2 and ε_2 are the values in the circumferential direction, and E is the modulus of elasticity of the pipe wall material. Of course, if the wall material is not homogeneous and isotropic, then a more complex analysis is required.

For water hammer pressure waves there is usually a stress and strain already resident in the pipe caused by the steady state flow. Hence we write the preceding equations in incremental form

$$\Delta\sigma_1 = \frac{\Delta\varepsilon_1 + \mu\Delta\varepsilon_2}{1-\mu^2}E \qquad \text{or} \qquad \Delta\varepsilon_1 = \frac{\Delta\sigma_1 - \mu\Delta\sigma_2}{E} \tag{8.15a}$$

$$\Delta\sigma_2 = \frac{\Delta\varepsilon_2 + \mu\Delta\varepsilon_1}{1-\mu^2}E \qquad \text{or} \qquad \Delta\varepsilon_2 = \frac{\Delta\sigma_2 - \mu\Delta\sigma_1}{E} \tag{8.15b}$$

The change in volume caused by circumferential stretching is

$$\delta\forall_c = \pi D\frac{\delta D}{2}\delta L \tag{8.16}$$

where $\pi\delta D = \pi D\Delta\varepsilon_2$. Combining the two equations gives

$$\delta\forall_c = \frac{1}{2}\pi D^2\delta L\Delta\varepsilon_2 \tag{8.17}$$

The change in volume caused by longitudinal stretching is

$$\delta\forall_l = \frac{\pi}{4}D^2\delta L\Delta\varepsilon_1 \tag{8.18}$$

Combining Eqs. 8.17 and 8.18 gives the total volume change due to pipe stretching as

$$\delta\forall = \frac{\pi}{4}D^2\delta L\left(\Delta\varepsilon_1 + 2\Delta\varepsilon_2\right) \tag{8.19}$$

We now begin the process of replacing the expressions for strain with those for the stress and pressure which cause the strain. The change in circumferential stress in the pipe wall under static conditions is

$$\Delta\sigma_2 = \frac{\Delta pD}{2e} \tag{8.20}$$

where e is the pipe wall thickness. However, the transient conditions of water hammer would in general cause the pipe to respond dynamically in a manner which can only be analyzed accurately by carefully considering the mass of the pipe and fitting materials as well as pipe restraints. That is, any valves, fittings, and other attachments in addition to the weight of the pipe must be displaced by pressure changes. These displacements are in turn affected by the type and elastic behavior of the pipe restraints. This type of analysis would be entirely too complex to accomplish in general, so we assume that the static conditions adequately approximate the dynamic behavior. Experimental results over the years have generally validated this approach. Substituting the above equation into the first of Eqs. 8.15b gives

$$\frac{\Delta pD}{2e} = \frac{\Delta\varepsilon_2 + \mu\Delta\varepsilon_1}{1-\mu^2}E \tag{8.21}$$

While the relation between circumferential stress and pressure is valid for all types of restraint, the relation between longitudinal stress and strain varies with restraint type. For example, if the pipe were anchored at one end and free to stretch longitudinally (much like a long slender pressure vessel), the longitudinal stress would be

$$\Delta\sigma_1 = \frac{\Delta pD}{4e} \tag{8.22}$$

under static conditions. On the other hand, if the pipe were rigidly anchored to prevent any axial strain, then $\Delta\sigma_1 = \mu\Delta\sigma_2$ because $\Delta\varepsilon_1 = 0$. However, if the pipe contained functioning expansion joints throughout its length, then $\Delta\sigma_1 = 0$ and $\Delta\varepsilon_1$ is of no interest. Following the nomenclature of Wylie and Streeter (1993), we identify the above cases as the following:

Case (a) pipe anchorage only at the upstream end;
Case (b) full pipe restraint from axial movement.
Case (c) longitudinal expansion joints along the pipeline.

In a practical sense the actual pipe restraint situation probably will not conform precisely to any of these cases but lies somewhere in this range of possibilities.

Here we will now choose one restraint case to develop relatively fully as an example of how the analysis of each case should proceed. Because buried pipelines are relatively common and might be expected to be fully restrained axially by soil friction and anchor blocks, we will examine Case (b) restraint next as we move ahead to compute a wave speed.

Wave Speed Solution for Case (b) Restraint

For this restraint choice $\Delta\varepsilon_1 = 0$ and Eqs. 8.15a become

$$\Delta\sigma_1 = \frac{\mu\Delta\varepsilon_2}{1-\mu^2}E \quad \text{or} \quad \Delta\sigma_1 = \mu\Delta\sigma_2 \tag{8.23}$$

and Eq. 8.21 becomes

$$\frac{\Delta pD}{2e} = \frac{\Delta\varepsilon_2}{1-\mu^2}E \tag{8.24}$$

Substituting this equation into Eq. 8.19 in place of $\Delta\varepsilon_2$ gives the total volume change as

$$\delta\maltese = \frac{\pi}{4}D^2\delta L\left(\frac{1-\mu^2}{E}\right)\left(\frac{\Delta pD}{e}\right) \tag{8.25}$$

Now recall that Eq. 8.11, based on conservation of mass, computes the incremental mass which has accumulated in the pipe control volume in δt seconds. If we subtract the mass in the pipe before the passage of the wave from the amount existing after the passage of the wave, we obtain

$$\delta M = (\rho + \delta\rho)(A\delta L + \delta\maltese) - \rho A\delta L \tag{8.26}$$

Equating this expression for δM with that of Eq. 8.11, expanding, and dropping small terms gives

$$\delta\rho A\delta L + \rho\delta\maltese = A\rho\Delta V\frac{\delta L}{a} \tag{8.27}$$

To arrange this equation into a more usable form, note for a given mass of material that an increase in pressure causes a decrease in volume and an increase in density. That is, $\rho\forall$ = constant so $\forall\delta\rho + \rho\delta\forall = 0$ and

$$\delta\rho = -\frac{\delta\forall}{\forall}\rho \tag{8.28}$$

Substituting this result into Eq. 8.13 gives

$$\delta\rho = \rho\left(\frac{\Delta p}{K}\right) \tag{8.29}$$

Replacing Δp with $\gamma\Delta H$ in the preceding equation, substituting it and Eq. 8.25 into Eq. 8.27 leads to

$$\gamma\,\Delta H\left[\frac{1}{K} + \left(\frac{1-\mu^2}{E}\right)\frac{D}{e}\right] = \frac{\Delta V}{a} \tag{8.30}$$

Combining this equation with Eq. 8.8 results in

$$a^2\rho\left[\frac{1}{K} + \frac{D}{e}\left(\frac{1-\mu^2}{E}\right)\right] = 1 \tag{8.31}$$

or, in the conventional form for wave speed,

$$a = \frac{\sqrt{K/\rho}}{\sqrt{1 + \frac{K}{E}\frac{D}{e}\left(1-\mu^2\right)}} \qquad \text{Case (b)} \tag{8.32}$$

It is now possible to compute the wave speed and pressure increase directly in simple situations where Eq. 8.8 applies.

Wylie and Streeter (1993) show that the equation for wave speed can be conveniently expressed in the general form

$$a = \frac{\sqrt{K/\rho}}{\sqrt{1 + \frac{K}{E}\frac{D}{e}(C)}} \tag{8.33}$$

where

for Case (a) restraint $\quad C = 5/4 - \mu \quad$ (8.34a)

for Case (b) restraint $\quad C = 1 - \mu^2 \quad$ (8.34b)

and

for Case (c) restraint $\quad C = 1.0 \quad$ (8.34c)

Keep in mind that this set of equations for wave speed applies only to thin-walled pipes. If the pipe walls are sufficiently thick, the above equations must be modified (see Section 8.3 for a discussion of what constitutes a thin-walled pipe).

The following table of values for E and μ should be useful in calculating wave speeds in pipes made of common materials. The bulk modulus K for water is approx-imately 300,000 lb/in^2, although some references cite K's as high as 320,000 lb/in^2.

Table 8.1 Moduli of Elasticity and Poisson Ratios for Common Pipe Materials

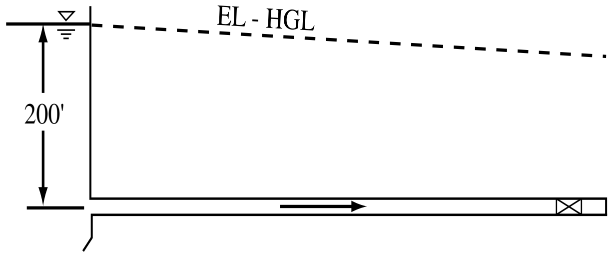
Material	**E (lb/in^2)**	**Poisson ratio μ**
Steel	30×10^6	0.30
Ductile cast iron	24×10^6	0.28
Copper	16×10^6	0.36
Brass	15×10^6	0.34
Aluminum	10.5×10^6	0.33
PVC	4×10^5	0.45
Fiberglass-reinforced plastic	$E_2 = 4 \times 10^6$	$\mu_2 = 0.27\text{-}0.30$
	$E_1 = 1.3 \times 10^6$	$\mu_1 = 0.20\text{-}0.24$
Asbestos Cement	3.4×10^6	0.30
Concrete	$57{,}000\sqrt{f_c'}$ f_c' = 28-day strength	dynamic value 0.24

However, even a small amount of free air or gas suspended in the liquid can drastically reduce the value of K and the resultant wave speed (see Section 8.4). Unfortunately, evaluating the amount of air, its distribution, its pervasiveness, and its effect on wave speed is most difficult. Consequently, in design situations it is common practice to take a conservative approach and assume there is no air present. This *generally* leads to a prediction of higher water hammer pressures. Any presence of entrained air or gas in the system would be a fortuitous circumstance, at least in the sense of reducing the wave speed.

One should note that a pipe can become infinitely strong ($E \to \infty$) without the wave speed also becoming infinite. From Eq. 8.33 it is clear that this situation causes the denominator of the equation to go to 1.0, resulting in a wave speed $a = \sqrt{K/\rho}$ which for water is about 4720 ft/s. This number has no practical significance in design because it is far too high to serve as even an approximate wave speed value for preliminary design. With even a limited amount of experience, a designer can make a far better estimate of the wave speed in the pipe under consideration.

Example Problem 8.1

To get a "feel" for the pressure head changes and the resultant elastic deformations of pipe and liquid caused by a typical water hammer situation and to demonstrate the effects of different pipe restraints, the following problem is analyzed.



Water flows in this 24-inch steel pipeline at a velocity of 6 ft/s. The pipeline has a wall thickness of 0.25 inches. First we will calculate the wave speed for the three types of restraint by using Eqs. 8.33 and 8.34:

Case (a) $$a = \frac{\sqrt{K/\rho}}{\sqrt{1+\frac{K}{E}\frac{D}{e}(C)}} = \frac{4720}{\sqrt{1+\frac{3\times 10^5}{3\times 10^7}\frac{24}{0.25}(5/4-0.30)}} = 3410 \text{ ft/s}$$

Case (b) $$a = \frac{4720}{\sqrt{1+\frac{3\times 10^5}{3\times 10^7}\frac{24}{0.25}\left(1-0.30^2\right)}} = 3450 \text{ ft/s}$$

Case (c) $$a = \frac{4720}{\sqrt{1+\frac{3\times 10^5}{3\times 10^7}\frac{24}{0.25}(1.00)}} = 3370 \text{ ft/s}$$

As one can see clearly here, the differences are for all practical purposes insignificant for this pipe.

Now we will compute with Eq. 8.8 the pressure head changes resulting from sudden valve closure for all three cases of restraint:

Case (a) $$\Delta H = -\frac{a}{g}\Delta V = -\frac{3410}{32.2}(-6) = 635 \text{ ft}$$

Case (b) $$\Delta H = -\frac{3450}{32.2}(-6) = 643 \text{ ft}$$

Case (c) $$\Delta H = -\frac{3370}{32.2}(-6) = 628 \text{ ft}$$

Because the head increase depends directly on the wave speed, the negligible difference in wave speed translates into a head difference of no more than 2%, which is usually a negligible difference.

We now calculate the change in pipe wall stress caused by these head increases for all three types of restraint:

Case (a) $$\Delta\sigma_2 = \frac{\Delta p D}{2e} = \frac{\left(635\times\frac{62.4}{144}\right)\times 24}{2\times 0.25} = 13,210 \text{ lb/in}^2$$

$$\Delta\sigma_1 = \frac{\Delta p D}{4e} = \frac{1}{2}\Delta\sigma_2 = 6600\ \text{lb}/\text{in}^2$$

Case (b) $$\Delta\sigma_2 = \frac{\Delta p D}{2e} = \frac{\left(643 \times \frac{62.4}{144}\right) \times 24}{2 \times 0.25} = 13{,}370\ \text{lb}/\text{in}^2$$

$$\Delta\sigma_1 = \mu\Delta\sigma_2 = 0.30 \times 13{,}370 = 4010\ \text{lb}/\text{in}^2$$

Case (c) $$\Delta\sigma_2 = \frac{\Delta p D}{2e} = \frac{\left(628 \times \frac{62.4}{144}\right) \times 24}{2 \times 0.25} = 13{,}060\ \text{lb}/\text{in}^2$$

$$\Delta\sigma_1 = 0$$

Next we calculate the percentage increase in the pipe diameter caused by the pressure head increase for all three cases of restraint. Using Eq. 8.15b,

$$\text{Percent change in } D = 100\frac{\delta D}{D} = 100 \times \Delta\varepsilon_2 = \frac{100}{E}\left(\Delta\sigma_2 - \mu\Delta\sigma_1\right)$$

Case (a) $$\text{Percent change} = \frac{100}{30 \times 10^6}\left(13{,}210 - 0.30 \times 6600\right) = 0.037\%$$

Case (b) $$\text{Percent change} = \frac{100}{30 \times 10^6}\left(13{,}370 - 0.30 \times 4010\right) = 0.041\%$$

Case (c) $$\text{Percent change} = \frac{100}{30 \times 10^6}\left(13{,}060 - 0.30 \times 0\right) = 0.044\%$$

These results, showing the relatively small elastic deformation of the pipe, substantiate many of the previous assumptions that were used in neglecting small terms in equations.

Finally, we will look at the water entering the pipe section during the passage of the pressure wave and determine what percentage is accommodated by pipe stretching and what portion is relegated to water compression. The fraction of liquid directed to liquid compression is given by the first term in Eq. 8.31.

$$\text{Percent change in water volume} = 100\rho a^2\left(\frac{1}{K}\right) = 100\frac{\rho a^2}{K}$$

The remainder is due to pipe stretching.

Case (a)
$$\text{Percent change in water volume} = 100\frac{1.94 \times 3410^2}{300{,}000 \times 144} = 52\%$$
$$\text{Percent accommodated by pipe stretching} = 48\%$$

Case (b)
$$\text{Percent change in water volume} = 100\frac{1.94 \times 3450^2}{300{,}000 \times 144} = 53\%$$
$$\text{Percent accommodated by pipe stretching} = 47\%$$

Case (c) Percent change in water volume $= 100\dfrac{1.94 \times 3370^2}{300,000 \times 144} = 51\%$

Percent accommodated by pipe stretching $= 49\%$

* * *

8.3 WAVE SPEEDS IN OTHER TYPES OF CONDUITS

The case of thin-walled pipes has been used previously to derive equations for wave speed which are essential to the computation of water hammer pressures. However, it is common practice to fabricate pipe of materials which result in the pipe having thick walls, e.g., concrete, asbestos cement, or ductile iron. Also, pipe can be manufactured of more than one material, the most common of which is reinforced concrete. Conduits may also be carved in rock and possibly lined with steel or concrete. We need to be able to calculate wave speed in all these cases.

8.3.1. THICK-WALLED PIPES

The analysis for thick-walled pipes proceeds along the same lines as for thin-walled pipes. However, for thick-walled pipes the variation in stress and strain across the pipe wall is taken into consideration. The analysis leads to the same equation for wave speed as for thin-walled pipes so long as we redefine the coefficients C for each case:

Case (a)
$$C = \frac{1}{1+\dfrac{e}{D}}\left[\left(\frac{5}{4}-\mu\right)+2\frac{e}{D}(1+\mu)\left(1+\frac{e}{D}\right)\right] \qquad (8.35a)$$

Case (b)
$$C = \frac{1}{1+\dfrac{e}{D}}\left[\left(1-\mu^2\right)+2\frac{e}{D}(1+\mu)\left(1+\frac{e}{D}\right)\right] \qquad (8.35b)$$

Case (c)
$$C = \frac{1}{1+\dfrac{e}{D}}\left[(1.0)+2\frac{e}{D}(1+\mu)\left(1+\frac{e}{D}\right)\right] \qquad (8.35c)$$

As e/D becomes vanishingly small, in every case the equation simplifies to the thin-walled equation. So when is it appropriate to use the thick-walled equations? In answering this question, remember that the wave speed is the key quantity, and it is affected not only by the e/D ratio but also by the other factors in Eq. 8.33. Watters (1984) concluded that the use of the thick-walled equations for $D/e > 40$ generally produces no significant improvement in the accuracy of the wave speed, except in cases where softer pipes such as PVC are in use. One could attempt to remove as much imprecision as possible by always using the thick-walled equations; the extra computation required is negligible. This approach will succeed whenever an application is represented well by one of the restraint cases, but the precision may otherwise be bogus when no one restraint case truly represents the application. Note also that the thin-walled equations lead to higher wave speeds and thus generally more conservative results.

Example Problem 8.2

To investigate the effect of wall thickness on wave speed, we will compute the wave speed for a steel pipe with an inside diameter of 9.522 in and a wall thickness of 0.239 in ($D/e = 40$) using both the thin-walled and thick-walled formulas. The results are shown in a table for all three restraint cases.

Wave Speed, ft/s

Restraint	Thin-walled	Thick-walled	Error %
Case (a)	4020	4000	0.5
Case (b)	4050	4020	0.7
Case (c)	4000	3970	0.7

* * *

8.3.2. CIRCULAR TUNNELS

The wave speed equations for circular tunnels can be derived from the thick-walled equations by letting the wall thickness go to infinity. When C for case (a) is inserted into the wave speed equation 8.33 and the *D/e* ratio is allowed to go to zero, the result is

$$a = \frac{\sqrt{K/\rho}}{\sqrt{1 + \frac{2K}{E}(1+\mu)}} \tag{8.36}$$

For tunnels which are concrete-lined or steel-lined with concrete backing, the analysis is a good deal more complex. Refer to Halliwell (1963) for the rather lengthy equations required for computing wave speed in this situation.

8.3.3. REINFORCED CONCRETE PIPE

For reinforced concrete pipe the transformed-section method can be used to convert the pipe into an equivalent homogeneous pipe. Then the computation of the wave speed can be accomplished by using the homogeneous pipe equations. To be assured of an accurate wave speed, it is necessary to know exactly how the pipe was fabricated. Only that concrete which can sustain load under pressure should be used in the computation. This type of concrete pipe is generally prestressed to assure this capability.

The transformed-section method replaces the concrete cross-sectional area with an equivalent cross-sectional area of steel using the formula

$$A_{st} = \frac{E_{con}}{E_{st}} A_{con} \tag{8.37}$$

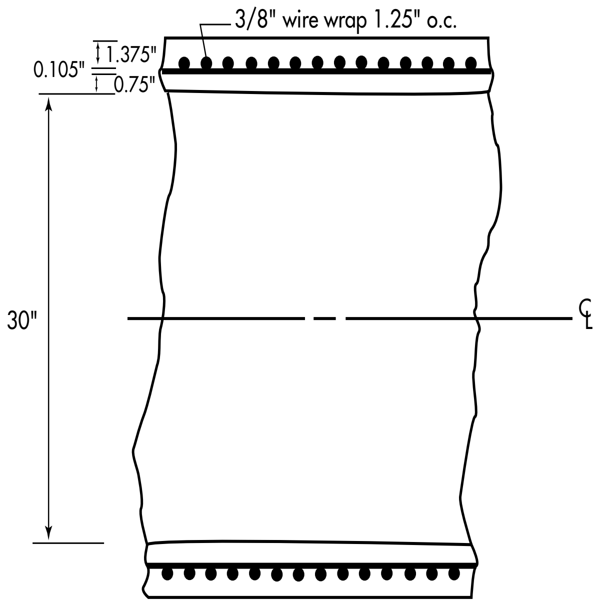
neglecting any variation in stress over the thickness of the concrete. We always convert the concrete to steel; doing the reverse would create an exceedingly thick-walled pipe.

In working with reinforced concrete pipe that is not prestressed, the concrete is assumed to carry no load. The reinforcing steel is regarded as a thin-walled steel pipe having the same cross-sectional area as the circumferential reinforcement has, and the wave speed is computed with the thin-walled equations.

If the reinforced concrete pipe is pretensioned or post-tensioned, the area of concrete placed in compression by the pre- or post-tensioning process must be included in the transformed section. This prestressing makes the pipe much stronger, but it also results in higher wave speeds which generally lead to higher water hammer pressures. The following illustrative problem demonstrates the transformed-section technique.

Example Problem 8.3

A 30-in. inside diameter reinforced concrete pipe is pretensioned using 3/8-in diameter wrapping wire placed 1.25 in on center. The pipe was manufactured by first fabricating a thin steel cylinder 0.105 in thick and then centrifugally placing a dense cement mortar lining 0.75 in thick inside the steel cylinder. After curing the liner, the pretensioning is accomplished by stressing the wire as it is wrapped around the steel cylinder. This process places the cement liner in compression. The ends of the wrapping wire are welded to the steel cylinder to maintain the pretensioning. A one-inch-thick concrete cover is placed over the wrapping wire as a protective coating. This cover carries no load. A longitudinal pipe section is shown below. Assume the 28-day strength of the concrete is 6000 lb/in^2.



Using the formula in Table 8.1 for concrete, E is

$$E_{con} = 57,000\sqrt{f_c'} = 57,000\sqrt{6000} = 4.4 \times 10^6 \text{ lb / in}^2$$

The area of steel wire per inch of pipe is $\frac{\pi}{4} \times 0.375^2 / 1.25 = 0.0884 \text{ in}^2/\text{ in}$.

The equivalent area of steel that is required to replace the cement lining which was prestressed during the wrapping process is

$$A_{st} = \frac{E_{con}}{E_{st}} A_{con} = \frac{4.4 \times 10^6}{30 \times 10^6} \times 0.75 = 0.110 \text{ in}^2/\text{ in}$$

Now the thickness of the equivalent steel pipe is

$$e_{eq} = 0.0884 + 0.110 + 0.105 = 0.303 \text{ in}$$

We compute the diameter of the equivalent pipe by locating the centroid of the section as follows:

$$\bar{r} = \frac{0.0884(15+0.75+0.105+0.375/2)+0.110(15+0.75/2)+0.105(15+0.75+0.105/2)}{0.303}$$

$$\bar{r} = 15.74 \text{ in.} \qquad \overline{D} = 31.5 \text{ in.}$$

Now the wave speed is computed using Case (b) restraint because that seems the most conservative:

$$a = \frac{4720}{\sqrt{1 + \dfrac{3 \times 10^5}{3 \times 10^7} \dfrac{31.5}{0.303}\left(1 - 0.30^2\right)}} = 3380 \text{ ft/s}$$

If the effect of the cement mortar lining is neglected, the wave speed drops to 2990 ft/s. It is the responsibility of the designer to make the judgment as to the proper wave speed to use or, as an alternative, analyze the system under both conditions to determine the more extreme behavior.

* * *

8.4 EFFECT OF AIR ENTRAINMENT ON WAVE SPEED*

When free air (or any other gas) is present in a pipeline, either as small bubbles or in larger volumes, the wave speed in the pipeline is decreased dramatically. As a consequence, the wave propagation patterns and the resulting pressures are substantially changed. We will demonstrate the effect by using the simplest model of air entrainment.

If the air-water mixture is assumed to be uniformly distributed throughout a portion of the pipeline, the wave speed in that portion of the pipeline can be computed by using Eq. 8.33. However, care must be taken to insure that the air-water *mixture* is used in determining the values of K and ρ. The bulk modulus K for the mixture is developed from Eq. 8.12 by replacing the relative change in overall volume by the sum of the relative changes in volume of the air and water. The result is

$$K_{mix} = \frac{K_{liq}}{1 + \alpha\left(\dfrac{K_{liq}}{K_{air}} - 1\right)} \tag{8.38}$$

where K_{liq} and K_{air} are the bulk moduli of elasticity for liquid and air, respectively, and α is the void fraction (volume of air ÷ total volume of mixture). For misture density is found by the same approach to be

$$\rho_{mix} = (1 - \alpha)\rho_{liq} \tag{8.39}$$

Substituting Eqs. 8.38 and 8.39 into Eq. 8.33 and recognizing that $K_{liq}/K_{air} >> 1$, we find the wave speed to be

$$a = \frac{\sqrt{K_{liq}/\rho_{mix}}}{\sqrt{1 + \dfrac{K_{liq}}{E}\dfrac{D}{e}C + \alpha\dfrac{K_{liq}}{K_{air}}}} \tag{8.40}$$

---

* This Section is adapted from Elementary Fluid Mechanics, by R. L. Street, G. Z. Watters, and J. K. Vennard, Ed. 7, Copyright 1996 by John Wiley & Sons, Inc. Reprinted by permission.

This same equation and a detailed description of the difficulties encountered in the solution of water hammer problems which have entrained air is given by Tullis et al. (1976).

It is clear from Eq. 8.40 that the wave speed in the pipeline depends on the pressure in the pipeline because the values of α and K_{air} depend on pressure. As a consequence, the wave speed varies with the passage of a pressure wave. This factor greatly complicates an analysis and makes the accurate prediction of water hammer pressures most difficult.

An example is presented below to demonstrate the dramatic effect that small fractions of entrained air can have on the wave speed. The first step which must be taken is to establish a method of determining K_{air}. As elementary fluid mechanics texts show, K_{air} depends on the thermodynamic process followed by the air as it compresses or expands. Wylie and Streeter (1993) suggest using an isothermal process with $K_{air} = p$. The other extreme is to use an isentropic process where $K_{air} = kp = 1.4p$. If some provision for heat transfer is made, then a polytropic process with $K_{air} = np = 1.2p$ (as used later with surge tanks) may be appropriate. The effects of these various alternatives are shown below.

Example Problem 8.4

Consider the pipeline of Example Problem 8.1 and consider air entrainment percentages of 0.10, 0.50, 1.0, and 2.0%. We assume the atmospheric pressure to be 14.7 lb/in^2 and use Case (b) restraint. For a polytropic process and an entrained air percentage of 0.10%, the wave speed from Eq. 8.40 is

$$a = \frac{\sqrt{\dfrac{300,000 \times 144}{1.94(1-0.001)}}}{\sqrt{1+\dfrac{3\times 10^5}{3\times 10^7}\dfrac{24}{0.25}\left(1-0.30^2\right)+0.001\times\dfrac{3\times 10^5}{1.2\times\left(\dfrac{200\times 62.4}{144}+14.7\right)}}} = 2270 \text{ ft/s}$$

The following table summarizes the wave speeds for the three separate thermodynamic processes:

% Air Entrainment	Isothermal Process	Polytropic Process	Isentropic Process
0.1	2150 ft/s	2270 ft/s	2360 ft/s
0.5	1160 ft/s	1260 ft/s	1340 ft/s
1.0	845 ft/s	920 ft/s	988 ft/s
2.0	610 ft/s	666 ft/s	777 ft/s

Regardless of the process, the differences in results among the assumed processes are not great in view of the other uncertainties.

* * *

8.5 DIFFERENTIAL EQUATIONS OF UNSTEADY FLOW

Up to this point we have seen for a given impulsive change in velocity ΔV at a pipeline section that we can compute the pressure head change ΔH which will result. This ability will now be extended so the velocity and pressure head at any pipe section at any time can be determined as the result of boundary and initial conditions imposed at any section of the system. To accomplish this, we will use Euler's equation from Chapter 7 and develop another equation based on conservation of mass.

8.5.1. CONSERVATION OF MASS

We apply conservation of mass to a control volume that coincides with the interior of the pipe and is of length *ds*:

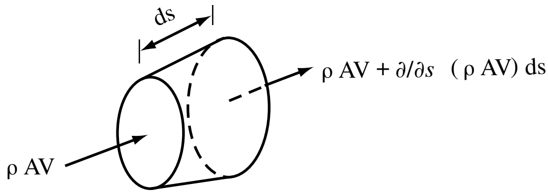


Figure 8.4 Control volume coinciding with the interior surface of the pipe.

The result of this application is

$$\rho AV - \left[\rho AV + \frac{\partial}{\partial s}(\rho AV)ds\right] = \frac{\partial}{\partial t}(\rho A ds) \tag{8.41}$$

or

$$-\frac{\partial}{\partial s}(\rho AV)ds = \frac{\partial}{\partial t}(\rho A ds) \tag{8.42}$$

At this point we employ a rather unconventional form of the control volume concept in that we require the sides of the control volume to be attached to the pipe wall. Thus the control volume will elongate as the pipe stretches longitudinally. The only exception is in Case (c) where we keep the control volume at a constant length even though the pipe elongates (the total length of the pipeline remains constant even as the pipe slips in its expansion joints). This technique is used because the pipe stretching affects the volume of storage, and the relation between pipe elasticity and the available volume for the liquid is identical to that in Section 8.2.

Expanding the parentheses of Eq. 8.42 yields

$$-\left(\rho A\frac{\partial V}{\partial s}ds + \rho V\frac{\partial A}{\partial s}ds + AV\frac{\partial \rho}{\partial s}ds\right) = \rho A\frac{\partial}{\partial t}(ds) + \rho ds\frac{\partial A}{\partial t} + Ads\frac{\partial \rho}{\partial t} \tag{8.43}$$

Regrouping and dividing by the control volume mass $\rho A ds$,

$$\frac{1}{\rho}\left(\frac{\partial \rho}{\partial t} + V\frac{\partial \rho}{\partial s}\right) + \frac{1}{A}\left(\frac{\partial A}{\partial t} + V\frac{\partial A}{\partial s}\right) + \frac{1}{ds}\frac{\partial}{\partial t}(ds) + \frac{\partial V}{\partial s} = 0 \tag{8.44}$$

Recognizing that $\frac{\partial \rho}{\partial t} + V\frac{\partial \rho}{\partial s} = \frac{d\rho}{dt}$ and $\frac{\partial A}{\partial t} + V\frac{\partial A}{\partial s} = \frac{dA}{dt}$, Eq. 8.44 becomes

$$\frac{1}{\rho}\frac{d\rho}{dt} + \frac{1}{A}\frac{dA}{dt} + \frac{\partial V}{\partial s} + \frac{1}{ds}\frac{d}{dt}(ds) = 0 \tag{8.45}$$

From Section 8.2, $K = -\frac{dp}{d\forall/\forall} = \frac{dp}{d\rho/\rho}$ so that

$$\frac{1}{\rho}\frac{d\rho}{dt} = \frac{1}{K}\frac{dp}{dt} \tag{8.46}$$

To develop a useful expression for dA/dt in terms of p, the elastic pipe deformations must be considered. For the change in cross-sectional area, Eq. 8.17 shows that

$$dA = \frac{d\forall_c}{dL} = \frac{1}{2}\pi D^2 d\varepsilon_2 = \frac{1}{2}\pi\frac{D^2}{E}\left(d\sigma_2 - \mu d\sigma_1\right) \tag{8.47}$$

$$\frac{1}{A}dA = \frac{2}{E}\left(d\sigma_2 - \mu d\sigma_1\right) \tag{8.48}$$

In evaluating these stresses we will again examine Case (b) restraint; hence

$$d\sigma_2 = \frac{D}{2e}dp \qquad \text{and} \qquad d\sigma_1 = \mu d\sigma_2 \tag{8.49}$$

so

$$d\sigma_2 - \mu d\sigma_1 = \left(1 - \mu^2\right)d\sigma_2 = \left(1 - \mu^2\right)\frac{D}{2e}\frac{dp}{dt} \tag{8.50}$$

Finally,

$$\frac{1}{A}\frac{dA}{dt} = \left(1 - \mu^2\right)\frac{D}{eE}\frac{dp}{dt} \tag{8.51}$$

Considering longitudinal expansion,

$$d(ds) = d\varepsilon_1 ds \tag{8.52}$$

which is zero for Case (b). Thus

$$\frac{1}{ds}\frac{d}{dt}(ds) = 0 \tag{8.53}$$

Combining all these results in Eq. 8.45 gives

$$\frac{1}{K}\frac{dp}{dt} + \left(1 - \mu^2\right)\frac{D}{eE}\frac{dp}{dt} + \frac{\partial V}{\partial s} = 0 \tag{8.54}$$

$$\frac{dp}{dt}\left[\frac{1}{K} + \left(1 - \mu^2\right)\frac{D}{eE}\right] + \frac{\partial V}{\partial s} = 0 \tag{8.55}$$

From Eq. 8.31 it is clear that the term in the brackets is $\frac{1}{a^2\rho}$. This statement is also correct for Case (a) and Case (c) pipe restraint. Making this substitution for the terms in brackets leads to

$$\frac{1}{\rho}\frac{dp}{dt} + a^2\frac{\partial V}{\partial s} = 0 \tag{8.56}$$

When we combine this result with the Euler equation of motion, Eq. 7.19, we have two independent partial differential equations for $p(s,t)$ and $V(s,t)$:

$$\frac{dV}{dt} + \frac{1}{\rho}\frac{\partial p}{\partial s} + g\frac{dz}{ds} + \frac{f}{2D}V|V| = 0 \tag{8.57}$$

$$a^2\frac{\partial V}{\partial s} + \frac{1}{\rho}\frac{dp}{dt} = 0 \tag{8.58}$$

8.5.2. INTERPRETATION OF THE DIFFERENTIAL EQUATIONS

Before moving to the solution of these equations in Chapter 9, we can learn about the nature of these solutions by looking at a linearized subset of the full equations. If we first express the pressure p in terms of the piezometric head H via the relation $p = \rho g(H - z)$, then Eqs. 8.57 and 8.58 become

$$\frac{dV}{dt} + g\frac{\partial H}{\partial s} + \frac{f}{2D}V|V| = 0 \tag{8.59}$$

and

$$a^2\frac{\partial V}{\partial s} + g\frac{dH}{dt} = 0 \tag{8.60}$$

in which the variation of the density ρ is presently assumed to be negligible. The derivative d/dt actually represents both the temporal and convective partial derivative terms. For example,

$$\frac{dV}{dt} = \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial s} \tag{8.61}$$

and similarly for dH/dt. Thus Eqs. 8.59 and 8.60 actually contain two nonlinear convective terms in addition to the nonlinear friction term.

Let us assume for the moment that the linear terms in Eqs. 8.59 and 8.60 are larger than the nonlinear terms and discard the nonlinear terms; we can evaluate later the consequences of this simplification. Then these equations become

$$\frac{\partial V}{\partial t} + g\frac{\partial H}{\partial s} = 0 \tag{8.62}$$

and

$$a^2\frac{\partial V}{\partial s} + g\frac{\partial H}{\partial t} = 0 \tag{8.63}$$

Since the equations are now linear, cross-differentiation and some algebra will allow us to eliminate either one of the two dependent variables V and H in favor of the other one. Thus, if we take the partial derivative of Eq. 8.62 with respect to s and the partial derivative of Eq. 8.63 with respect to t, the algebra leads to

$$\frac{\partial^2 H}{\partial t^2} = a^2\frac{\partial^2 H}{\partial s^2} \tag{8.64}$$

By interchanging the differentiation roles of s and t, we can demonstrate that V is also governed by this equation.

Equation 8.64 is a basic equation of mathematical physics called the wave equation. The parameter a in Eq. 8.64 is known as the wave propagation speed. Hence we expect the solutions of Eq. 8.64 for either H or V to display the behavior of waves. By means of a change in the independent variables, we can deduce the general solution of Eq. 8.64 for H; the procedure is identical for V. We begin with $H = H(s, t)$ and $V = V(s, t)$, and we choose as new independent variables $u = t + s/a$ and $v = t - s/a$. Application of the chain rule of differentiation to compute the new partial derivatives of H then leads to

$$4\frac{\partial^2 H}{\partial u \partial v} = 0 \tag{8.65}$$

as the new form of the governing equation. The general solution immediately follows as

$$H - H_0 = F_1\left(t + \frac{s}{a}\right) + F_2\left(t - \frac{s}{a}\right) \tag{8.66}$$

in which F_1 and F_2 are each an entirely general function of their one argument, and H_0 is an additive constant of integration which fixes the reference level for the head H.

We turn now to the interpretation of the result expressed in Eq. 8.66. Functions F_1 and F_2 are each wave forms; either or both may exist in a particular problem, depending on the particular initial and boundary conditions. We focus first on F_1: at any instant t_1 the wave form H described by the function F_1 can be any function of the distance variable s,

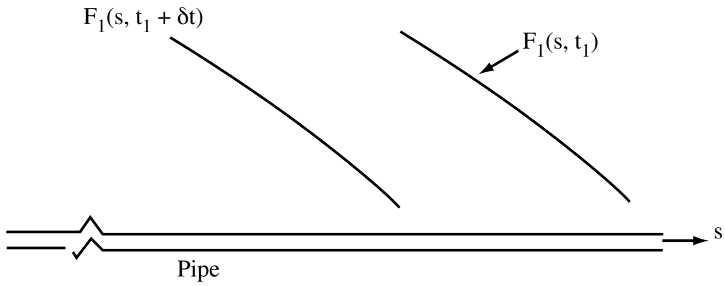


Figure 8.5 Motion of the wave form F_1.

as we see on the right side of Fig. 8.5. So long as the argument $t + s/a$ is unchanged, the
wave form is unchanged. But as the time t advances, the argument $t + s/a$ can only remain constant if s/a decreases by δt as t increases by δt; thus the constant wave form moves in the negative s-direction (to the left), as Fig. 8.5 also shows. We conclude that F_1 describes a left-moving wave. By similar reasoning we find that F_2 describes a right-moving wave form. The general solution to Eq. 8.66 is then a superposition of any number, one or many, of left- and/or right-moving waves.

Is the neglect of the nonlinear convective acceleration terms justified? These are the terms

$$V\frac{\partial V}{\partial s} \quad and \quad V\frac{\partial H}{\partial s} \tag{8.67}$$

If we apply the scaling $s \sim at$ to the terms in Eq. 8.67, we quickly find

$$\begin{aligned} V\frac{\partial V}{\partial s} &\sim \frac{V}{a}\frac{\partial V}{\partial t} \\ V\frac{\partial H}{\partial s} &\sim \frac{V}{a}\frac{\partial H}{\partial t} \end{aligned} \tag{8.68}$$

Since it is almost always true that $V/a << 1$, the convective terms, initially dropped, are nearly always much smaller than the linear terms that were retained. In later solutions the neglect of these terms will often be an acceptable approximation, although in some ways the quality of the resulting solution is less precise. Only in rare problems where V/a is not much smaller than unity is it essential to retain these terms.

8.6 PROBLEMS*

8.1 A high-pressure water system is being designed for use in a lumber mill to remove bark from logs. The main portion of the pipe system is 6-inch steel pipe with walls 0.219 inches thick. The inside diameter of the pipe is 6.187 inches, and the working water pressure in the pipe is 1094 lb/in^2. The allowable stress in the steel pipe walls is 15,000 lb/in^2. Under steady flow conditions the water pressure in the pipe is about 750 lb/in^2 with a flow velocity of 10 ft/s. Because of the nature of the process, there is a need for a very rapid valve closure.

(a) Compute the wave speed for all three types of restraint using the thin-walled pipe formulas.

(b) Make a recommendation as to whether the water hammer pressures developed under sudden valve closure could overstress the pipe.

8.2 A 12-inch PVC line is laid above ground using bell-and-spigot joints. It is restrained laterally to prevent buckling but is free to strain longitudinally. However, concrete anchors at each bend prevent the pipe from blowing apart. The inside diameter of the pipe is 12.09 inches and the wall thickness is 0.311 inches.

(a) Choose the proper restraint case, and compute the wave speed using the thin-walled pipe formula.

(b) Also calculate the percent increase in pipe volume that is caused when a water velocity of 10 ft/s is brought suddenly to rest.

(c) What change in diameter does this represent?

8.3 A long water line is to be constructed of T-30 Transite pipe (asbestos cement). The pipe is rated for a maximum pressure of 300 lb/in^2. The inside diameter of the pipe is 18 inches, and the outside diameter is 19.70 inches. Lengths of pipe are joined with couplings and ring gaskets.

(a) Compute the wave speed for all three types of restraint, assuming the thin-walled pipe formulas apply.

(b) Which wave speed would you recommend using? Why?

8.4 When water hammer occurs in the pipe of Problem 8.3, the water is compressed and the pipe is stretched. What percentage of the volume change can be attributed to water compression and what percentage to pipe expansion? Assume Case (b) restraint applies.

8.5 Calculate the wave speed in the following situations for Case (b) restraint using the thin-walled pipe formulas:

(a) Steel pipe, 36-inch inside diameter, 0.375-inch wall thickness

(b) Ductile cast iron pipe, 18-inch inside diameter, 0.50-inch wall thickness

(c) Aluminum pipe, 4-inch inside diameter, 0.10-inch wall thickness

(d) Asbestos cement, 11.56-inch inside diameter, 1.26-inch wall thickness

(e) Class 125 PVC, 6.22-inch inside diameter, 0.20-inch wall thickness

Note the variation in a for this wide range of pipe sizes and materials.

8.6 For the PVC pipe of Problem 8.5, compute the percent change in cross-sectional area caused by a sudden velocity change of 10 ft/s.

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* Problems 8.1, 8.5, and 8.7-8.11 are adapted from Elementary Fluid Mechanics, by R. L. Street, G. Z. Watters, and J. K. Vennard, Ed. 7, Copyright 1996 by John Wiley & Sons, Inc. Reprinted by permission.

8.7 The wall of a steel pipe 8000 ft long, with a 6 ft inside diameter and a wall thickness of 0.50 inches, is stressed at 7000 lb/in^2. Water is flowing at 5 ft/s.

(a) For Case (a) restraint and sudden valve closure, what amount of water enters the pipe after valve closure?

(b) How is this volume distributed among radial stretching, longitudinal stretching, and water compression?

8.8 Calculate the wave speed in a water-filled copper tube that is installed without longitudinal restraint. The tube has a 0.375 inch inside diameter, is 75 ft long and has a wall thickness of 0.03 inches. The steady state pressure in the tube is 73 lb/in^2.

8.9 A Class 51 ductile iron pipe conveys water between two reservoirs. The outside diameter is 15.30 inches and, for this class of pipe, the wall thickness is 0.36 inches. Assuming the pipe can be considered to be thin-walled, compute the wave speed for all three types of restraint.

8.10 For the pipe of Problem 8.9, compute the percent change in pipe volume that occurs as the result of a sudden stoppage of a 10 ft/s flow of water. Use Case (b) restraint.

What is the percent change in the density of the water?

8.11 A plastic supply pipe in a building water system is anchored at both ends and has expansion joints along its entire length. The line is 800 ft long, 6.00 inches inside diameter with a 0.200 inch wall thickness. The modulus of elasticity for this material is 500,000 lb/in^2. Water in the pipe normally flows at 10 ft/s, and the system valves are designed to close very quickly.

(a) If $\mu = 0.5$ for this material, what is the wave speed?

(b) If the steady state pressure in the pipe is about 100 lb/in^2, estimate the maximum pressure that could occur in the system under the worst water hammer conditions?

(c) What would be the stresses in the pipe walls under these conditions?

8.12 An aluminum irrigation pipeline laid on level ground consists of a series of 20-ft pieces connected rigidly together. The two ends of the pipeline are anchored solidly to the ground. If the pipe is 8 inches in inside diameter with 0.10-inch walls, calculate the wave speed in the pipe.

8.13 Class 100 PVC pipe has a nominal diameter of 8 in but has an actual inside diameter of 8.205 in and a wall thickness of 0.210 in.

(a) Calculate the wave speed in this pipe for all three types of restraint using the thin-walled pipe formulas.

(b) Also calculate the percent change in cross-sectional area of this pipe for Case (b) restraint when a flow of velocity of 8 ft/s is suddenly brought to rest.

8.14 For the pipe of Problem 8.13, calculate the wave speed for Case (b) restraint using the thick-walled pipe formulas.

8.15 In Problem 8.1 the wave speeds for the three restraint conditions were computed, assuming thin-walled pipe, as 4190 ft/s, 4210 ft/s, and 4170 ft/s for Cases (a), (b), and (c), respectively.

(a) Use the thick-walled pipe formulas to recompute the wave speeds, and compute the percent error caused by using the thin-walled pipe formulas.

(b) For sudden stoppage of a 10 ft/s flow, what is the error in pressure head increase when the thin-walled pipe formulas are used?

(c) Is the more accurate figure conservative?

8.16 In Problem 8.3 the wave speeds were computed with the thin-walled pipe formulas for T-30 Transite pipe as 2830 ft/s, 2870 ft/s, and 2790 ft/s for Cases (a), (b), and (c), respectively. Recompute the three wave speeds using the thick-walled pipe formulas and find the percent change. Will the use of more accurate wave speeds result in more conservative pressure head changes?

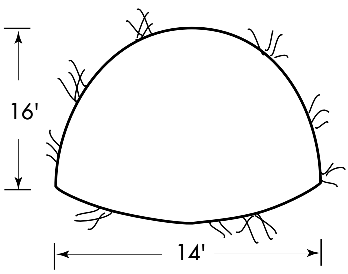
8.17 A Class 200 PVC pipe has an inside diameter of 3.146 inches and an outside diameter of 3.500 inches. The pipe sections are to be connected by the bell-and-spigot method using ring gaskets. When placed in a trench, the pipeline will be anchored at the ends and at all bends by concrete anchor blocks. Compute the wave speed for the pipeline.

8.18 An unlined power tunnel is to be excavated through limestone between a diversion dam and hydroelectric power plant penstocks. If the tunnel is approximately circular in cross section and 14.5 ft in diameter, what wave speed is appropriate for use in water hammer calculations?

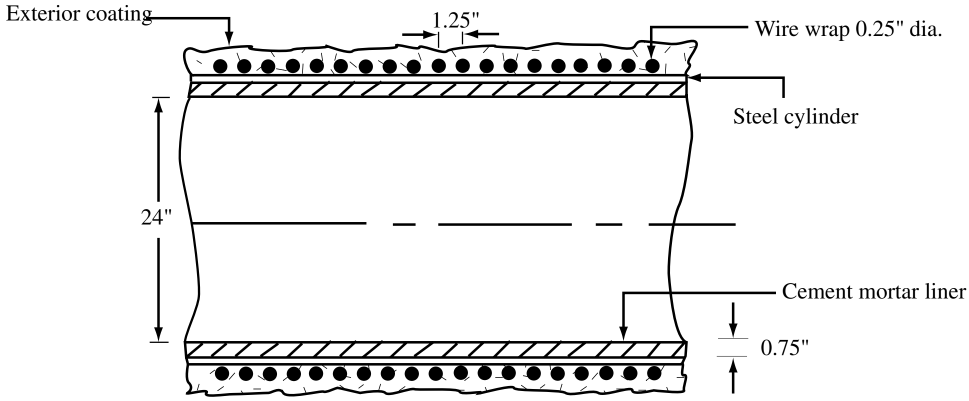
8.19 A 12-inch-diameter hydraulic conduit is drilled through a massive part of a concrete gravity dam. Estimate the wave speed in this situation for concrete with a 28-day strength of 4000 lb/in^2.

8.20 A circular tunnel which is 18 ft in diameter is being cut through Sierra granite as part of a hydroelectric power development. For purposes of a water hammer study, compute the wave speed that would apply in this circumstance.

8.21 The power tunnel for a hydroelectric power plant is unlined and cut through quartzite. The tunnel has the dimensions shown below. Estimate the wave speed to be used for water hammer analysis.

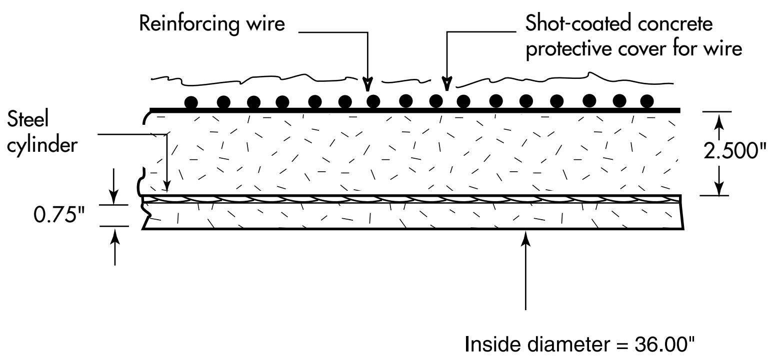


8.22 Pretensioned concrete cylinder pipe is used in a long water-supply transmission line. The pipe shown below has 24-in inside diameter, and the steel cylinder is 14 gage (0.0747 in). The steel wrapping wire is 0.25 in in diameter and spaced on 1.25-in centers. The cement-mortar liner is 0.75 in thick and has a 28-day strength of 4500 lb/in^2. The exterior coating is 0.75 in thick over the wire and is for the protection of the wire only. Calculate the wave speed in the pipe.



8.23 A 36-in inside diameter reinforced concrete pipe has steel reinforcing rod wrapped around the pipe midway within a 4-in wall. The steel rod has an average area of 0.80 in^2 per lineal foot of pipe, and each wrap is spaced 3 in on center. Assuming the gasket joints act as expansion joints, estimate the wave speed in the pipe, both including and excluding the concrete. The 28-day strength of the concrete is 6000 lb/in^2.

8.24 A water project under design will be using embedded-cylinder, prestressed concrete pipe. The cylinder is to be fabricated from 10 gage (0.1345 in) steel, and the 7/16-in wire will be wrapped on 1.00-in centers. A sketch of a longitudinal section of the pipe is shown below. All of the concrete in the pipe will have a 28-day strength of 5000 lb/in^2. Recommend a wave speed to be used in a water hammer analysis.



8.25 A pretensioned 48-inch inside diameter reinforced concrete pipe similar in configuration to that of Problem 8.22 is manufactured using a 10 gage (0.1345 inch) steel cylinder wrapped with 0.244-inch diameter steel wire on 1.00-inch centers. The total wall thickness of the pipe is 5.0 inches with a 3.00-inch cement-mortar lining. The cement liner has a 28-day strength of 6000 lb/in^2. Assuming the gasket joints act as expansion joints, calculate the wave speed for the pipe.

CHAPTER 9

SOLUTION BY THE METHOD OF CHARACTERISTICS

The history of water hammer analysis is marked by various clever and practical techniques for solving the Euler and conservation of mass equations derived in Chapters 7 and 8. Those methods were a reflection of the level of sophistication of the numerical analysis capabilities of their time as well as the ingenuity of the practitioners. In recent years the availability of low-cost, high-performance desktop computers has led to the creation of solution methods for these equations which are numerically very accurate and are capable of incorporating a wide range of boundary and initial conditions.

At this time the most general and widely-used technique for solving these equations is the *method of characteristics*. It is no coincidence that this method is very compatible with numerical solution by digital computer. For this reason we consider only the method of characteristics approach to problem solving in this and following chapters.

9.1 METHOD OF CHARACTERISTICS, APPROXIMATE GOVERNING EQUATIONS

9.1.1. DEVELOPMENT OF THE CHARACTERISTIC EQUATIONS

Anticipating that many engineers are even today unfamiliar with the method of characteristics as a solution technique, we first introduce the method using approximate versions of Eqs. 8.57 and 8.58. These approximate equations are obtained by neglecting the spatial variation of V and p whenever both space- and time-varying terms appear in the same equation. We do this because, in general, the spatial variations are much less significant in determining the solution behavior than are the time-varying terms.

Following this approach, Eq. 8.57 becomes

$$\frac{\partial V}{\partial t} + \frac{1}{\rho}\frac{\partial p}{\partial s} + g\frac{dz}{ds} + \frac{f}{2D}V|V| = 0 \tag{9.1}$$

and Eq. 8.58 becomes

$$a^2\frac{\partial V}{\partial s} + \frac{1}{\rho}\frac{\partial p}{\partial t} = 0 \tag{9.2}$$

The essence of the method of characteristics is the successful replacement of a pair of *partial* differential equations by an equivalent set of *ordinary* differential equations. The development of the method begins by presuming that the pair of Eqs. 9.1 and 9.2 may be replaced by some linear combination of themselves. Using l as a constant linear scale factor, sometimes called a Lagrange multiplier, one possible combination is

$$\lambda\left(\frac{\partial V}{\partial t} + \frac{1}{\rho}\frac{\partial p}{\partial s} + g\frac{dz}{ds} + \frac{f}{2D}V|V|\right) + \left(a^2\frac{\partial V}{\partial s} + \frac{1}{\rho}\frac{\partial p}{\partial t}\right) = 0 \tag{9.3}$$

Regrouping terms,

$$\left(\lambda \frac{\partial V}{\partial t} + a^2 \frac{\partial V}{\partial s}\right) + \left(\frac{1}{\rho}\frac{\partial p}{\partial t} + \frac{\lambda}{\rho}\frac{\partial p}{\partial s}\right) + \lambda g \frac{dz}{ds} + \frac{\lambda f}{2D} V|V| = 0 \qquad (9.4)$$

We note that if $\lambda \frac{\partial V}{\partial t} + a^2 \frac{\partial V}{\partial s}$ is to be replaced by $\lambda \frac{dV}{dt}$, then $\lambda \frac{ds}{dt} = a^2$. Further, if $\frac{1}{\rho}\frac{\partial p}{\partial t} + \frac{\lambda}{\rho}\frac{\partial p}{\partial s}$ is to be replaced by $\frac{1}{\rho}\frac{dp}{dt}$, then $\frac{\lambda}{\rho} = \frac{1}{\rho}\frac{ds}{dt}$. To satisfy these two requirements for $\frac{ds}{dt}$, we discover that $\lambda^2 = a^2$ or

$$\lambda = \pm a \qquad (9.5)$$

The scale factor l is linear and constant, as required, so long as a is constant, and we have succeeded in combining Eqs. 9.1 and 9.2. We first rewrite Eq. 9.3 with $l = +a$ as a replacement for the first equation. Then we rewrite Eq. 9.3 with $l = -a$ as a replacement for the second equation. Upon dividing the resulting equations by the wave speed a, we have a pair of ordinary differential equations rather than partial differential equations:

$$\frac{dV}{dt} + \frac{1}{a\rho}\frac{dp}{dt} + g\frac{dz}{ds} + \frac{f}{2D}V|V| = 0 \qquad (9.6)$$

$$\frac{dV}{dt} - \frac{1}{a\rho}\frac{dp}{dt} + g\frac{dz}{ds} + \frac{f}{2D}V|V| = 0 \qquad (9.7)$$

However, there are now special restrictive conditions on the independent variables in each equation. Equation 9.6 is subject to the requirement that $\lambda \frac{ds}{dt} = a^2$, so $\frac{ds}{dt} = \frac{a^2}{\lambda} = +a$. Therefore, Eq. 9.6 is valid only when $\frac{ds}{dt} = +a$. Similarly, Eq. 9.7 is valid only when $\frac{ds}{dt} = -a$. Thus we have replaced two partial differential equations by two pairs of ordinary differential equations, and we must follow these rules which relate the independent variables s and t. As we believe it is easier to visualize the propagation of pressure waves in terms of the piezometric head $p = g(H - z)$ the height of the EL-HGL above a horizontal datum (commonly sea level), we convert from p to H.

The new form of Eqs. 9.6 and 9.7 is now

$$\frac{dV}{dt} + \frac{g}{a}\frac{dH}{dt} + \frac{f}{2D}V|V| = 0 \quad \text{only when} \quad \frac{ds}{dt} = +a \qquad (9.8)$$

$$\frac{dV}{dt} - \frac{g}{a}\frac{dH}{dt} + \frac{f}{2D}V|V| = 0 \quad \text{only when} \quad \frac{ds}{dt} = -a \qquad (9.9)$$

From the fact that special relations must be maintained between s and t in Eqs. 9.8 and 9.9, the equations $\frac{ds}{dt} = +a$ and $\frac{ds}{dt} = -a$ have come to be called the *characteristics* of Eqs. 9.8 and 9.9, hence the name of the analysis procedure.

To see how we use these characteristic equations in a solution, we work with a graph having s as the abscissa and t as the ordinate, referred to as the s-t plane. Figure 9.1 shows how the s-t plane is related to the physical problem. Here the s-coordinate is the

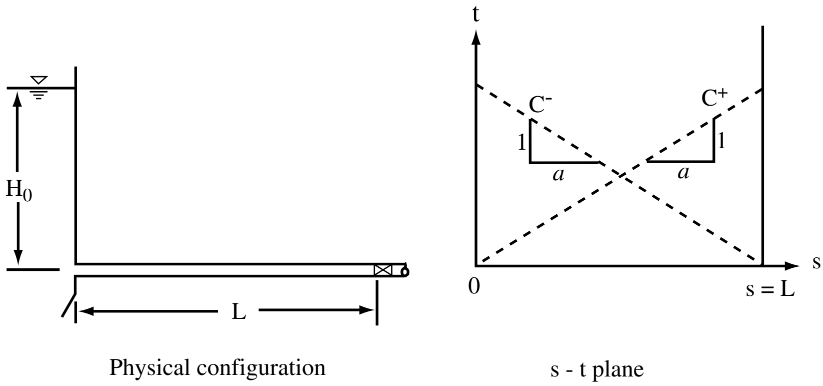


Figure 9.1 The s-t plane for the simple flow of Chapter 7.

distance along the pipe from the upstream end. With a as a constant, the characteristic equation for Eq. 9.8, $\frac{ds}{dt} = +a$, can easily be integrated (after inverting to get into proper form) to yield $t = s/a + \text{constant}$. This equation describes a family of straight lines of slope $1/a$ on the s-t plane; the position of any one line depends on the constant of integration. Because these lines are associated with the characteristic equation having the $+$ sign for a, they are referred to as C^+ characteristics. Figure 9.1 depicts a C^+ characteristic passing through the origin. Similarly, the characteristic equation for Eq. 9.9 describes a family of straight lines on the s-t plane with a slope $-1/a$. The characteristics of Eq. 9.9 are referred to as C^- characteristics; one is plotted in Fig. 9.1 passing through the point $s = L$.

Let us revisit the simple water hammer illustration of Chapter 7 to understand further the concept of characteristics. In that example a valve at the downstream end of a pipeline was suddenly closed, causing a pressure wave to propagate upstream at speed a. Friction was neglected. For this situation Eqs. 9.8 and 9.9, with negligible friction, can be written

$$\frac{dV}{dt} \pm \frac{g}{a}\frac{dH}{dt} = 0 \tag{9.10}$$

Multiplying by dt and rearranging,

$$dV = \pm\frac{g}{a}dH \quad \text{or} \quad dH = \pm\frac{a}{g}dV \tag{9.11}$$

This equation has the same form (replacing dV and dH with ΔV and ΔH) as the equation for pressure head increment that was derived earlier as Eq. 8.8. Tracing the C^- characteristic which has a slope of $1/a$ from right to left, we note from Fig. 9.1 that the wave reaches the origin (upstream end of the pipe) at $t = L/a$. Both of these results validate the simple water hammer analysis of Chapter 7 and show that pressure waves propagate along the characteristic lines. As we will see later, this important physical fact is crucial in obtaining reliable results from the numerical analysis.

With a physical grasp of Eqs. 9.8 and 9.9 now in hand, we will proceed to formalize the solution process more carefully. To compute values of H and V at various locations along the pipe as functions of time, we must begin with a knowledge of *initial* conditions along the s-axis of the s-t plane and *boundary* conditions for all time at the pipe ends $s = 0$ and $s = L$. Then a solution for values of H and V can "march" forward (upward) in the s-t plane.

To see how this marching process works, refer to Fig. 9.2, which is an s-t plane for some as yet undefined problem. At any point on the s-t plane, say point P, the values

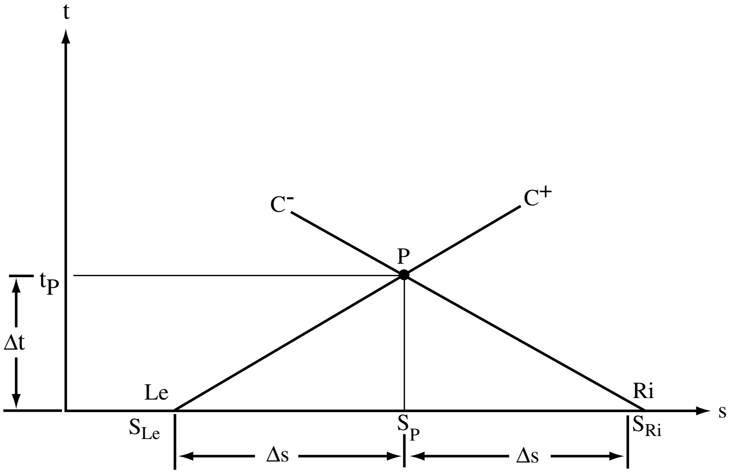


Figure 9.2 The s-t plane showing characteristics for Eqs. 9.8 and 9.9.

of the continuous variables H and V are unique (i.e., the H and V values are independent of the characteristic with which they are associated). We next draw the C^+ and C^- characteristic lines through point P and extend them to intersect the s-axis at points on the left and right sides of P, here called Le and Ri, respectively. Note that these two points are in this approximate case each the same distance Δs from P. Equation 9.8 applies along the C^+ characteristic, and Eq. 9.9 applies along the C^- characteristic. The information which determines H and V will propagate forward in time along these two characteristics from Le and Ri.

9.1.2. THE FINITE DIFFERENCE REPRESENTATION

In seeking a numerical solution to our problem, we write Eqs. 9.8 and 9.9 in finite difference form. Equation 9.8 becomes

$$\frac{V_P - V_{Le}}{t_P - 0} + \frac{g}{a}\frac{H_P - H_{Le}}{t_P - 0} + \frac{f}{2D}V_{Le}\left|V_{Le}\right| = 0 \tag{9.12}$$

and Eq. 9.9 becomes

$$\frac{V_P - V_{Ri}}{t_P - 0} - \frac{g}{a}\frac{H_P - H_{Ri}}{t_P - 0} + \frac{f}{2D}V_{Ri}\left|V_{Ri}\right| = 0 \tag{9.13}$$

We have made two significant assumptions in developing these equations. First we assume that the velocity at the beginning of the time interval, rather than an average velocity over the interval, adequately represents the frictional effect. The computational implications are significant. If we were to include the unknown value V_P in the friction term, the difference equations would become nonlinear and require an iterative solution. In view of the tremendous number of times that we will solve these equations and the sometimes troublesome nature of nonlinear solution techniques, we choose not to employ that approach. With the generally small time increments in the solution of transient problems, we intuitively expect that this simplification will not cause significant inaccuracies in our results.

We also assume that the steady-state friction coefficient can adequately represent friction losses in a transient flow. The assumption of a non-transient constant friction coefficient in transient analyses has always been an approximation. The use of the steady-state Darcy-Weisbach f implies that the flow in the pipe is behaving as a wholly rough flow. That is, the f which would normally be changing with Reynolds number as the transient velocity changes, is kept constant. Use of the Hazen-Williams C partially compensates for this problem by letting an equivalent "f" adjust somewhat with the transient velocity change. However, neither method is based on the fundamental behavior of transient flow.

As the velocity changes relatively rapidly, even reversing direction, the velocity profile becomes quite complex. The calculation of the shear stress and energy dissipation is difficult. Silva-Araya and Chaudhry (1997) provide a state-of-the-art assessment of this problem; they retain the friction coefficient (the Darcy-Weisbach f) but multiply it by an energy dissipation factor to account for the additional friction loss in a transient flow. The method employs a dissipation function for axisymmetric flow which includes the effects of both viscous and turbulent stresses. While this approach shows promise, the extra computational effort currently appears excessive for practical use. Consequently we continue to use the traditional approach of employing steady-state friction coefficients.

We now replace $t_P - 0$ with Δt in the above equations so the analysis will apply to more than the first time interval. Multiplying these equations by Δt gives

$$C^+ : \; \left(V_P - V_{Le}\right) + \frac{g}{a}\left(H_P - H_{Le}\right) + \frac{f\Delta t}{2D} V_{Le}\left|V_{Le}\right| = 0 \tag{9.14}$$

and

$$C^- : \; \left(V_P - V_{Ri}\right) - \frac{g}{a}\left(H_P - H_{Ri}\right) + \frac{f\Delta t}{2D} V_{Ri}\left|V_{Ri}\right| = 0 \tag{9.15}$$

These equations will be referred to as the C^+ and C^- equations, respectively.

The characteristic equations can also be written in finite difference form as

$$\Delta s = \pm\, a\,\Delta t \tag{9.16}$$

Now, proceeding with the finite difference numerical solution, we must select a spatial interval in the s-direction, i.e., the number of sections into which the pipe will be divided. If we decide to divide the pipe into N sections, then each section will be of length $\Delta s = L/N$. This decision fixes Δs, and Eq. 9.16 is then used to compute Δt. We can now construct the *grid of characteristics* shown in Fig. 9.3 atop the next page.

Grid points along the s-axis represent points spaced Δs apart along the pipe axis, and the values of H and V at these points are *initial conditions*. Usually these initial conditions are a set of values of H and V which describe a steady flow in the pipeline at the moment a transient begins. With the known values from points Le and Ri we can now solve Eqs. 9.14 and 9.15 simultaneously to obtain the values H_P and V_P at points 2 through N at time $t = \Delta t$. The boundary conditions at $s = 0$ and $s = L$ must be used in conjunction with the appropriate C^+ or C^- equation to compute the values of H_{P_1} and $H_{P_{N+1}}$. This completes the solution for all the values of H and V at time $t = \Delta t$. We next compute the values of H and V at time $t = 2\Delta t$ using the just-computed values at $t = \Delta t$ as the known values in Eqs. 9.14 and 9.15. This process is repeated continuously as we march ahead in the s-t plane.

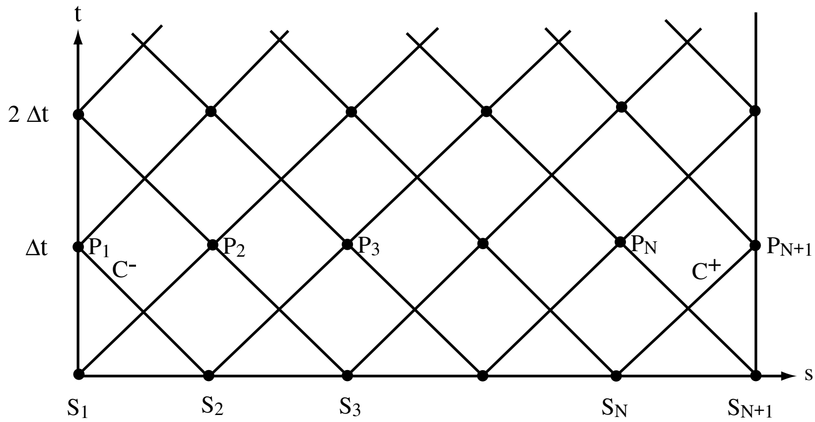


Figure 9.3 The characteristic grid for a single pipe.

Finally, we should emphasize an important conceptual point arising from our analysis. Any change in the velocity or head at a point in the pipeline cannot be sensed at another point in the pipeline until the pressure wave has had time to propagate at the wave speed to that section. This effect is illustrated in Fig. 9.4 showing where and when a disturbance at

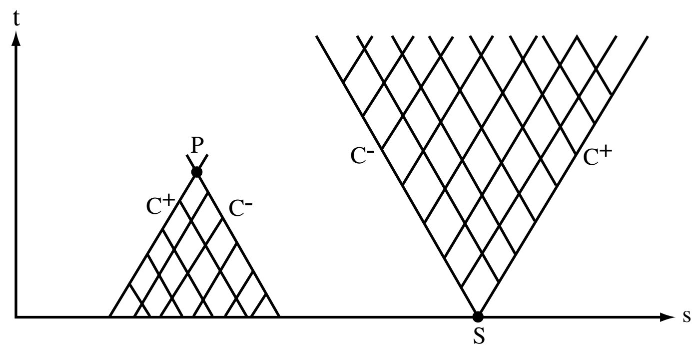


Figure 9.4 Disturbance propagation in the s-t plane.

S can be sensed at subsequent times. A corollary to this concept is also illustrated in Fig. 9.4; the values of H and V at a point P can only be affected by events contained within the zone formed by the subtended C^+ and C^- characteristics.

9.1.3. SETTING UP THE NUMERICAL PROCEDURE

In the previous section we have developed finite difference equations which permi t us to calculate H and V at predetermined intersections of the C^+ and C^- characteristics. The values of H and V at the ends of the pipe were determined by using boundary conditions. Now we will arrange the solution procedure so it can be conveniently implemented on a computer.

First we develop a pair of equations to find H and V at the interior points (points 2 through N). We do this by solving Eqs. 9.14 and 9.15 simultaneously to obtain

$$V_P = \frac{1}{2}\left[\left(V_{Le} + V_{Ri}\right) + \frac{g}{a}\left(H_{Le} - H_{Ri}\right) - \frac{f\Delta t}{2D}\left(V_{Le}\left|V_{Le}\right| + V_{Ri}\left|V_{Ri}\right|\right)\right] \tag{9.17}$$

$$H_P = \frac{1}{2}\left[\frac{a}{g}\left(V_{Le} - V_{Ri}\right) + \left(H_{Le} + H_{Ri}\right) - \frac{a}{g}\frac{f\Delta t}{2D}\left(V_{Le}\left|V_{Le}\right| - V_{Ri}\left|V_{Ri}\right|\right)\right] \tag{9.18}$$

The boundary conditions at each end of the pipe describe externally-imposed conditions on velocity and/or pressure head. To aid the reader in understanding how boundary conditions are applied, we will examine a few common ones now.

Reservoir boundary condition (upstream end of pipe)

Where a pipe exits from a reservoir, the head H assumes the value corresponding to the head of the reservoir water surface. If the water surface elevation is constant in time, then H is constant. If the reservoir water surface elevation changes with time, so too does H, if the local pipe entrance loss is neglected. This is represented in equation form as

$$H_{P_1} = H_0 \tag{9.19}$$

This value for H_{P_1} is substituted (R = 2) into Eq. 9.15 to yield an expression for velocity:

$$V_{P_1} = V_2 + \frac{g}{a}\left(H_0 - H_2\right) - \frac{f\Delta t}{2D} V_2\left|V_2\right| \tag{9.20}$$

If the reservoir were at the downstream end of the pipe, the same approach using the C^+ equation would give a similar expression for $V_{P_{N+1}}$.

Velocity boundary condition (downstream end of pipe)

When the velocity is known at the downstream end of a pipe, this information can be combined with the C^+ characteristic equation to develop an equation for $H_{P_{N+1}}$. For example, suppose a valve is closed so that the velocity decreased linearly from V_0 to zero in T_c seconds. The velocity behavior is

$$\begin{aligned} V_{P_{N+1}} &= V_0\left(1 - \frac{t}{T_c}\right), \quad && 0 \le t \le T_c \\ V_{P_{N+1}} &= 0, && t \ge T_c \end{aligned} \tag{9.21}$$

The equation for $H_{P_{N+1}}$ can be found by substituting Eqs. 9.21 into Eq. 9.14 to give

$$H_{P_{N+1}} = H_N - \frac{a}{g}\left(V_{P_{N+1}} - V_N\right) - \frac{a}{g}\frac{f\Delta t}{2D} V_N\left|V_N\right| \tag{9.22}$$

for any value of $V_{P_{N+1}}$, including zero.

Constant speed pump boundary condition (upstream end of pipe)

This boundary condition offers the added complexity of having both H_{P_1} and V_{P_1} in the boundary equation. Consequently the boundary equation must be solved simultaneously with Eq. 9.15 to produce equations for H_{P_1} and V_{P_1}.

We now choose a way to represent the pump boundary condition. The simplest approach that is reasonably general is to represent the pump discharge characteristics by a quadratic equation of the form

$$h_p = A_p{}'\, Q^2 + B_p{}'\, Q + C_p{}' \tag{9.23}$$

in which Q is the pump discharge and h_p is the head increase across the pump. These variables are not identical to those in the C^- equation, so we make some adjustments. We replace Q with $V_{P_1}A$ and h_p with $H_{P_1} - H_{sump}$. Incorporating H_{sump} into $C_p{}'$ and A into $A_p{}'$ and $B_p{}'$ leads to

$$H_{P_1} = A_p V_{P_1}^2 + B_p V_{P_1} + C_p \tag{9.24}$$

We note for future use that if this curve is to be concave down and always sloping downward for increasing Q (generally it should do this), then $A_p<0$, $B_p<0$, and $C_p>0$. This information is needed in computerizing this boundary condition.

When Eq. 9.24 is solved simultaneously with the C^- characteristic equation, Eq. 9.15, the elimination of H_{P_1} leads to the following equation for V_{P_1};

$$V_{P_1} - V_2 - \frac{g}{a}\left(A_p V_{P_1}^2 + B_p V_{P_1} + C_p\right) + \frac{g}{a} H_2 + \frac{f\Delta t}{2D} V_2 \left|V_2\right| = 0 \tag{9.25}$$

Rearranging, we get

$$\left(\frac{g}{a} A_p\right) V_{P_1}^2 + \left(\frac{g}{a} B_p - 1\right) V_{P_1} + \left(V_2 + \frac{g}{a} C_p - \frac{g}{a} H_2 - \frac{f\Delta t}{2D} V_2 \left|V_2\right|\right) = 0 \tag{9.26}$$

This quadratic equation can now be solved for V_{P_1}. Then a back substitution into Eq. 9.24 will yield H_{P_1}.

Incidentally, if a (loss-free) check valve were installed downstream of the pump, we could model it mathematically by first computing V_{P_1} from Eq. 9.26 and then checking the sign of the velocity; if it were negative, we would set V_{P_1} to zero before calculating H_{P_1} from the C^- characteristic equation, Eq. 9.15.

9.1.4. COMPUTERIZING THE NUMERICAL PROCEDURE

The problem-solving approach we have developed is relatively easy to program for the computer. Since we divided the pipe into N sections, the node points between sections can be numbered sequentially from 1 to $N+1$, beginning at the upstream end of the pipe. Keeping in mind the connection between the subscripts in our equations and the indices of the subscripted variables in computer programs, we rewrite Eqs. 9.14 and 9.15 as

$$C^+ : \left(V_{P_i} - V_{i-1}\right) + \frac{g}{a}\left(H_{P_i} - H_{i-1}\right) + \frac{f\Delta t}{2D} V_{i-1}\left|V_{i-1}\right| = 0 \tag{9.27}$$

$$C^- : \left(V_{P_i} - V_{i+1}\right) - \frac{g}{a}\left(H_{P_i} - H_{i+1}\right) + \frac{f\Delta t}{2D} V_{i+1}\left|V_{i+1}\right| = 0 \tag{9.28}$$

The solutions for the interior values of H_P and V_P (Eqs. 9.17 and 9.18) are now

$$V_{P_i} = \frac{1}{2}\left[\left(V_{i-1} + V_{i+1}\right) + \frac{g}{a}\left(H_{i-1} - H_{i+1}\right) - \frac{f\Delta t}{2D}\left(V_{i-1}\left|V_{i-1}\right| + V_{i+1}\left|V_{i+1}\right|\right)\right] \tag{9.29}$$

$$H_{P_i} = \frac{1}{2}\left[\frac{a}{g}\left(V_{i-1} - V_{i+1}\right) + \left(H_{i-1} + H_{i+1}\right) - \frac{a}{g}\frac{f\Delta t}{2D}\left(V_{i-1}\left|V_{i-1}\right| - V_{i+1}\left|V_{i+1}\right|\right)\right] \tag{9.30}$$

for $2 \le i \le N$.

All boundary conditions must also be written in a form that is consistent with the subscripted-variable approach. The boundary conditions for the reservoir and the linearly-varying velocity are already in the proper form; however, the constant-speed pump boundary condition, which requires the solution of two simultaneous equations, needs further work. The practitioner must perform the algebra and then program the computer. We will now examine this process in detail because boundary conditions which lead to pairs of equations which must be solved are very common in transient problems. This example also employs a technique for handling the bulky C^+ and C^- equations in an efficient way which can be directly transported to the computer code.

Constant-speed pump revisited

For subsequent algebraic manipulation it is convenient to simplify the equations by representing a collection of known terms by a single symbol. Here we can write Eq. 9.28, applicable to the upstream end of the pipe, as

$$V_{P_1} = C_1 + C_2 H_{P_1} \tag{9.31}$$

where

$$C_1 = V_2 - \frac{g}{a}H_2 - \frac{f\Delta t}{2D}V_2\left|V_2\right| \qquad C_2 = \frac{g}{a} \tag{9.32}$$

In the computer C_1 and C_2 will just be numbers because they were calculated by using known values from the previous time.

Combining Eq. 9.31 with Eq. 9.24 to eliminate H_{P_1} gives

$$\frac{V_{P_1} - C_1}{C_2} = A_p V_{P_1}^2 + B_p V_{P_1} + C_p \tag{9.33}$$

Preparing the equation in standard quadratic form,

$$V_{P_1}^2 + \frac{B_p - 1/C_2}{A_p}V_{P_1} + \frac{C_p + C_1/C_2}{A_p} = 0 \tag{9.34}$$

Letting $C_3 = \dfrac{B_p - 1/C_2}{A_p}$ and $C_4 = \dfrac{C_p + C_1/C_2}{A_p}$, this equation becomes

$$V_{P_1}^2 + C_3 V_{P_1} + C_4 = 0 \tag{9.35}$$

The solution is

$$V_{P_1} = \frac{C_3}{2}\left[-1 \pm \sqrt{1 - \frac{4C_4}{C_3^2}}\right] \tag{9.36}$$

It only remains to determine which of the $\pm$ signs to use.

This sign decision must be made many more times, so we examine the process in detail now. We begin by determining the sign (where possible) of the C-terms. From Eq. 9.32 one can see that

$$C_1 = \text{ unknown sign} \qquad C_2 = (+) \tag{9.37}$$

Assuming the usual behavior for the coefficients of the pump model, Eq. 9.23, one has

$$A_p = (-) \qquad B_p = (-) \qquad C_p = (+) \tag{9.38}$$

From the definition equations for C_3 and C_4,

$$C_3 = \frac{(-)-(+)}{(-)} = (+) \qquad C_4 = \frac{(+)+(\text{unknown})}{(-)} = (\text{unknown}) \tag{9.39}$$

We conclude that C_3 is always positive, and we are not sure of the sign of C_4. Equation 9.36 can be written in terms of signs as

$$\left(\text{sign } V_{P_1}\right) = (+)\left[-1 \pm \sqrt{1 - \frac{(?)}{(+)}}\right] \tag{9.40}$$

At the beginning of a steady flow process V_{P_1} is positive; thus we must be able to obtain some positive values from this equation. This can only happen if the term in the brackets is positive. Because there must also be the possibility of negative velocities (see Chapter 7), the term in the brackets must also take on negative values. Because the square root must be positive, the only possibility that could lead to [ ] being either positive or negative would occur when a $+$ sign is selected from the $\pm$ option. We are left with the equation for V_{P_1} as

$$V_{P_1} = \frac{C_3}{2}\left[-1 + \sqrt{1 - \frac{4C_4}{C_3^2}}\right] \tag{9.41}$$

and

$$H_{P_1} = \frac{V_{P_1} - C_1}{C_2} \tag{9.42}$$

Revisiting the issue of backflow, we might assume a check valve is installed. To simulate the check valve, we would test V_{P_1} and, if it were negative, set $V_{P_1} = 0$. We then go ahead and calculate H_{P_1} from Eq. 9.42. As before, we could not use Eq. 9.24 to compute H_{P_1} because the check valve has isolated the pump from the pipeline.

9.1.5. ELEMENTARY COMPUTER PROGRAMS

The first elementary computer program is presented in Fig. 9.5 to demonstrate the structure of a water hammer analysis computer code. The program is written in FORTRAN and provided in dynamic array dimensional form both as source code and executable elements on the enclosed CD ROM. The source listing here will enable the reader to study the various blocks of code comprising the analysis.

The program has NAMELIST input which makes the input data file easier to read. The input parameters are identified in the program under the NAMELIST /SPECS/ statement. Each input parameter is defined in the COMMENT statements at the beginning of the program listing.

In this basic program the user must perform the steady-state hydraulic calculations required to provide some of the input data to the program. These steady-state values are the initial conditions which are entered into a data file created by the user; it is to be read by the program at execution time. The boundary conditions in this program are written into the source code and consist of a constant-head reservoir at the upstream end and a linearly-decreasing velocity at the downstream end. To use the program for any other boundary conditions would require the user to modify the code itself and then recompile it, as will be explained further with the second program in this section.

The program will simulate the water hammer process until the time of simulation reaches TMAX. At that time the program will cease execution; it will then determine and print the maximum and minimum values of pressure head, *H*, and *V* that occurred at each node. The output from the analysis will be printed in an output file designated by the user in response to a prompt during execution.

```
  PROGRAM PROG1
*************************************************************************
*   PROGRAM NO. 1
*   APPROXIMATE-METHOD WATER HAMMER PROGRAM FOR A SINGLE STRAIGHT PIPE.
*
*   THIS PROGRAM HAS BEEN INCLUDED FOR THE CONVENIENCE OF THE READER.
*   THE AUTHOR ACCEPTS NO RESPONSIBILITY FOR ITS CORRECTNESS.
*   USERS OF THIS PROGRAM DO SO AT THEIR OWN RISK.
*************************************************************************
*                CONSTANT-HEAD RESERVOIR AT UPSTREAM END
*   VELOCITY DECREASES LINEARLY WITH TIME TO ZERO AT DOWNSTREAM VALVE
*
*                 *****  DATA DESCRIPTION  *****
*   TITLE1 =  FIRST JOB DESCRIPTION.  ANY INFORMATION, 80 COLUMNS MAXIMUM
*   TITLE2 = SECOND JOB DESCRIPTION.  ANY INFORMATION, 80 COLUMNS MAXIMUM
*   IOUT = PRINT OUTPUT INDEX. GIVES PRINTED OUTPUT EVERY IOUT-TH TIME
*          STEP. FOR EXAMPLE, IF IOUT = 3, THEN OUTPUT IS PRINTED EVERY
*          THIRD TIME STEP.
*   NPARTS = NUMBER OF PIPE SEGMENTS INTO WHICH PIPE IS DIVIDED
*   D = PIPE DIAM, IN        L = PIPE LENGTH, FT
*   F = DARCY-WEISBACH  F-VALUE OR HAZEN-WILLIAMS  C-VALUE
*   A = WAVE SPEED, FT/S     VZERO = INITIAL STEADY STATE VELOCITY, FT/S
*   HZERO = UPSTEAM RESERVOIR ELEVATION, FT
*   ELEVUP = ELEVATION OF UPSTREAM END OF PIPE, FT
*   ELEVDN = ELEVATION OF DOWNSTREAM END OF PIPE, FT
*   TMAX = MAXIMUM REAL TIME OF SIMULATION, SEC
*   TCLOSE = TIME REQUIRED FOR VALVE CLOSURE, SEC
      DIMENSION X[ALLOCATABLE](:),V[ALLOCATABLE](:),H[ALLOCATABLE](:),
     $HLOW[ALLOCATABLE](:),HHIGH[ALLOCATABLE](:),HEAD[ALLOCATABLE](:),
     $VNEW[ALLOCATABLE](:),HNEW[ALLOCATABLE](:),PIPEZ[ALLOCATABLE](:)
      REAL L,NEXP
```

Figure 9.5 An elementary computer program for the approximate method.

```
  CHARACTER TITLE1*80,TITLE2*80,ROUGH*3,CH,CH12,CH78,CH79,CHN
      CHARACTER FNAME*12
      NAMELIST /SPECS/ IOUT,NPARTS,D,L,F,A,VZERO,HZERO,ELEVUP,
     $ELEVDN,TMAX,TCLOSE
*-----------------------------------------------------------------------
      WRITE(*,791)
      READ(*,100) FNAME
      OPEN(5,FILE=FNAME)
      READ(5,100) TITLE1
      READ(5,100) TITLE2
      READ(5,SPECS)
      WRITE(*,792)
      READ(*,100) FNAME
      OPEN(6,FILE=FNAME,STATUS='NEW')
*-----------------------------------------------------------------------
      CH=CHAR(27)
      CH12=CHAR(12)
      CH78=CHAR(78)
      CH79=CHAR(79)
      CHN=CHAR(3)
      NP=NPARTS+1
      ALLOCATE (X(NP),V(NP),H(NP),HLOW(NP),HHIGH(NP),HEAD(NP),
     $VNEW(NP),HNEW(NP),PIPEZ(NP))
      WTT=L/A
      DELL=L/NPARTS
      T=0.
      NEXP=1.0
      ROUGH='F ='
      IF(F.GT.10.) NEXP=0.85
      IF(F.GT.10.) ROUGH='C ='
      DELT=DELL/A
      C=32.2/A
      INDEX=TMAX/DELT + 1
      DELEL=(ELEVDN-ELEVUP)/NPARTS
      NODES=NPARTS+1
      WRITE(6,101) CH,CH78,CHN
      WRITE(6,200)
      WRITE(6,203) TITLE1
      WRITE(6,203) TITLE2
      WRITE(6,201) IOUT,NPARTS,L,A,D,ROUGH,F,VZERO,HZERO,ELEVUP,
     $ELEVDN,WTT,TCLOSE,TMAX,DELT
      AK=12.*F*DELT/(2.0*D)
      IF(F.GT.10.) AK=12.*DELT*195./(2.0*D*(F**1.85)*(D/12.)**.17)
      IF(F.GT.10.) F=195./((F**1.85)*(VZERO**.15)*((D/12.)**.17))
      DELHF=12.*F*DELL*VZERO**2/(64.4*D)
*
      DO 300 I=1,NODES
      V(I)=VZERO
      H(I)=HZERO-(I-1)*DELHF
      HLOW(I)=H(I)
      HHIGH(I)=H(I)
      X(I)=(I-1)*DELL/L
      PIPEZ(I)=ELEVUP+(I-1)*DELEL
      HEAD(I)=H(I)-PIPEZ(I)
  300 CONTINUE
      WRITE(6,101) CH12
      WRITE(6,202)
      WRITE(6,204) T,(X(I),HEAD(I),H(I),V(I),I=1,NODES)
```

Figure 9.5, cont'd. An elementary computer program for the approximate method.

```
      DO 99 II=1,INDEX
      T=T+DELT
*  ** COMPUTE H AND V AT INTERIOR NODES **
      DO 20 I=2,NPARTS
      VNEW(I)=0.5*(V(I-1)+V(I+1)+C*(H(I-1)-H(I+1))-AK*(V(I-1)*ABS(V(I-1)
     $)**NEXP+V(I+1)*ABS(V(I+1))**NEXP))
   20 HNEW(I)=0.5*(H(I-1)+H(I+1)+(V(I-1)-V(I+1))/C-AK*(V(I-1)*ABS(V(I-1)
     $)**NEXP-V(I+1)*ABS(V(I+1))**NEXP)/C)
*  ** COMPUTE H AND V AT UPSTREAM END **
*     THIS BOUNDARY CONDITION IS FOR A CONSTANT-HEAD RESERVOIR
      HNEW(1)=HZERO
      VNEW(1)=V(2)+C*(HNEW(1)-H(2))-AK*V(2)*ABS(V(2))**NEXP
*  ** COMPUTE H AND V AT DOWNSTREAM END **
*     THIS BOUNDARY CONDITION IS FOR LINEARLY DECREASING VELOCITY
      IF(T.GT.TCLOSE) GO TO 30
      VNEW(NODES)=VZERO*(1.-T/TCLOSE)
      GO TO 31
   30 VNEW(NODES)=0.0
   31 HNEW(NODES)=H(NPARTS)+(V(NPARTS)-VNEW(NODES)-AK*V(NPARTS)*
     $ABS(V(NPARTS))**NEXP)/C
      DO 50 I=1,NODES
      IF(HNEW(I).LT.HLOW(I)) HLOW(I)=HNEW(I)
      IF(HNEW(I).GT.HHIGH(I)) HHIGH(I)=HNEW(I)
   50 HEAD(I)=HNEW(I)-PIPEZ(I)
      IF(MOD(II,IOUT).EQ.0) WRITE(6,204) T,(X(I),HEAD(I),HNEW(I),VNEW(I)
     $,I=1,NODES)
      IF(T.GT.TMAX) GO TO 400
      DO 40 I=1,NODES
      V(I)=VNEW(I)
   40 H(I)=HNEW(I)
   99 CONTINUE
*
  400 WRITE(6,101) CH12
      WRITE(6,205)
      DO 401 I=1,NODES
      HEADMX=HHIGH(I)-PIPEZ(I)
      HEADMN=HLOW(I)-PIPEZ(I)
  401 WRITE(6,206) X(I),HEADMX,HEADMN,HHIGH(I),HLOW(I)
      WRITE(6,101) CH,CH79
  100 FORMAT(A)
  101 FORMAT(3A)
  200 FORMAT(///20X,33('*')/20X,'* WATER HAMMER IN A SINGLE PIPE *'/
     $20X,33('*')//)
  201 FORMAT(//29X,'INPUT DATA'/29X,10('-')//28X,'IOUT =',I4/26X,
     $'NPARTS =',I4//31X,'L =',F7.1,' FT'/31X,'A =',F7.1,' FT/S'/
     $31X,'D =',F7.2,' IN'/31X,A,F9.4//
     $27X,'VZERO =',F7.2,' FT/S'/27X,'HZERO =',F7.1,' FT'/
     $26X,'ELEVUP =',F7.1,' FT'/26X,'ELEVDN =',F7.1,' FT'//
     $29X,'L/A =',F7.3,' SEC'//26X,'TCLOSE =',F7.2,' SEC'/
     $28X,'TMAX =',F7.2,' SEC'/28X,'DELT =',F7.3,' SEC')
  202 FORMAT(//5X,'PRESSURE HEADS, H-VALUES AND VELOCITIES AS FUNCTIONS
     $OF TIME'/5X,60('-'))
  203 FORMAT(10X,A)
  204 FORMAT(//11X,2(4X,'  X   HEAD,FT  H,FT  V,FT/S ')/'T =',F6.3,
     $' SEC',2(2X,'----- ------- -----  ------  ')/(10X,2(5X,F5.3,2F6.0,
     $F7.2)))
  205 FORMAT(///18X,27('*')/18X,'* TABLE OF EXTREME VALUES *'/18X,27('*
     $')//13X,'X    MAX HEAD  MIN HEAD   MAX H   MIN H'/11X,5('-'),2X,8(
```

Figure 9.5, cont'd. An elementary computer program for the approximate method.

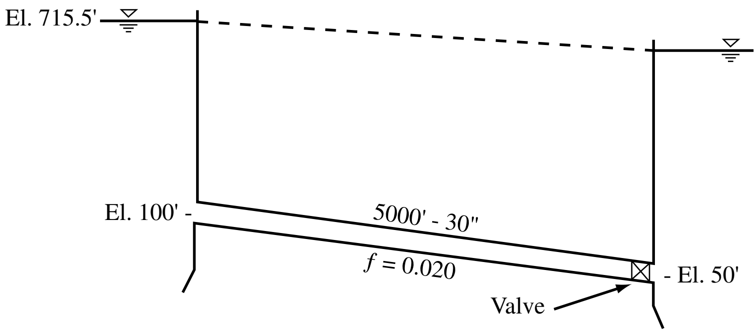
```
     $'-'),2X,8('-'),2X,6('-'),2X,6('-'))
 206 FORMAT(11X,F5.3,2X,F7.0,3X,F7.0,3X,F6.0,2X,F6.0)
 791 FORMAT(/' ENTER THE NAME OF YOUR INPUT DATA FILE:  '\)
 792 FORMAT(/' ENTER THE NAME OF THE FILE ON WHICH THE OUTPUT IS TO BE
     $WRITTEN: '\)
      END
```

Figure 9.5, concluded. An elementary computer program for the approximate method.

The following example problem demonstrates the input data file and the output of the program.

Example Problem 9.1

The 30-in steel pipeline is 5000 ft long with a wave speed of 2500 ft/s. The steady-state velocity is 5 ft/s. The valve at the downstream end is closed in such a manner that the velocity at the valve decreases linearly to zero in 10 sec. There is initially a negligible head loss in the valve. Find the maximum and minimum pressure heads in the system during this transient, and state their location.



Using PROG1 to solve the problem, we will divide the pipe into 8 sections and set up the following data file via the text editor:

```
DEMONSTRATION OF PROGRAM NO. 1 - INPUT DATA FILE "EP91.DAT"
CONSTANT-HEAD RESERVOIR UPSTREAM & LINEARLY DECREASING VELOCITY
DOWNSTREAM
&SPECS IOUT=4,NPARTS=8,D=30.,L=5000.,F=.020,A=2500.,VZERO=5.00,
       HZERO=715.5,ELEVUP=100.,ELEVDN=50.,TMAX=20.00,TCLOSE=10.00/
```

The output from the analysis is shown below. To save space only the beginning and end of the output data file are printed. A scan of the Table of Extreme Values shows that a maximum pressure head of 810 ft occurs at the valve, and the minimum pressure head of 569 ft occurs at the midpoint of the line.

```
                    ********************************
                    * WATER HAMMER IN A SINGLE PIPE *
                    ********************************

DEMONSTRATION OF PROGRAM NO. 1 - INPUT DATA FILE "EP91.DAT"
CONSTANT-HEAD RESERVOIR UPSTREAM & LINEARLY DECREASING VELOCITY DOWNSTREAM
```

```
                         INPUT DATA
                        ----------

                     IOUT =   4
                   NPARTS =   8

                        L = 5000.0 FT
                        A = 2500.0 FT/S
                        D =  30.00 IN
                        F =    .0200
                    VZERO =   5.00 FT/S
                    HZERO =  715.5 FT
                   ELEVUP =  100.0 FT
                   ELEVDN =   50.0 FT
                      L/A =  2.000 SEC
                   TCLOSE =  10.00 SEC
                     TMAX =  20.00 SEC
                     DELT =   .250 SEC


     PRESSURE HEADS, H-VALUES AND VELOCITIES AS FUNCTIONS OF TIME
     ------------------------------------------------------------

                  X   HEAD,FT  H,FT  V,FT/S     X   HEAD,FT  H,FT   V,FT/S
T =  .000 SEC  ----- ------- -----  ------    ----- ------- -----  ------
                .000   616.   716.   5.00      .125   620.   714.   5.00
                .250   624.   712.   5.00      .375   628.   710.   5.00
                .500   633.   708.   5.00      .625   637.   706.   5.00
                .750   641.   704.   5.00      .875   646.   702.   5.00
               1.000   650.   700.   5.00


                  X   HEAD,FT  H,FT  V,FT/S     X   HEAD,FT  H,FT   V,FT/S
T = 1.000 SEC  ----- ------- -----  ------    ----- ------- -----  ------
                .000   616.   716.   5.00      .125   620.   714.   5.00
                .250   624.   712.   5.00      .375   628.   710.   5.00
                .500   633.   708.   5.00      .625   647.   715.   4.88
                .750   661.   723.   4.75      .875   675.   731.   4.63
               1.000   689.   739.   4.50

                  .     .      .      .          .     .      .       .

                  .     .      .      .          .     .      .       .

                  .     .      .      .          .     .      .       .
                  X   HEAD,FT  H,FT  V,FT/S     X   HEAD,FT  H,FT   V,FT/S
T =20.000 SEC  ----- ------- -----  ------    ----- ------- -----  ------
                .000   616.   716.   -.95      .125   630.   724.   -.82
                .250   645.   733.   -.70      .375   660.   742.   -.58
                .500   675.   750.   -.46      .625   690.   759.   -.34
                .750   705.   767.   -.23      .875   720.   776.   -.11
               1.000   735.   785.    .00


                  ***************************
                  * TABLE OF EXTREME VALUES *
                  ***************************
```

```
 X    MAX HEAD  MIN HEAD   MAX H   MIN H
      -----  --------  --------  ------  ------
        .000     616.      616.     716.    716.
        .125     641.      603.     735.    697.
        .250     666.      592.     753.    679.
        .375     691.      580.     772.    661.
        .500     715.      569.     790.    644.
        .625     740.      575.     808.    644.
        .750     763.      581.     826.    643.
        .875     787.      586.     843.    642.
       1.000     810.      592.     860.    642.
```

*   *   *

The second elementary program PROG1P is a modification of the first program which contains the boundary conditions for a constant-speed pump at the upstream end of the line. In addition, the valve at the downstream end has been altered to permit the velocity to change linearly from the initial steady-state value to a prescribed final steady-state value (including zero). To accommodate the pump at the upstream end, the constant-head reservoir boundary conditions are replaced with those of Eqs. 9.41 and 9.42. The downstream boundary conditions are adjusted to permit the velocity to decrease to any lower constant value. The input data file must be expanded to provide the three pump curve coefficients $A_p^{'}$, $B_p^{'}$, and $C_p^{'}$ and the final velocity at the downstream valve. Shown below are selected portions of the new code that are required in PROG1 to implement these changes.

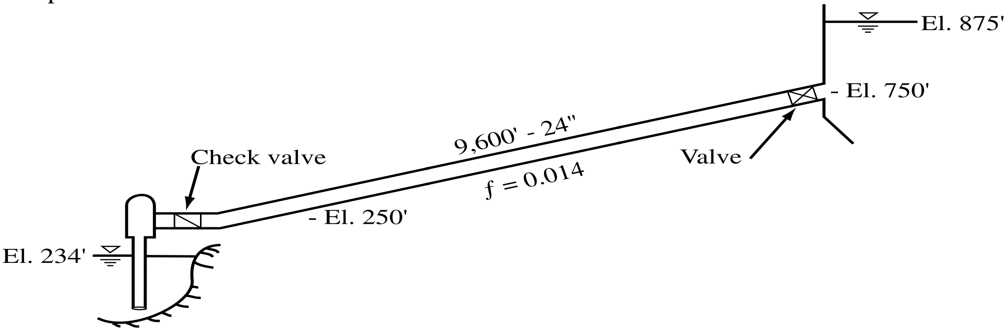
```
   .   .    .    .    .    .    .    .
   .   .    .    .    .    .    .    .
*   HSUMP = PUMP SUMP ELEVATION, FT
*   ELEVUP = ELEVATION OF UPSTREAM END OF PIPE, FT
*   ELEVDN = ELEVATION OF DOWNSTREAM END OF PIPE, FT
*   TMAX = MAXIMUM REAL TIME OF SIMULATION, SEC
*   TCLOSE = TIME REQUIRED FOR VALVE CLOSURE, SEC
*   VFINAL = FINAL VELOCITY, FT/S
*   THE VALUES OF APRIME, BPRIME, AND CPRIME ARE COMPUTED WITH THE
*      DISCHARGE IN GAL/MIN AND TOTAL HEAD FOR ALL STAGES IN FT
*      APRIME = FIRST COEFFICIENT IN PUMP CHARACTERISTIC EQUATION
*      BPRIME = SECOND COEFFICIENT IN PUMP CHARACTERISTIC EQUATION
*      CPRIME = THIRD COEFFICIENT IN PUMP CHARACTERISTIC EQUATION
   .   .    .    .    .    .    .    .
   .   .    .    .    .    .    .    .
   .   .    .    .    .    .    .    .
      NAMELIST /SPECS/ IOUT,NPARTS,D,L,F,A,VZERO,HSUMP,ELEVUP,
     $ELEVDN,TMAX,TCLOSE,VFINAL,APRIME,BPRIME,CPRIME
   .   .    .    .    .    .    .    .
   .   .    .    .    .    .    .    .
   .   .    .    .    .    .    .    .
      AREA=0.7854*D*D/144.
      APRIME=APRIME*(AREA*449.)**2
      BPRIME=BPRIME*AREA*449.
      CPRIME=CPRIME+HSUMP
      HPUMP=APRIME*VZERO*VZERO+BPRIME*VZERO+CPRIME
```

```
   .    .      .      .      .      .      .      .
   .    .      .      .      .      .      .      .
   .    .      .      .      .      .      .      .
*   ** COMPUTE H AND V AT UPSTREAM END **
*      THIS BOUNDARY CONDITION IS FOR A CONSTANT-HEAD PUMP
       C1=V(2)-C*H(2)-AK*V(2)*ABS(V(2))
       C3=(BPRIME-1.0/C)/APRIME
       C4=(CPRIME+C1/C)/APRIME
       CHEK=4.*C4/(C3*C3)
       IF(CHEK.GT.0.) GO TO 25
       VNEW(1)=0.5*C3*(-1.+SQRT(1.0-CHEK))
       GO TO 26
    25 VNEW(1)=0.
    26 HNEW(1)=(VNEW(1)-C1)/C
*   ** COMPUTE H AND V AT DOWNSTREAM END **
*      THIS BOUNDARY CONDITION IS FOR LINEARLY DECREASING VELOCITY
       IF(T.GT.TCLOSE) GO TO 30
       VNEW(NODES)=VZERO-(T/TCLOSE)*(VZERO-VFINAL)
       GO TO 31
    30 VNEW(NODES)=VFINAL
    31 HNEW(NODES)=H(NPARTS)+(V(NPARTS)-VNEW(NODES)-AK*V(NPARTS)*
      $ABS(V(NPARTS))**NEXP)/C
   .    .      .      .      .      .      .      .
   .    .      .      .      .      .      .      .
```

Example Problem 9.2

A four-stage Johnston 20 CC turbine pump (the pump characteristic diagram is in Appendix B) with 15 3/4 in impellers is used to pump water from a river to an elevated storage reservoir. The welded-steel pipeline is 9600 ft long, 24 in inside diameter, with a wall thickness of 0.1875 in and a friction factor of 0.014. A special valve is located at the downstream end of the line to cause the velocity at the valve to vary linearly with time.

The design engineer would like to close the valve in 30 sec before shutting down the pump. Determine the maximum and minimum pressure heads which would occur under this plan.



We will use PROG1P and divide the pipeline into 10 parts to solve the problem. But first we must calculate the wave speed and the coefficients of the parabolic equation which will model the pump characteristics.

The pipe has a D/e ratio of 128, so we can use Eq. 8.33 for thin-walled pipes to compute the wave speed. Using Case (b) restraint because it gives the highest wave speed, we find

$$a = \frac{4720}{\sqrt{1 + \dfrac{3\times10^5}{30\times10^6}\dfrac{24}{0.1875}\left(1-0.3^2\right)}} = 3200 \quad \text{ft/s}$$

To determine the parabolic equation coefficients, we select three points on the pump characteristic diagram, write three equations with the unknown coefficients in them, and then solve the equations. We will use the following points:

Q = 0, 4000, and 7000 gal/min and h = 254, 200, and 137 ft/stage

The coefficients $A_p' = -1.071\times10^{-6}$, $B_p' = -9.215\times10^{-3}$, and $C_p' = 254$ are the result. However, this is a four-stage pump, so each of the coefficients must be multiplied by four before they are inserted into the program. The data file created by the text editor follows:

```
SOLUTION FOR EXAMPLE PROBLEM 9.2 - INPUT DATA FILE "EP92.DAT"
JOHNSTON 20 CC PUMP UPSTREAM, VALVE DOWNSTREAM CLOSING IN 30 SEC
&SPECS IOUT=1000, NPARTS=10, D=24.00, L=9600., F=0.014, A=3200.,
     VZERO=4.11, HSUMP=234., ELEVUP=250., ELEVDN=750., TCLOSE=30.,
     TMAX=60., VFINAL=0., APRIME=-4.28E-6, BPRIME=-0.03686, CPRIME=1016./
```

The results of the analysis are shown below; they reveal that the maximum pressure head of 1029 ft occurs at the pump, while the minimum pressure head of 125 ft occurs at the valve.

```
         **********************************
                        * WATER HAMMER IN A SINGLE PIPE *
                        **********************************

SOLUTION FOR EXAMPLE PROBLEM 9.2 - INPUT DATA FILE "EP92.DAT"
JOHNSTON 20 CC PUMP UPSTREAM, VALVE DOWNSTREAM CLOSING IN 30 SEC


                              INPUT DATA
                              ----------

                             IOUT =1000
                           NPARTS =   10

                                L = 9600.0 FT
                                A = 3200.0 FT/S
                                D =   24.00 IN
                                F =     .0140

                           VFINAL =     .00 FT/S
                            VZERO =    4.11 FT/S
                            HSUMP =   234.0 FT
                           ELEVUP =   250.0 FT
                           ELEVDN =   750.0 FT
```

```
                 L/A =  3.000 SEC

              TCLOSE =  30.00 SEC
                TMAX =  60.00 SEC
                DELT =    .300 SEC
              APRIME =-.4280E-05
              BPRIME =-.3686E-01
              CPRIME = .1016E+04
```

PRESSURE HEADS, H-VALUES AND VELOCITIES AS FUNCTIONS OF TIME

T = .000 SEC

X	HEAD,FT	H,FT	V,FT/S	X	HEAD,FT	H,FT	V,FT/S
.000	642.	892.	4.11	.100	591.	891.	4.11
.200	539.	889.	4.11	.300	487.	887.	4.11
.400	435.	885.	4.11	.500	384.	884.	4.11
.600	332.	882.	4.11	.700	280.	880.	4.11
.800	228.	878.	4.11	.900	177.	877.	4.11
1.000	125.	875.	4.11				

```
****************************
* TABLE OF EXTREME VALUES *
****************************
```

X	MAX HEAD	MIN HEAD	MAX H	MIN H
.000	1029.	642.	1279.	892.
.100	977.	591.	1277.	891.
.200	926.	539.	1276.	889.
.300	874.	487.	1274.	887.
.400	822.	435.	1272.	885.
.500	770.	384.	1270.	884.
.600	722.	332.	1272.	882.
.700	674.	280.	1274.	880.
.800	626.	228.	1276.	878.
.900	577.	177.	1277.	877.
1.000	529.	125.	1279.	875.

*   *   *

9.2 COMPLETE METHOD OF CHARACTERISTICS

In solving the complete equations we can proceed in a manner similar to that for the approximate method. However, in this case, we will use the original Eqs. 8.57 and 8.58.

9.2.1. THE COMPLETE EQUATIONS

We once again use the linear constant multiplier l to combine the Euler and conservation of mass equations. Multiplying Eq. 8.57 by l and adding the result to Eq. 8.58 gives

$$\lambda \frac{dV}{dt} + \frac{\lambda}{\rho}\frac{\partial p}{\partial s} + \lambda g \frac{dz}{ds} + \lambda \frac{f}{2D} V|V| + a^2 \frac{\partial V}{\partial s} + \frac{1}{\rho}\frac{dp}{dt} = 0 \qquad (9.43)$$

To repeat the same procedure, we must separate $d\mathrm{V}/\mathrm{dt}$ and dp/dt into their component parts. The result is

$$\left(\lambda\frac{\partial V}{\partial t}+\lambda V\frac{\partial V}{\partial s}\right)+\frac{\lambda}{\rho}\frac{\partial p}{\partial s}+\lambda g\frac{dz}{ds}+\lambda\frac{f}{2D}V|V|+a^2\frac{\partial V}{\partial s}+\left(\frac{1}{\rho}\frac{\partial p}{\partial t}+\frac{V}{\rho}\frac{\partial p}{\partial s}\right)=0 \qquad (9.44)$$

Regrouping the terms in the equation gives

$$\left[\lambda\frac{\partial V}{\partial t}+\left(\lambda V+a^2\right)\frac{\partial V}{\partial s}\right]+\left[\frac{1}{\rho}\frac{\partial p}{\partial t}+\left(\frac{\lambda}{\rho}+\frac{V}{\rho}\right)\frac{\partial p}{\partial s}\right]+\lambda g\frac{dz}{ds}+\lambda\frac{f}{2D}V|V|=0 \qquad (9.45)$$

As before,

$$\left[\lambda\frac{\partial V}{\partial t}+\left(\lambda V+a^2\right)\frac{\partial V}{\partial s}\right]=\lambda\frac{dV}{dt} \quad \text{if} \quad \lambda\frac{ds}{dt}=\lambda V+a^2 \qquad (9.46)$$

and

$$\left[\frac{1}{\rho}\frac{\partial p}{\partial t}+\left(\frac{\lambda}{\rho}+\frac{V}{\rho}\right)\frac{\partial p}{\partial s}\right]=\frac{1}{\rho}\frac{dp}{dt} \quad \text{if} \quad \frac{1}{\rho}\frac{ds}{st}=\frac{\lambda}{\rho}+\frac{V}{\rho} \qquad (9.47)$$

Thus we require for *ds/dt* that

$$\frac{ds}{dt}=V+\frac{a^2}{\lambda} \quad \text{and} \quad \frac{ds}{dt}=\lambda+V \qquad (9.48)$$

Equating these two expressions to eliminate *ds/dt* and then solving for *l* leads to

$$\lambda = \pm a \qquad (9.49)$$

With *l* again equal to the wave speed, we find that the equations for the characteristics are

$$\frac{ds}{dt}=V+a \quad \text{and} \quad \frac{ds}{dt}=V-a \qquad (9.50)$$

Finally we replace the pressure in favor of total head using $p=\gamma(H-z)$.

The final set of equations, which are the analogs of Eqs. 9.8 and 9.9, is

$$C^+: \quad \frac{dV}{dt}+\frac{g}{a}\frac{dH}{dt}-\frac{g}{a}V\frac{dz}{ds}+\frac{f}{2D}V|V|=0 \quad \text{only when} \quad \frac{ds}{dt}=V+a \qquad (9.51)$$

$$C^-: \quad \frac{dV}{dt}-\frac{g}{a}\frac{dH}{dt}+\frac{g}{a}V\frac{dz}{ds}+\frac{f}{2D}V|V|=0 \quad \text{only when} \quad \frac{ds}{dt}=V-a \qquad (9.52)$$

These ordinary differential equations are quite similar to those for the approximate case. However, the characteristic lines in the *s-t* plane, which were of constant slope for the approximate method, are now *curved*, their slope a function of $V(s,t)$. This is an important distinction because it introduces some complications into the numerical solution procedure which we must address.

9.2.2. THE NUMERICAL SOLUTION

We first assume that the characteristic curves can be approximated as straight lines over each single Δt interval. This assumption is attractive because (1) Δt may be made as small as one wishes, and (2) usually $a>>V$, causing *ds/dt* to be nearly constant. But the slopes of the C^+ and C^- characteristic lines are no longer the same in magnitude.

The problem this creates in the finite difference approximations to the differential equations can be seen in Fig. 9.6. Assume now that the grid intervals Δs and Δt have been chosen (we will see shortly how this is done), and once again we seek to find the values of H and V at P. The curved characteristics intersecting at P are approximated by straight lines, whose slopes have been determined by the known values of velocity at the earlier time. We now see in Fig. 9.6 that the characteristics intersecting at P no longer pass through the grid points Le and Ri, but instead they pass through the $t =$

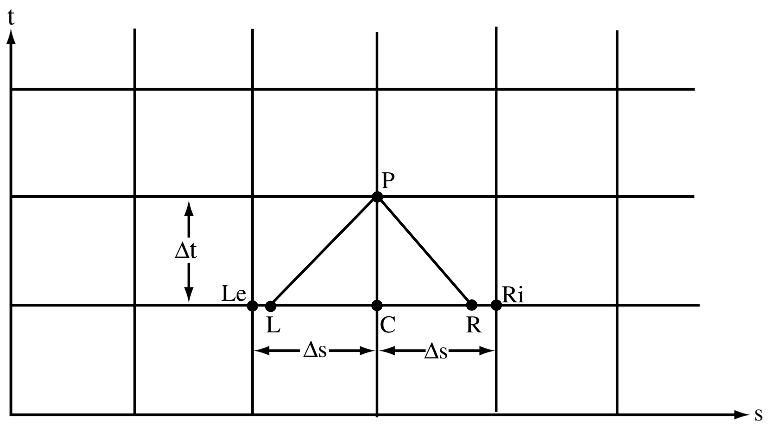


Figure 9.6 Interpolation of H and V values on the Δs-Δt grid.

constant line at points identified as L and R somewhere between Le and Ri.

For this situation the finite difference approximations to Eqs. 9.51 and 9.52 become

$$\frac{V_P - V_L}{\Delta t} + \frac{g}{a}\frac{H_P - H_L}{\Delta t} - \frac{g}{a}V_L\frac{dz}{ds} + \frac{f}{2D}V_L|V_L| = 0 \tag{9.53}$$

$$\frac{V_P - V_R}{\Delta t} - \frac{g}{a}\frac{H_P - H_R}{\Delta t} + \frac{g}{a}V_R\frac{dz}{ds} + \frac{f}{2D}V_R|V_R| = 0 \tag{9.54}$$

The new difficulty here is that the values of V_L, H_L, V_R, and H_R are not known, thereby causing Eqs. 9.53 and 9.54 to include six unknowns rather than the two unknowns which occurred in the approximate method. We overcome this problem by choosing Δt so that point L is near Le and R is near Ri; now linear interpolation becomes an accurate way to evaluate the values of H and V at points L and R. Figure 9.7 defines the parameters needed in the interpolation procedure.

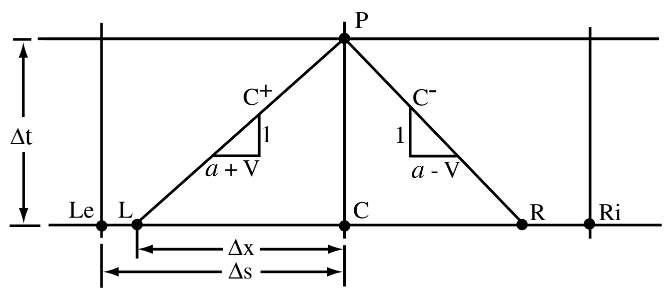


Figure 9.7 Parameters in the interpolation procedure.

Along the C^+ characteristic,

$$\frac{\Delta x}{\Delta s} = \frac{V_L - V_C}{V_{Le} - V_C} = \frac{H_L - H_C}{H_{Le} - H_C} \tag{9.55}$$

with

$$\frac{\Delta x}{\Delta t} = \frac{a + V_L}{1} \tag{9.56}$$

Solving these two equations for V_L and H_L yields

$$V_L = (V_{Le} - V_C)\frac{\Delta x}{\Delta s} + V_C \quad \text{and} \quad H_L = (H_{Le} - H_C)\frac{\Delta x}{\Delta s} + H_C \tag{9.57}$$

Replacing Δx in these equations using the same relation for $\Delta x/\Delta t$ now produces

$$V_L = \frac{V_C + a\dfrac{\Delta t}{\Delta s}(V_{Le} - V_C)}{1 - \dfrac{\Delta t}{\Delta s}(V_{Le} - V_C)} \tag{9.58}$$

and

$$H_L = H_C + \frac{\Delta t}{\Delta s}(H_{Le} - H_C)(a + V_L) \tag{9.59}$$

A similar analysis along the C^- characteristic gives

$$V_R = \frac{V_C + a\dfrac{\Delta t}{\Delta s}(V_{Ri} - V_C)}{1 - \dfrac{\Delta t}{\Delta s}(V_{Ri} - V_C)} \tag{9.60}$$

and

$$H_R = H_C + \frac{\Delta t}{\Delta s}(H_{Ri} - H_C)(a - V_R) \tag{9.61}$$

Because $\dfrac{\Delta t}{\Delta s}(V_{Le} - V_C)$ is on the order of $\dfrac{V}{a+V}$, which is very small compared to 1, it is a good approximation to neglect the second terms in the denominators of Eqs. 9.58 and 9.60. The results are

$$V_L = V_C + a\frac{\Delta t}{\Delta s}(V_{Le} - V_C) \tag{9.62}$$

and

$$V_R = V_C + a\frac{\Delta t}{\Delta s}(V_{Ri} - V_C) \tag{9.63}$$

Since we now have known values for V_L, H_L, V_R, and H_R, we can solve Eqs. 9.53 and 9.54 simultaneously for V_P and H_P. The solutions are

$$V_P = \frac{1}{2}\left[(V_L + V_R) + \frac{g}{a}(H_L - H_R) + \frac{g}{a}\Delta t(V_L - V_R)\sin\theta - \frac{f\Delta t}{2D}\left(V_L|V_L| + V_R|V_R|\right)\right] \tag{9.64}$$

$$H_P = \frac{1}{2}\left[\frac{a}{g}\left(V_L - V_R\right) + \left(H_L + H_R\right) + \Delta t\left(V_L + V_R\right)\sin\theta - \frac{a}{g}\frac{f\Delta t}{2D}\left(V_L|V_L| - V_R|V_R|\right)\right] \quad (9.65)$$

in which $\sin\theta = dz/ds$ is positive for pipes sloping upward in the downstream direction. Our next step is to determine how the Δs-Δt grid is established.

9.2.3. THE Δs-Δt GRID

The non-constant slope of the curving characteristics and the decision to approximate each as a straight line over a small time interval now force us to face the problem of finding appropriate values of Δs and Δt which will yield accurate and numerically stable solutions. Actually it is always possible to find a pair of values for Δs and Δt which will require no interpolations *for a given section* of the characteristic lines. However, seeking these values would lead to a confusing array of Δs's and Δt's which would make it impossible to keep track of where and when things are happening.

One solution for this problem is to select a uniform rectangular grid on the s-t plane where Δs and Δt are fixed for all time at values which minimize the interpolation and simplify the programming. This method is called the *rectangular grid* method. To establish the grid dimensions, we proceed in the same manner as with the approximate method. We decide how many parts in which to divide the pipeline, thereby fixing Δs. Then the integrated characteristic equations are used to select Δt. The resulting integrated characteristic equations (assuming constant V) are

$$\begin{aligned} \Delta t &= \frac{\Delta s}{V+a} \qquad \text{for the } C^+ \text{ characteristic} \\ \Delta t &= \frac{\Delta s}{V-a} \qquad \text{for the } C^- \text{ characteristic} \end{aligned} \quad (9.66)$$

Because this interpolation procedure implies that the points L and R are between points Le and Ri, we must limit Δt to assure this is always so. The preceding equations suggest the criterion

$$\Delta t \le \frac{\Delta s}{\max|a+V|} \quad (9.67)$$

in which $\max|a+V|$ is the maximum expected absolute value of the sum of the wave speed and velocity. If the location of points L and R fall "outside" the grid points Le and Ri, numerical stability and accuracy problems will develop, as we demonstrate later. These problems are related to the earlier discussion of how "messages" are transmitted along the pipeline. When points L and R are outside the grid points, the procedure uses information in computing H_P and V_P which hasn't physically had enough time to reach point P. This is improper numerical procedure which will lead at best to inaccurate results and at worst to numerical instability. We must guard against this happening in our computer programs.

A computer program (not included) which would incorporate the complete method of characteristics would be very similar to PROG1 for the approximate method. It is only necessary to add a few lines of code to compute the proper Δt and $\sin\theta$ and a few more lines to compute V_L, H_L, V_R, and H_R from Eqs. 9.59, 9.61, 9.62, and 9.63.

A comparison of results from the two methods when $a >> V$ shows for typical situations that the two methods produce essentially the same answers. This indicates that the basic assumption behind the approximate method is sound, namely that the time variation in V and H is more significant that the spatial variation. So one might wonder why the complete method would ever be used. It turns out that the matching of internal boundary conditions in more complex pipe systems between pipes of different sizes and wave speeds requires the interpolation procedure we have just derived. In these more complex problems we will use the complete equations because they already include the interpolation procedure.

9.3 SOME PARAMETER EFFECTS ON SOLUTION RESULTS

It is both informative and useful to examine some effects of the parameters of the problems on solution results. The results here include the effects of friction, the number of parts into which the pipe is divided, the slope of the pipe, and the effect of rate of velocity change. An example of numerical instability and inaccuracy is presented. The pipeline in Fig. 9.8 is used to demonstrate these effects.

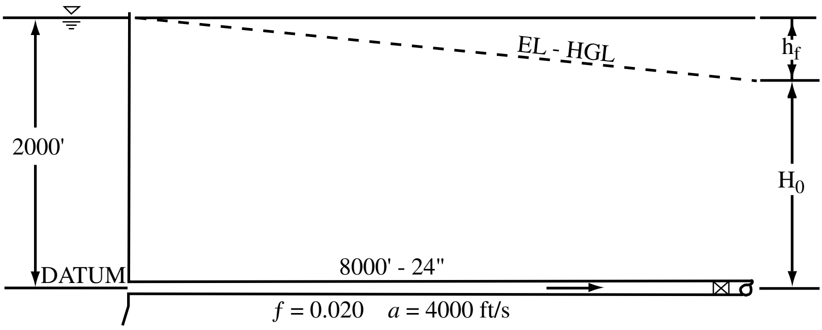


Figure 9.8 Model system for the investigation of sensitivity to system parameters.

9.3.1. THE EFFECT OF FRICTION

The effect of friction on solutions is studied by introducing initial velocities of 2.5 ft/s, 5.0 ft/s, and 10.0 ft/s into the pipeline of Fig. 9.8. We will do this for sudden valve closure. The results are listed in Table 9.1. The *increase* in pressure at the valve seems to

Table 9.1
The Effects of Friction on the Maximum Water Hammer Pressure at the Valve for Sudden Valve Closure (N = 6)

Steady Velocity ft/s	**Computed H_{max}** ft	h_f ft	H_0 ft	$\boxed{\Delta H = -a\,\Delta V/g}$ ft	$\boxed{h_f + H_0 + \Delta H}$ ft
2.5	2311	8	1992	311	2311
5.0	2621	31	1969	621	2621
10.0	3242	124	1876	1242	3242

be the sum of the friction loss in the pipe and ΔH from Eq. 8.8. In fact it appears that the maximum pressure occurring at the valve may be estimated by the formula

$$H_{\max} \approx H_0 + h_f + \Delta H \tag{9.68}$$

Keep in mind this is an approximation which seems to work in this case but should be applied with caution to other situations.

To develop a grasp of how friction affects the results in a water hammer situation, we show in Fig. 9.9 the position of the *EL-HGL* at successive times as the wave propagates through the pipe. The pipeline of Fig. 9.8 will be used with an initial steady velocity of 10 ft/s and sudden valve closure. The increase in head ΔH propagates upstream at approximately the wave velocity, increasing the steady state head at each point by an amount ΔH. It might seem after a time L/a that the *EL-HGL* would be a line parallel to the original steady state *EL-HGL* but positioned ΔH above it. However, a chain of events occurs, beginning at the first time step, which causes the pressure head at each point in the pipeline to continue to creep upward even though the pressure wave has already passed. This happens because the fluid in the pipe is not in equilibrium. Even though the velocity is zero, there is a pressure gradient caused by the sloping *EL-HGL*. As a consequence, there is a small downstream velocity which develops to eliminate the pressure gradient and bring the system into equilibrium. This process of upward adjustment continues until it is interrupted by the returning pressure wave. At the valve the

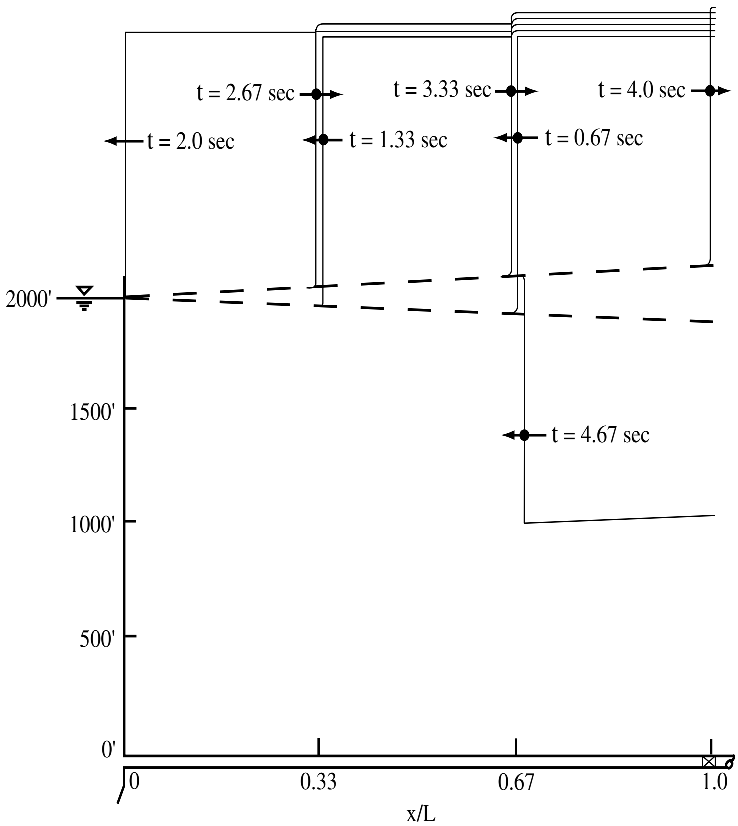


Figure 9.9 The progression of frictional effects in a single pipe with sudden valve closure.

cumulative upward pressure adjustment is equal in amount to the original steady state friction loss in the pipeline. This adjustment explains why Eq. 9.68 works in this case. Keep in mind that the purpose of this example is to create for the reader a physical feeling for the process; it is not intended to be a substitute for an analysis. It does, however, illustrate the value of a computer program in determining accurately the effects of friction.

9.3.2. THE EFFECT OF THE SIZE OF *N*

It seems reasonable to expect the accuracy of results to increase as the number of pipeline segments N increases since $\Delta s = L/N$. It is surprising then to discover that the choice of N has relatively little effect on the solution. For example, for sudden valve closure with an initial velocity of 5 ft/s in Fig. 9.8, the maximum and minimum values of H differ by less than 10 ft between solutions for $N = 3$ and $N = 18$, as shown in Table 9.2. For lower initial velocities, the difference is even less significant.

There are two points to be made here. First, except in the case of a rapidly-varying velocity, there is little to be gained by using a larger N than is necessary. Second, for a given simulation time the number of grid points and the subsequent computer execution time varies as N^2. However, we must also select Δt sufficiently small to capture accurately such time-varying boundary conditions as the movement of a valve, and smaller values for Δt are directly linked to larger values of N.

Table 9.2
Effects of N-value on Pressure Head (ft) at the Valve for Rapid and Slow Velocity Change (ΔV = 5 ft/s)

	Sudden Valve Closure		Valve Closure in $4L/a$ sec	
N	H_{max}	H_{min}	H_{max}	H_{min}
3	2611	1417	2288	1969
6	2616	1412	2290	1969
18	2619	1409	2292	1969

9.3.3. THE EFFECT OF PIPE SLOPE

Flows in pipelines ranging in slope over $\pm 25\%$ were simulated to determine the effect of slope. Results were nearly identical for both extremes of slope. While the slope of the pipe should not be ignored (it is needed in computing the pressure head, anyway), the reader should be comfortable when "smoothing" a pipeline system profile to reduce the number of series pipes to a manageable number of constant-slope pipes. By this means we can use the series pipe program of Chapter 10 to obtain accurate estimates of pressure head along the pipeline while still exercising control over Δs and Δt.

9.3.4. NUMERICAL INSTABILITY AND ACCURACY

Earlier it was stated that Δt should be limited in size to insure that points L and R in Fig. 9.6 remain between the grid points Le and Ri at all times. If not, numerical instability was presumed to occur as well as inaccuracy in the computed results.

To demonstrate these two problems, we again use the pipeline of Fig. 9.8 with sudden valve closure. While Δs was computed and held constant, Δt was assigned four different values to illustrate the two problems. The first Δt was chosen so that L and Le coincide, a case requiring minimum interpolation and leading to maximum accuracy. We will refer to this value as Δt_0. Higher values of Δt would cause L to move outside Le, leading to numerical instability. Lower values of Δt would lead to poorer linear interpolations, resulting in inaccurate results. The four computer simulations are identified in the legend in Fig. 9.10.

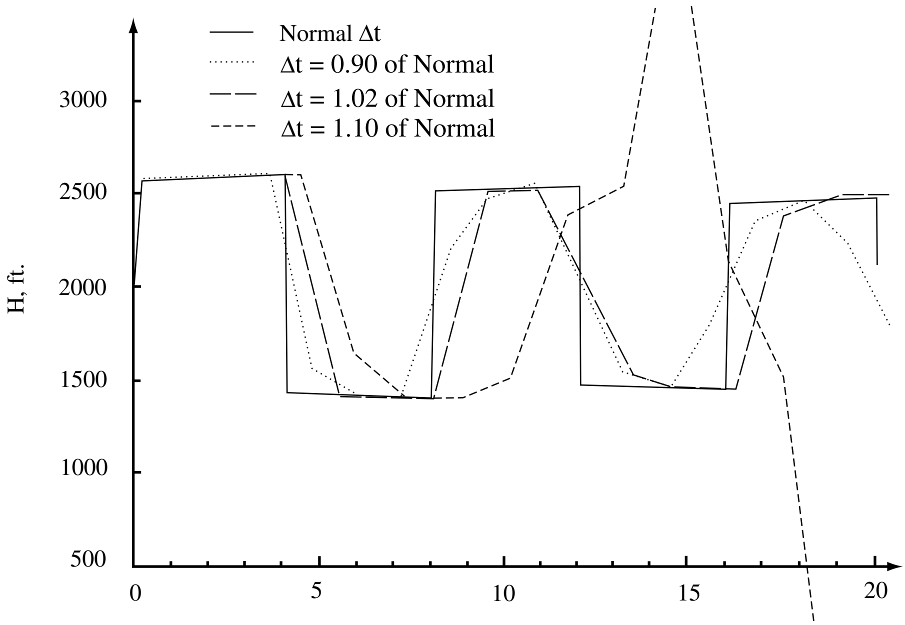


Figure 9.10 Numerical instability and inaccuracy in a single pipe with sudden valve closure.

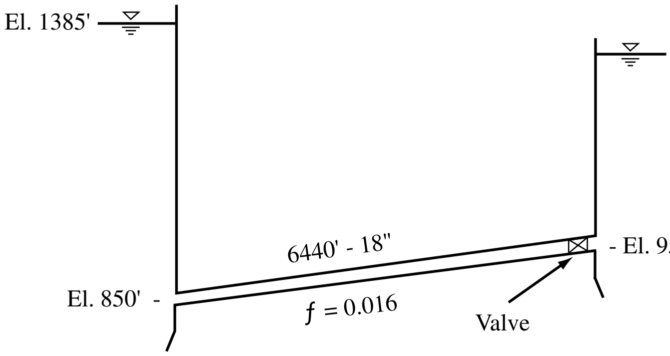
For the normal value of Δt, the plot shows the typical nearly-square wave from Chapter 7. When Δt is reduced to 90% of normal, the effect is to round off the sharp corners and distort the timing of events, a diffusive process. At 102% of the normal Δt, we see a gradual deterioration of the simulation with time. Finally, at 110% of the normal Δt, the resulting numerical stability is strikingly evident.

In summary, it is important to locate the points L and R inside of and as close to the grid intersections as possible. This minimizes interpolation errors and retains the numerical accuracy of the simulation. It is even more important to insure that the points L and R never are outside the grid points. This care will prevent inaccuracies associated with numerical instability which could be insidiously present in calculations, especially where the duration of the simulation is not so long that the more dramatic effects of instability can become obvious.

9.4 PROBLEMS

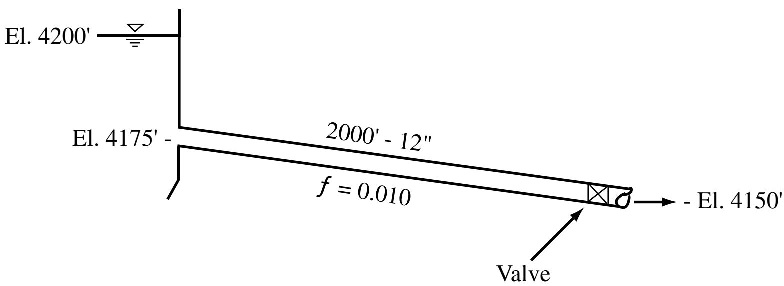
9.1 The ductile iron pipeline shown below is 6440 ft long, 18 inches inside diameter, with a wave speed of 3220 ft/s. It carries 3970 gal/min of water with $f = 0.016$. The valve at the downstream end can cause the velocity to vary linearly with time.

Calculate the maximum and minimum pressure heads and their locations for closure times of 0, 4, 8, and 12 sec. Use PROG1 with *NPARTS* = 10 in your analysis.



9.2 A 12-inch PVC pipeline with an inside diameter of 12.091 in, a wall thickness of 0.311 in, and an f of 0.010 carries 2150 gal/min of irrigation water from a supply reservoir to an irrigation network. A valve which can vary the velocity linearly with time is at the downstream end of the pipeline.

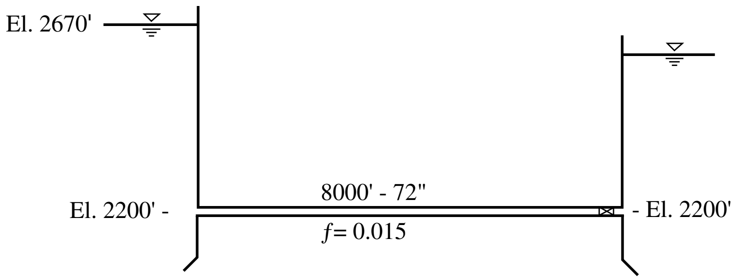
Find the minimum time of valve closure that can be used if no negative pressure can be permitted in the line. Neglect any effect of the system downstream of the valve. Use PROG1 with Case (c) restraint for the pipeline, and divide it into 10 parts for the analysis.



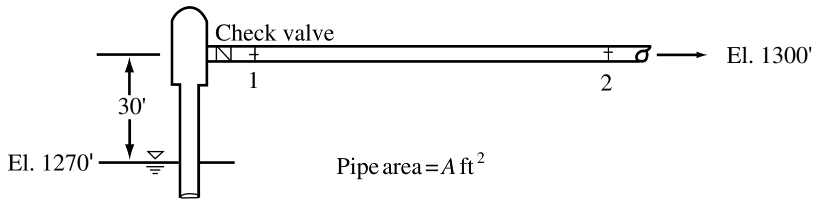
9.3 The pipeline of Problem 8.7 has $f = 0.015$ and carries water between two reservoirs, as shown below. The valve at the downstream end of the pipe is capable of varying the velocity linearly with time.

(a) Can the valve be closed suddenly without causing a negative pressure in the pipeline?
(b) What would the maximum and minimum pressure heads in the line be if the valve were closed in $4L/a$ seconds?

Use PROG1 with *NPARTS* = 10 in your analysis.



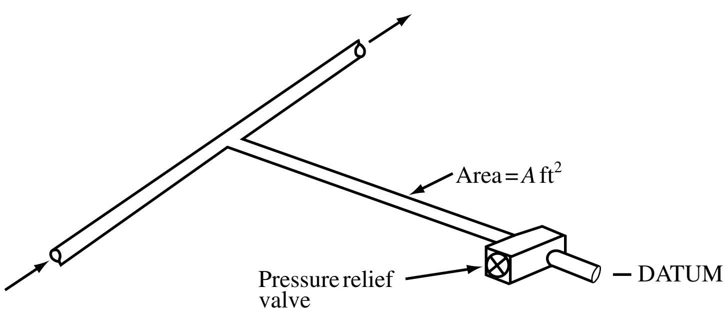
9.4 The characteristics of the pump below can be approximated by $h_p = A_p Q + B_p$. Using the C$^-$ equation in the form $H_P = C_1 + C_2 V_P$, write the boundary conditions for the pump, using a datum of sea level, so that you provide the equations needed to determine H_P and V_P.



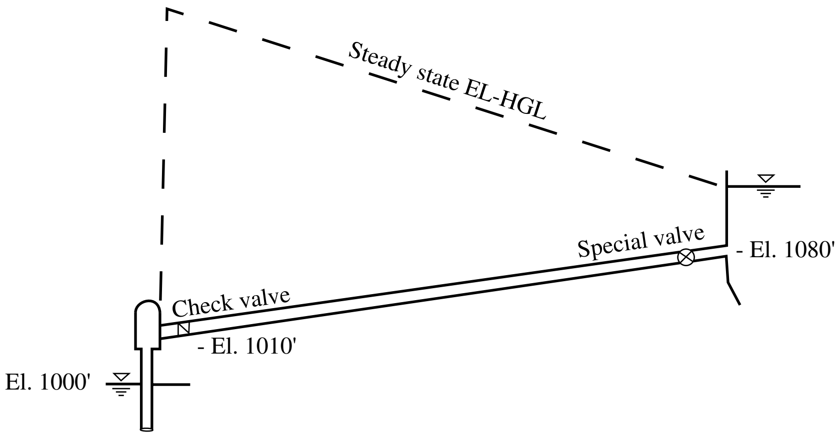
9.5 The lateral line extending from the main pipeline in the sketch below ends in a pressure relief valve which will open if the pressure exceeds a maximum value p_{max}. When the valve is open, the discharge from the valve into the atmosphere is

$$Q\left(\text{ft}^3/\text{s}\right) = K\sqrt{\text{pressure head (ft) in pipe just upstream of valve}}$$

A water hammer analysis requires that the boundary condition at the valve be found. Your task is to combine the above equation with the C$^+$ equation $V_P = C_1 - C_2 H_P$ to develop an equation for V_P at the upstream side of the valve.



9.6 An eight-stage Johnston 14 BC turbine pump (the pump characteristic diagram is in Appendix B) with 11-inch impellers pumps 850 gal/min through the pipeline below. The welded steel pipeline is one mile long, 6 in inside diameter with a wall thickness of 0.135 in. Case (b) restraint applies and $f = 0.016$ may be used. A special valve at the downstream end of the line reduces the discharge from 850 gal/min to 250 gal/min linearly with time.



The engineer in charge is considering four separate closure times of 3, 6, 9, and 12 seconds and wants to know the maximum and minimum pressures occurring in each case. You are to complete the following tasks:

(a) Develop with program PUMPC a parabolic equation for the pump characteristics of the form

$$h_p = A_p' Q^2 + B_p' Q + C_p'$$

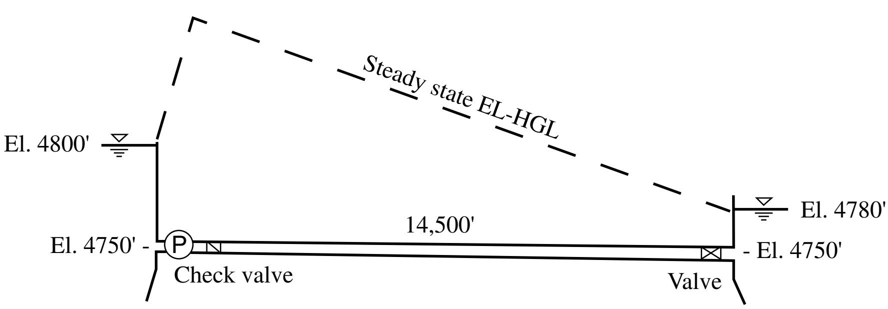
(b) Plot this equation on the pump characteristic diagram to demonstrate how well it fits over the range from 0 to 1200 gal/min.

(c) Calculate the wave speed in the pipe.

(d) For the four closure times, find the maximum and minimum pressures at the valve and the check valve and the times they occur.

(e) Plot pressure head as a function of time at both locations for a time at least $4L/a$ seconds past the last valve movement. Use PROG1P with $NPARTS = 5$ in your analysis.

9.7 The T-30 Transite pipe of Problem 8.3 is installed in a pumping system. To determine f for the pipe, assume it is hydraulically smooth. The pipeline configuration is shown below. The pump is a Johnston 20 CC single-stage turbine pump (see Appendix B) with a 15 3/4-in impeller. The pump runs at constant speed while the downstream valve is closed to decrease the velocity at the valve linearly with time to 2 ft/s in 15 sec.



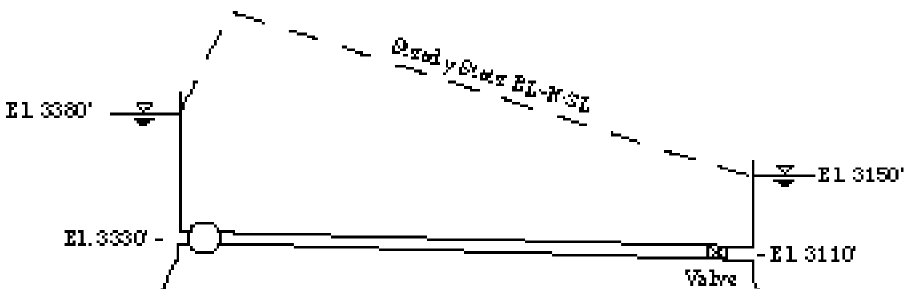
Complete the following tasks:

(a) Develop with program PUMPC a parabolic equation for the pump characteristic of the form

$$h_p = A_p' Q^2 + B_p' Q + C_p'$$

(b) Plot this equation on the pump characteristic diagram to demonstrate how well it fits over the range from 0 to 7000 gal/min.
(c) Calculate the wave speed in the pipe.
(d) Find the maximum and minimum pressures at the valve and the check valve and the times they occur.
(e) Plot pressure head as a function of time at both locations for a time at least $4L/a$ seconds after the last valve movement. Use PROG1P with *NPARTS* = 5.

9.8 A single-stage Johnston 18 DC turbine pump (see Appendix B) with a $13\frac{3}{16}$-in impeller is used to increase the discharge in the gravity flow pipeline below. The welded steel line is 12 in inside diameter and three miles long with $f = 0.014$. The wall



thickness is 0.179 in and Case (b) restraint applies. While the pump runs at constant speed, the downstream valve closes in a manner that causes the discharge in the pipe to decrease linearly with time to zero in 30 sec. If the pressure head at the pump exceeds the shutoff head during the shutdown procedure, flow backward through the pump will occur according to the relation

$$Q(\text{gal/min}) = 6.0\left(H_{P_1} - H_{shutoff}\right)$$

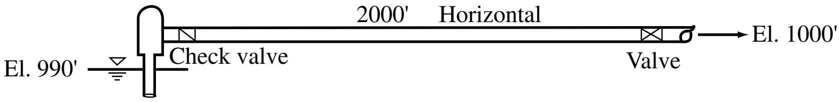
Your tasks:

(a) Calculate the steady state discharge and velocity in the pipeline.
(b) Develop with program PUMPC a parabolic equation for the pump characteristics of the form

$$h_p = A'_p Q^2 + B'_p Q + C'_p$$

(c) Plot this equation on the pump characteristic diagram to demonstrate how well it fits over the range from 0 to 6000 gal/min.
(d) Calculate the wave speed in the pipe.
(e) Formulate the new boundary conditions at the pump.
(f) Modify and recompile PROG1P with the revised boundary conditions.
(g) Find the maximum and minimum pressures at the valve and the pump and the times they occur.
(f) Plot pressure head versus time at both locations for a time extending at least $4L/a$ seconds after the last valve movement. Use PROG1PR with *NPARTS* = 5 in your analysis.

9.9 The pump in the horizontal pipeline below is a two-stage Johnston 18 DC pump (see Appendix B) with $13\frac{3}{4}$-in impellers that provides a flow of 4600 gal/min in the line. The valve at the downstream end is closed rapidly so that the velocity decreases linearly with time to 20% of its steady state value in 2 sec. The pipe is 24 in inside diameter with $f = 0.022$ and a wave speed of 3500 ft/s.



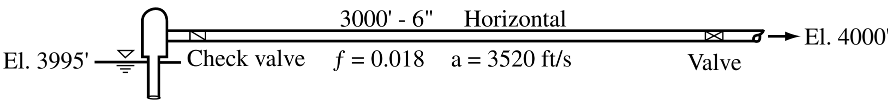
Your tasks:

(a) Develop with program PUMPC a parabolic equation for the pump characteristics of the form

$$h_p = A_p' Q^2 + B_p' Q + C_p'$$

(b) Plot this equation on the pump characteristic diagram to how it fits over the range from 0 to 6000 gal/min.
(c) Find the maximum and minimum pressures at the valve and the check valve and the times they occur.
(d) Plot pressure head as a function of time at both locations for at least $4L/a$ seconds past the end of valve movement. Use program PROG1P with *NPARTS* = 4 in your analysis.

9.10 The pump in this pipeline is a four-stage Johnston 14 BC pump (see Appendix B)



with an 11-in impeller. Under steady flow conditions with the valve open, the discharge is 850 gal/min. The valve will be closed so that the velocity at the valve decreases linearly with time to zero.

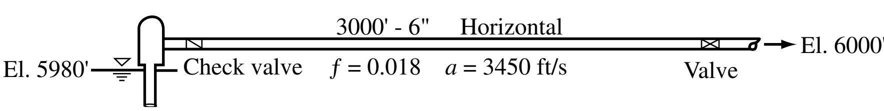
The engineer in charge is considering three separate closure times of 3, 5, and 7 seconds and wants to know the maximum and minimum pressures occurring in each case. Your assignment is to complete these tasks:

(a) Develop with program PUMPC a parabolic equation for the pump characteristics of the form

$$h_p = A_p' Q^2 + B_p' Q + C_p'$$

(b) Plot this equation on the pump characteristic diagram to see how it fits over the range from 0 to 1200 gal/min.
(c) For the three closure times find the maximum and minimum pressures at the valve and the check valve and the times they occur.
(d) Plot pressure head as a function of time at both locations at least $4L/a$ seconds past the ending of valve movement. Use PROG1P with *NPARTS* = 5.

9.11 The pump in the pipeline below is a five-stage Johnston 14 BC pump (see Appendix B) with an 11-inch diameter impeller. The steady state discharge in the system



is 850 gal/min. The valve at the downstream end of the line moves so the velocity at the valve decreases linearly to 4.0 ft/s in 4 sec.

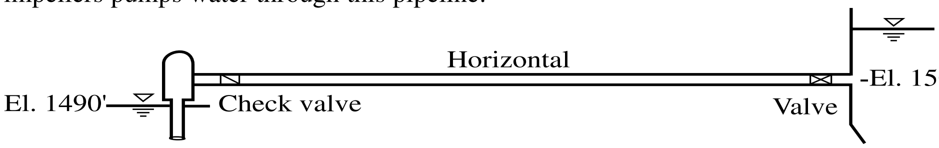
Your tasks:
(a) Develop with program PUMPC a parabolic equation for the pump characteristics of the form

$$h_p = A_p^{'} Q^2 + B_p^{'} Q + C_p^{'}$$

(b) Plot this equation on the pump characteristic diagram to show how it fits over the range from 0 to 1200 gal/min.
(c) Find the maximum and minimum pressures at the valve and the check valve and the times they occur.
(d) Plot pressure head as a function of time at both locations for at least $4L/a$ seconds after the ending of valve movement. Use program PROG1P with $NPARTS = 5$.

9.12 Solve Problem 9.11 but
(a) only reduce the velocity to 5.0 ft/s;
(b) use a wave speed of 3750 ft/s.

9.13 A five-stage Johnston 12 ES turbine pump (see Appendix B) with $7\frac{13}{16}$-in impellers pumps water through this pipeline:



With steady flow and the valve open, the flow is 1600 gal/min. The welded steel line has $f = 0.017$, is 5000 ft long and has a 10-in outside diameter with a wall thickness of 0.135 in. The pipe is installed so that Case (b) restraint most nearly applies. The special valve at the downstream end permits the velocity to be varied linearly with time. Now the engineer wishes to reduce the steady state flow from 1600 gal/min to 250 gal/min. Two closure times of 3 sec and 6 sec are under consideration. To choose between the two, it is desired to know the extreme pressures developed under each closure time.

Your tasks are as follows:
(a) Develop with program PUMPC a parabolic equation for the pump characteristics of the form

$$h_p = A_p^{'} Q^2 + B_p^{'} Q + C_p^{'}$$

(b) Plot this equation on the pump characteristic diagram to see how it fits over the range from 0 to 2000 gal/min.
(c) Calculate the wave speed in the pipe.
(d) Find the maximum and minimum pressures at the valve and the check valve and the times they occur for both closure times.
(e) Plot pressure head as a function of time at both locations for at least $4L/a$ seconds past the last valve movement. Use program PROG1P with $NPARTS = 5$.

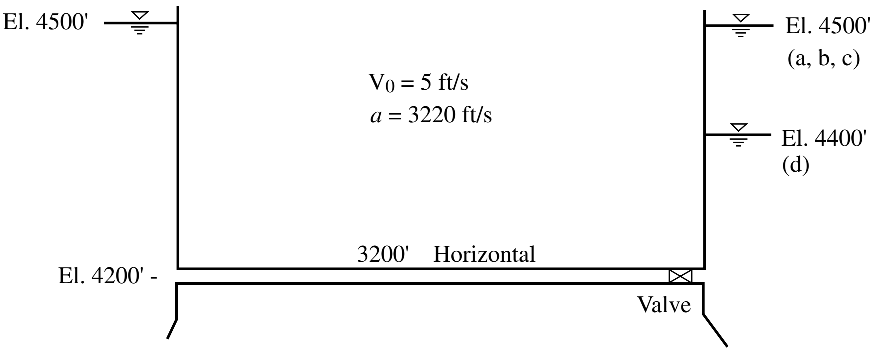
9.14 Answer the first three questions *neglecting friction* in the pipeline shown below.

(a) What is the maximum pressure head which will result from sudden valve closure?

(b) What is the minimum pressure head which will result from sudden valve closure?

(c) Sketch pressure head vs. time at the center of the 3220-ft pipeline for the first $4L/a$ seconds.

(d) If there is a 100-ft *friction loss* in the pipeline, what would be the maximum pressure head resulting from sudden valve closure?

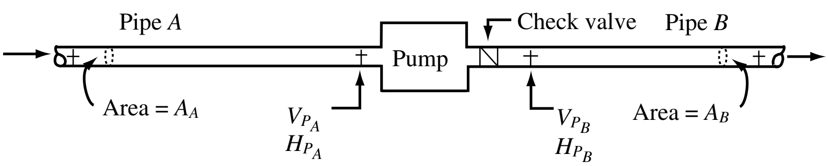


9.15 The booster pump in the pipeline below provides a head increase h_p which can be described by

$$h_p = A_p^{'} Q^2 + B_p^{'} Q + C_p^{'}$$

Develop a set of boundary condition equations which can be solved for V_{P_A}, V_{P_B}, H_{P_A}, and H_{P_B} under all water hammer conditions. Then solve this set of equations to give individual formulas for the calculation of each of these variables under any condition. Assume no negative pressures will occur.

$$C^+ : \; V_{P_A} = C_1 - C_2 H_{P_A} \qquad C^- : \; V_{P_B} = C_3 + C_4 H_{P_B}$$

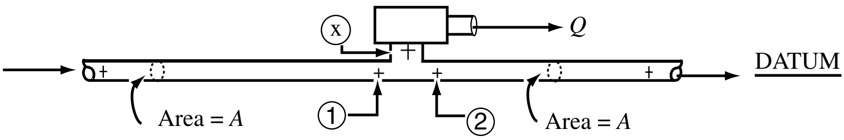


9.16 The surge relief valve in the pipeline shown atop the next page is designed to open when the pressure at the valve exceeds a specified value. When this occurs, the valve opens suddenly and discharges water into the atmosphere according to the equation

$$Q\left(\mathrm{ft}^3/\mathrm{s}\right) = C\sqrt{\text{Pressure head at } x}$$

Using the centerline of the horizontal pipe as the datum, develop an equation for $H_P \left(H_P = H_{P_1} = H_{P_2} = H_{P_x}\right)$ when the surge valve is open. The C^+ and C^- equations are

$$C^+ : \; V_P = C_3 - C_4 H_P \qquad C^- : \; V_P = C_1 + C_2 H_P$$



9.17 The pipeline shown below discharges into the atmosphere just downstream of the gate valve. The surge relief valve just upstream of the gate valve will open when the pressure head in the pipe exceeds H_{max}. When this occurs, the relief valve opens suddenly, and the discharge is then given by

$$Q\left(\mathrm{ft}^3/\mathrm{s}\right) = K\sqrt{\text{Pressure head in pipe at surge valve}}$$

Your tasks:

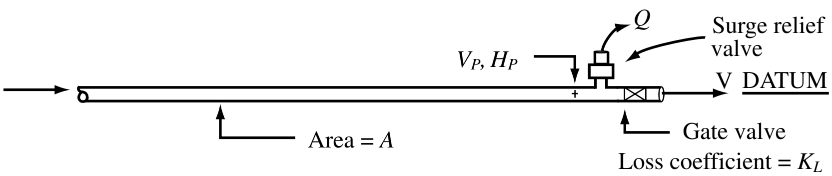
(a) Write a set of equations which can be used to solve for all the variables if
  (1) the surge valve is not open,
  (2) both the surge valve and the gate valve are open,
  (3) the surge valve is open and the gate valve is closed.

(b) Arrange the equations for each condition so they involve H_P only.

(c) For condition (2) above, solve the equation for H_P.

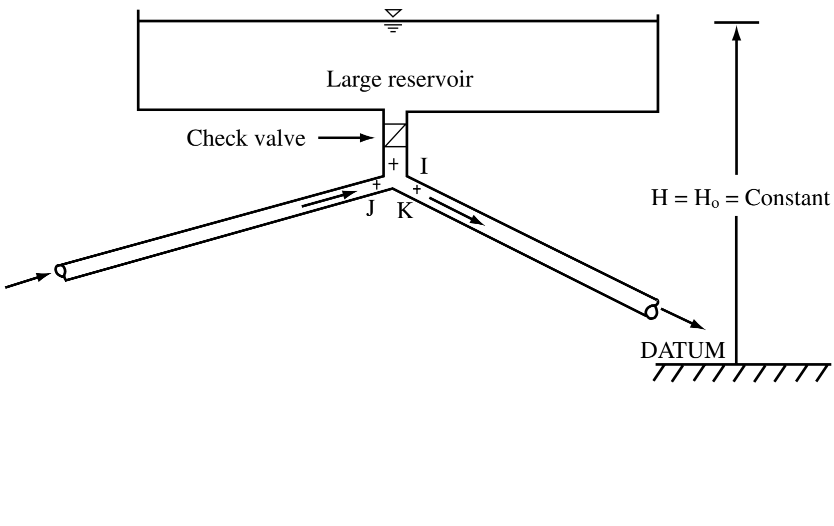
(d) Explain how you would decide which of the conditions would apply.

The C+ equations is $C^+ :\ V_P = C_3 - C_4 H_P$



9.18 The large reservoir is a one-way surge tank connected to the pipeline through a very short pipe I with a check valve. The check valve in the short pipe prevents flow from the pipeline into the reservoir. When H in the pipeline at the junction drops below the value of H_0 in the reservoir, the check valve opens and flow enters the pipeline from the reservoir. The equation for discharge from the reservoir into the pipeline is

$$Q = KA\sqrt{2g\left(H_0 - H_{P_I}\right)}$$



(a) How many unknowns exist at the junction? List them.
(b) Write down the independent equations that contain these unknowns.
(c) From these equations develop an equation containing only H_P as an unknown.
(d) Solve the equation for H_P, explaining how you would select the proper sign in the quadratic solution. Also explain how you would know whether the check valve is open.

All pipes have the same area A. All pipes have the same wave speed a. The C^+ and C^- equations are $C^+\!: \ V_P = C_3 - C_4 H_P \qquad C^-\!: \ V_P = C_1 + C_2 H_P$.

CHAPTER 10

PIPE SYSTEM TRANSIENTS

A natural extension of the analysis of single-pipe systems is to more elaborate pipe systems. In practice, design situations almost always confront systems that are larger and more complex than single, straight pipelines. We now have already been introduced to most of the analysis techniques that are needed for these systems, so we can immediately begin with the simplest type, series pipe systems. We will then move on to branching pipe systems and also examine how to represent actual valve behavior in a realistic way, as opposed to the artificial linear-varying-velocity approach used in Chapter 9. This chapter will prepare us to analyze gravity-flow pipeline transient situations successfully.

10.1 SERIES PIPES

In a series pipe system each pipe (pipeline segment) in the series carries the same steady-flow discharge, but each pipe may have its own velocity, diameter, wave speed, and so on. Each segment must be straight and have constant properties and geometry. These restrictions include the very important case of a single, constant-diameter pipe which can be divided into segments, thereby creating a series pipe system, in order to analyze a pipeline with a profile containing changes in grade.

10.1.1. INTERNAL BOUNDARY CONDITIONS

The method of characteristics solution for each pipe in series proceeds as in Chapter 9. The interior nodes are treated with equations similar to Eqs. 9.64 and 9.65. Boundary conditions at the upstream and downstream ends are again represented by a suitable combination of the C^+ and C^- equations along with a reservoir, valve, or other special condition. The principal difference is the need now for *internal boundary conditions* at the series pipe junctions.

Figure 10.1 portrays a typical series pipe internal boundary condition. There are two points, P_1 and P_2, one on each side of the junction, which are very close together and

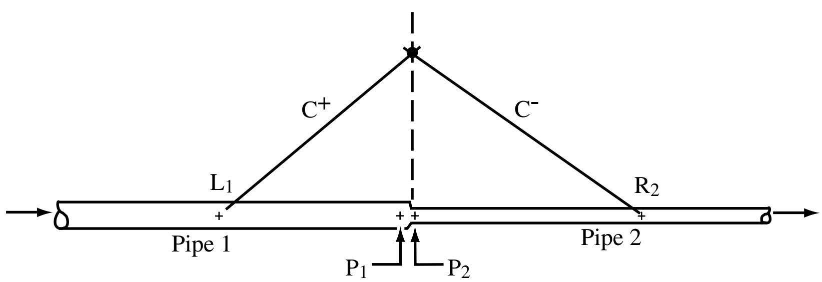


Figure 10.1 Boundary conditions at a typical series pipe junction.

represent the location of four unknown quantities, H_{P_1}, H_{P_2}, V_{P_1}, and V_{P_2}, which must be calculated. Therefore, we must find four equations to solve for these unknowns. For the upstream pipe the C^+ equation can be written, from Eq. 9.53, as

$$V_{P_1} = V_{L_1} + \frac{g}{a_1} H_{L_1} - \frac{f_1 \Delta t}{2 D_1} V_{L_1} \left| V_{L_1} \right| + \frac{g}{a_1} \Delta t V_{L_1} \sin \theta_1 - \frac{g}{a_1} H_{P_1} \tag{10.1}$$

or in short form

$$V_{P_1} = C_3 - C_4 H_{P_1} \tag{10.2}$$

Similarly, for the downstream pipe the C^- equation, Eq. 9.54, yields

$$V_{P_2} = V_{R_2} - \frac{g}{a_2} H_{R_2} - \frac{f_2 \Delta t}{2 D_2} V_{R_2} \left| V_{R_2} \right| - \frac{g}{a_2} \Delta t V_{R_2} \sin \theta_2 + \frac{g}{a_2} H_{P_2} \tag{10.3}$$

or in short form

$$V_{P_2} = C_1 + C_2 H_{P_2} \tag{10.4}$$

It is clear from Eqs. 10.2 and 10.4 that we have four unknowns in the two equations. The two additional required equations are obtained from the conservation of mass and work-energy principles. Assuming there is a negligible mass of fluid between points 1 and 2, conservation of mass gives

$$V_{P_1} A_1 = V_{P_2} A_2 \tag{10.5}$$

Applying the work-energy equation between these same two points 1 and 2, again with negligible mass between the points, and neglecting the difference in velocity heads and any local loss of head across the junction,

$$H_{P_1} = H_{P_2} \tag{10.6}$$

If the head loss at the junction were significant (for example, a closing valve or a pressure reducing valve), then the head loss across the valve must be included in Eq. 10.6.

Solving Eqs. 10.2, 10.4, 10.5, and 10.6 simultaneously leads to the following equations for the heads H at the junction:

$$H_{P_1} = H_{P_2} = \frac{C_3 A_1 - C_1 A_2}{C_2 A_2 + C_4 A_1} \tag{10.7}$$

Once these heads have been computed, the velocities can be found by back-substitution into Eqs. 10.2 and 10.4.

10.1.2. SELECTION OF Δt

In the previous section we presumed that our chosen C^+ and C^- characteristics intersected at the pipe junction, as Fig. 10.1 shows. This is rarely true because the slope of each characteristic line depends on the wave speed and fluid velocity in the pipe and the horizontal location of the node depends on the number of sections into which the pipe is divided. That is to say, if we extend the C^+ and C^- characteristics from adjacent nodes at a particular instant in time, they usually will not intersect at the junction. So we need a strategy to overcome this problem. If we are successful, then Eqs. 10.2, 10.4, and 10.7 will apply, as before.

We begin by rewriting Eq. 9.67 for deriving the value of Δt:

$$\Delta t = \frac{\Delta s}{\max|V+a|} = \frac{\Delta s}{(V+a)} = \frac{L}{N(V+a)} \tag{10.8}$$

For a given N it is clear from Eq. 10.8 that we will usually find a different Δt for each pipe in a series. Figure 10.2 illustrates this situation for two typical pipes in series. At a

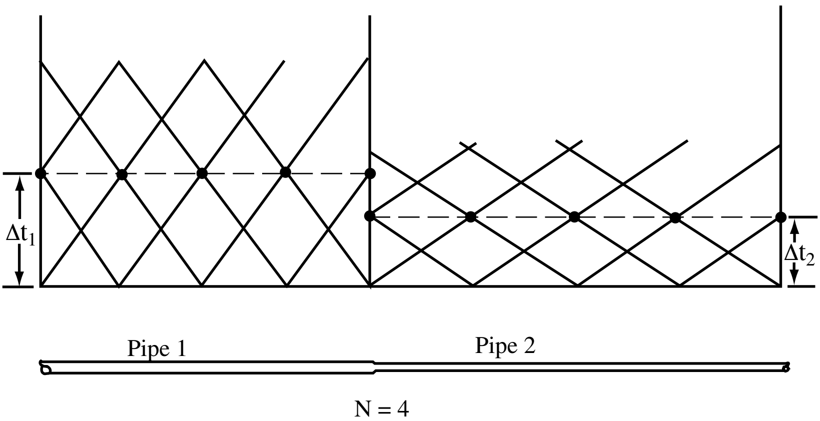


Figure 10.2 The s-t plane for a two-pipe system with equal N's.

pipe junction the result is a pair of characteristic lines that do not meet at the common end of the two pipes. However, we can bring the two characteristics closer to meeting at the junction by choosing a different N for each pipe; however, because N must be an integer, we cannot guarantee this will work. In fact, the chance of success is so small that we must discard this approach.

Another approach which shows more promise is demonstrated in Fig. 10.3. We reduce Δt for all pipes to that value for the pipe with the smallest Δt, called the "controlling"

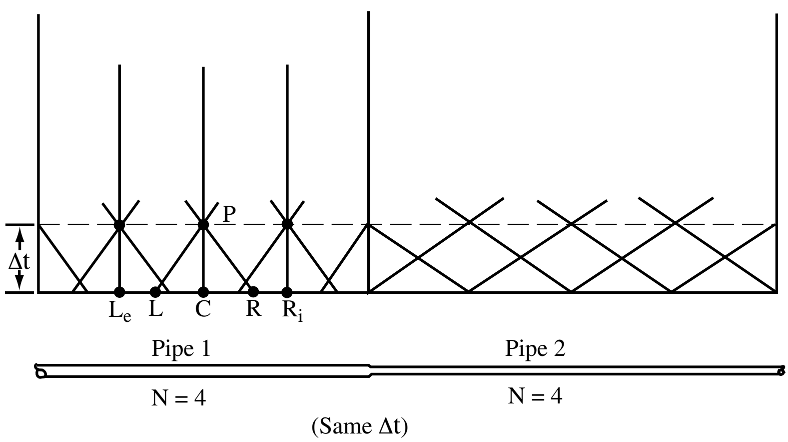


Figure 10.3 The s-t plane with equal Δt's and large interpolations.

pipe. This will force the characteristics to meet at the pipe junction. However, the other end of each characteristic line will no longer intersect at the rectangular grid points. To overcome this problem, we adopt the interpolation procedures developed in Chapter 9. Unfortunately, as can be seen from Fig. 10.3, we may then need some very large interpolations, far too large to assure the accuracy of the numerical analysis.

A technique does exist to reduce this interpolation error while still causing the characteristics to meet at the junction. Here we increase N in all pipes which initially had Δt's larger than that of the "controlling" pipe. As we increase each N, the interpolation gets successively smaller. Eventually, as shown in Fig. 10.4, we obtain a situation in each pipe where a further increase in N will cause the characteristic lines to intersect

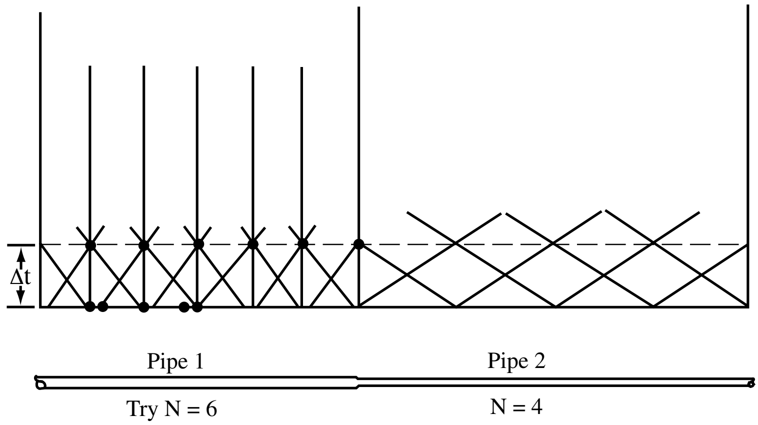


Figure 10.4 The s-t plane with variable N values and minimal interpolation.

outside the rectangular grid point. We stop increasing N just before this happens because this represents the optimum interpolation situation. We now have each of the series pipes divided into a different number of sections with minimum interpolation, but all pipes now have a common Δt.

These values N_i for each pipe i are computed by the recipe

$$N_i = \frac{L_i}{\Delta t_{\min}\left(V_i + a_i\right)} \tag{10.9}$$

where Δt_{min} is associated with the controlling pipe. Integer truncation in the computer will give the proper maximum N_i for each of the pipes in series. For the controlling pipe the original N is retained.

Because the minimization of interpolation error is so significant in the preservation of accuracy, we actually compute the amount of interpolation in the computer program for each series pipe. If the amount of interpolation remains too large, there are ways to reduce it. The easiest way is to increase the base value of N because this will cause all pipes to be divided into more parts and may lead to less interpolation. The only disadvantage is a substantial increase in the amount of computation, which is caused by the larger number of grid points.

Probably it is best to pursue some preliminary computations to determine which pipes in the series system produce the most trouble. Then move a few internal junctions (change the length of individual pipe sections without changing the total length of the system) to generate segment lengths which cause less interpolation error. Another popular technique is to adjust the wave speed to reduce interpolation error. The rationale behind this approach is that the wave speed cannot be determined very precisely anyway, so why not use this uncertainty to improve the numerical simulation accuracy? Karney and Ghidaoui (1997) recently reviewed the interpolation issue and propose a new, more flexible technique. However, they also state that no one method is best in all cases. Whichever technique minimizes interpolation error in a particular application should be used.

Another "trick" which may work well in conjunction with the previous technique is to "simplify" the pipeline profile. Often a pipeline has literally dozens of changes in grade, and following these grades precisely would require a series pipe analysis with dozens of pipe segments. This problem formulation, along with the minimization of interpolation error, could lead to prohibitive computation times. However, since experience has shown that pipeline slope has little effect on water hammer pressures, the actual pipeline profile can be replaced with a model containing only a few segments and a simplified profile without seriously affecting the results of the analysis. This feature permits short pipe segments to be combined with longer segments and/or several short segments to be combined into a single long segment. This can be done with concurrent attention to opportunities to minimize interpolation error. Care should be taken, however, to attempt to include the high and low points along the pipeline as junctions, since they tend to be the critical pressure points. With some experience and care in approximating a pipeline profile, the user will be able to analyze a system accurately with a minimum of computation.

10.1.3. THE COMPUTER PROGRAM

The computer program PROG2 for the solution of series pipe problems is included on the CD. It is an extension of PROG1 which accounts for the differing properties in the series pipes. The input parameters are again defined with COMMENT statements in the program listing, and the user must develop the steady-state input conditions outside the program.

In this program a double subscripting of variables is required since parameters now vary from pipe to pipe. Some caution is appropriate in choosing a base value for N. If a relatively short pipe occurs in the system, it will probably be the controlling pipe and produce the minimum Δt. If other considerably longer pipe segments exist in the system, they will have a large number of sections. A short preliminary computer run ($TMAX = 2\Delta t$) with a small N will permit you to examine the effect of parameters on the analysis and see how much interpolation, which is displayed, is required.

This program assumes a reservoir at the upstream end and a closing valve and reservoir at the downstream end. Head loss coefficients must be entered for the valve which can be closed at two different rates. The program also provides a printer plot of maximum and minimum pressure heads along the pipeline, a printer plot of pressure head vs. time for up to four points along the pipeline, a table of pressure head and velocity vs. time for the same four points, and a data file which can be read by an external graphing program to make traditional graphs of pressure head vs. time. The subroutines PGRAPH and PROFILE are used to accomplish these tasks; input data requirements and input parameter descriptions are included as COMMENT statements in the subroutine source listings.

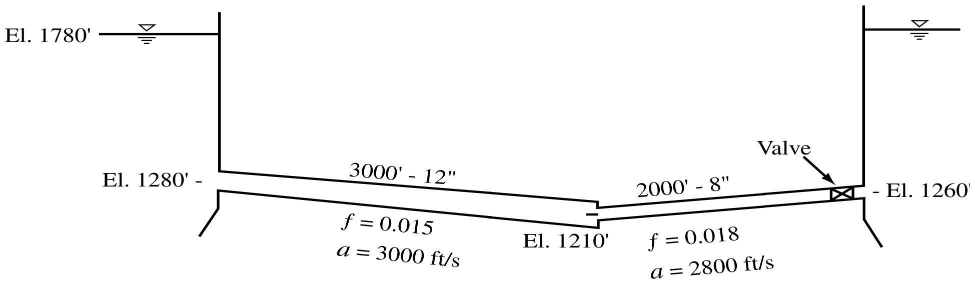
To demonstrate the use of this program and its various features, we now look at a simple example. Though we have not yet discussed how real valve input data is developed, we include such a valve treatment in this example; the details of modeling the valve head loss are included later in this chapter.

Example Problem 10.1

The series pipe system shown on the next page conveys 800 gal/min and has a valve at the downstream end which closes at a uniform rate until it is fully closed after 5 sec. Find the maximum and minimum pressures in the system and their points and times of occurrence. Also provide printer plots of the extreme pressure heads along the pipeline and pressure head vs. time at the valve and at the pipe junction, tables of pressure head vs. time at these two points, and a traditional plot of pressure head vs. time at the two points.

The input data file to accomplish these tasks follows:

```
DEMONSTRATION OF PROGRAM NO. 2 - INPUT DATA FILE "EP101.DAT"
RESERVOIR UPSTREAM, VALVE CLOSING LINEARLY IN 5 SEC DOWNSTREAM
&SPECS NPIPES=2,NPARTS=5,IOUT=10,QZERO=800.,HZERO=1780.,ZEND=1260.,
       HATM=32.,TMAX=15.00,DTNEW=0.,TC1=0.,TC2=5.00,PC1=100.,
       PFILE=T,HVPRNT=T,PPLOT=T,GRAPH=T,RERUN=F/
0.   .0167  .0313  .0556  .100  .1787  .3333  .625  1.25  2.50  5.27
12.00  3000.  .015  3000.  1280.
 8.00  2000.  .018  2800.  1210.
&GRAF NSAVE=2,IOUTSA=2,PIPE=2,2,0,0,NODE=1,999,0,0/
```



The initial portion of the output file follows. The printout is too lengthy to reproduce in its entirety but is contained on the CD as OUT101 and may be reviewed with the text editor. The page accompanying the abbreviated printout shows the traditional graph of pressure head vs. time as produced by Axum software from Trimetrix. This software reads the plot file created at the end of execution when GRAPH=T is in the input data (see above data file).

```
DEMONSTRATION OF PROGRAM NO. 2 - INPUT DATA FILE "IP101.DAT"
RESERVOIR UPSTREAM, VALVE CLOSING LINEARLY IN 5 SEC DOWNSTREAM

                          INPUT DATA
                          ----------

                         IOUT =  10
                       NPARTS =   5
                       NPIPES =   2

                        QZERO =  800.0 GPM
                        HZERO = 1780.0 FT

                         ZEND = 1260.0 FT
                         HATM =   32.0 FT

                         TMAX =  15.00 SEC
                         DELT =   .143 SEC
                          TC1 =    .00 SEC
                          PC1 = 100.00 PERCENT OPEN
                          TC2 =   5.00 SEC, VALVE IS CLOSED
```

VALVE LOSS COEFFICIENTS

% OPEN	1.0/KL
0.	.000E+00
10.	.167E-01
20.	.313E-01
30.	.556E-01
40.	.100E+00
50.	.179E+00
60.	.333E+00
70.	.625E+00
80.	.125E+01
90.	.250E+01
100.	.527E+01

PIPE INPUT DATA

PIPE	DIAM,IN	LENGTH,FT	WAVE SPD,FT/S	PIPEZ,FT	F	VEL,FT/S
1	12.00	3000.0	3000.	1280.	.0150	2.27
2	8.00	2000.0	2800.	1210.	.0180	5.10

PIPE	DELT,SEC	PARTS	SINE	L/A,SEC	INTERPOLATION
1	.200	7	-.02333	1.00	.002
2	.143	5	.02500	.71	.002

PRESSURE HEADS, H-VALUES AND VELOCITIES AS FUNCTIONS OF TIME

T = .000 SEC	X	HEAD,FT	H,FT	V,FT/S	X	HEAD,FT	H,FT	V,FT/S
PIPE 1	.000	500.	1780.	2.27	.143	509.	1779.	2.27
	.286	519.	1779.	2.27	.429	528.	1778.	2.27
	.571	538.	1778.	2.27	.714	547.	1777.	2.27
	.857	557.	1777.	2.27	1.000	566.	1776.	2.27
PIPE 2	.000	566.	1776.	5.10	.200	552.	1772.	5.10
	.400	538.	1768.	5.10	.600	523.	1763.	5.10
	.800	509.	1759	5.10	1.000	495.	1755.	5.10

T = 1.426 SEC	X	HEAD,FT	H,FT	V,FT/S	X	HEAD,FT	H,FT	V,FT/S
PIPE 1	.000	500.	1780.	2.27	.143	509.	1779.	2.27
	.286	519.	1779.	2.27	.429	528.	1778.	2.27
	.571	538.	1778.	2.27	.714	547.	1777.	2.27
	.857	557.	1777.	2.27	1.000	566.	1776.	2.27
PIPE 2	.000	566.	1776.	5.10	.200	552.	1772.	5.10
	.400	538.	1768.	5.10	.600	524.	1764.	5.10
	.800	509.	1759.	5.10	1.000	495.	1755.	5.10

```
                    **************************
                    * TABLE OF EXTREME VALUES *
                    **************************
```

	X	MAX HEAD	TIME	MIN HEAD	TIME	MAX H	MIN H
PIPE 1							
	.000	500.0	15.1	500.0	15.1	1780.	1780.
	.143	746.9	6.7	346.4	10.1	2017.	1616.
	.286	777.7	6.6	341.1	10.1	2038.	1601.
	.429	794.1	11.8	332.1	15.1	2044.	1582.
	.571	820.4	11.8	280.1	15.1	2060.	1520.
	.714	834.2	11.8	289.1	15.1	2064.	1519.
	.857	858.7	11.4	256.7	14.8	2079.	1477.
	1.000	878.4	11.4	248.0	14.8	2088.	1458.
	X	MAX HEAD	TIME	MIN HEAD	TIME	MAX H	MIN H
PIPE 2							
	.000	878.4	11.4	248.0	14.8	2088.	1458.
	.200	961.5	5.7	92.7	9.1	2182.	1313.
	.400	962.4	5.6	60.4	9.1	2192.	1290.
	.600	955.5	5.6	47.3	9.1	2196.	1287.
	.800	946.0	5.7	34.2	9.4	2196.	1284.
	1.000	936.3	5.6	15.6	9.4	2196.	1276.

```
MAX HEAD = 962.4 FT (416.6 PSI) IN PIPE 2 AT X = .400 AT T = 5.56 SEC

MIN HEAD =  15.6 FT (  6.7 PSI) IN PIPE 2 AT X =1.000 AT T = 9.41 SEC
```

*   *   *

10.2 BRANCHING PIPES

A new feature, not occurring in direct series pipes, is found when three or more pipes join at a junction. We will call these branching pipe systems. As a practical measure, we will consider only three-pipe and four-pipe junctions.

10.2.1. THREE-PIPE JUNCTIONS

The typical three-pipe junction is shown in Fig. 10.5, with the initial flow directions indicated by arrows whose directions are established by the steady state conditions. That is, the signs of terms in the equations to be written and the characteristic lines to be followed will be determined by the steady flow behavior (Since the direction of flow was readily apparent in our earlier problems, there was no previous need to emphasize its determination).

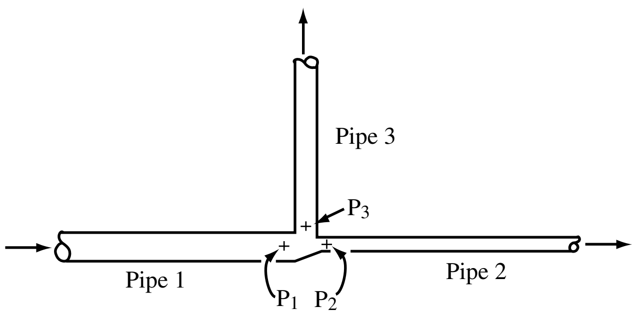
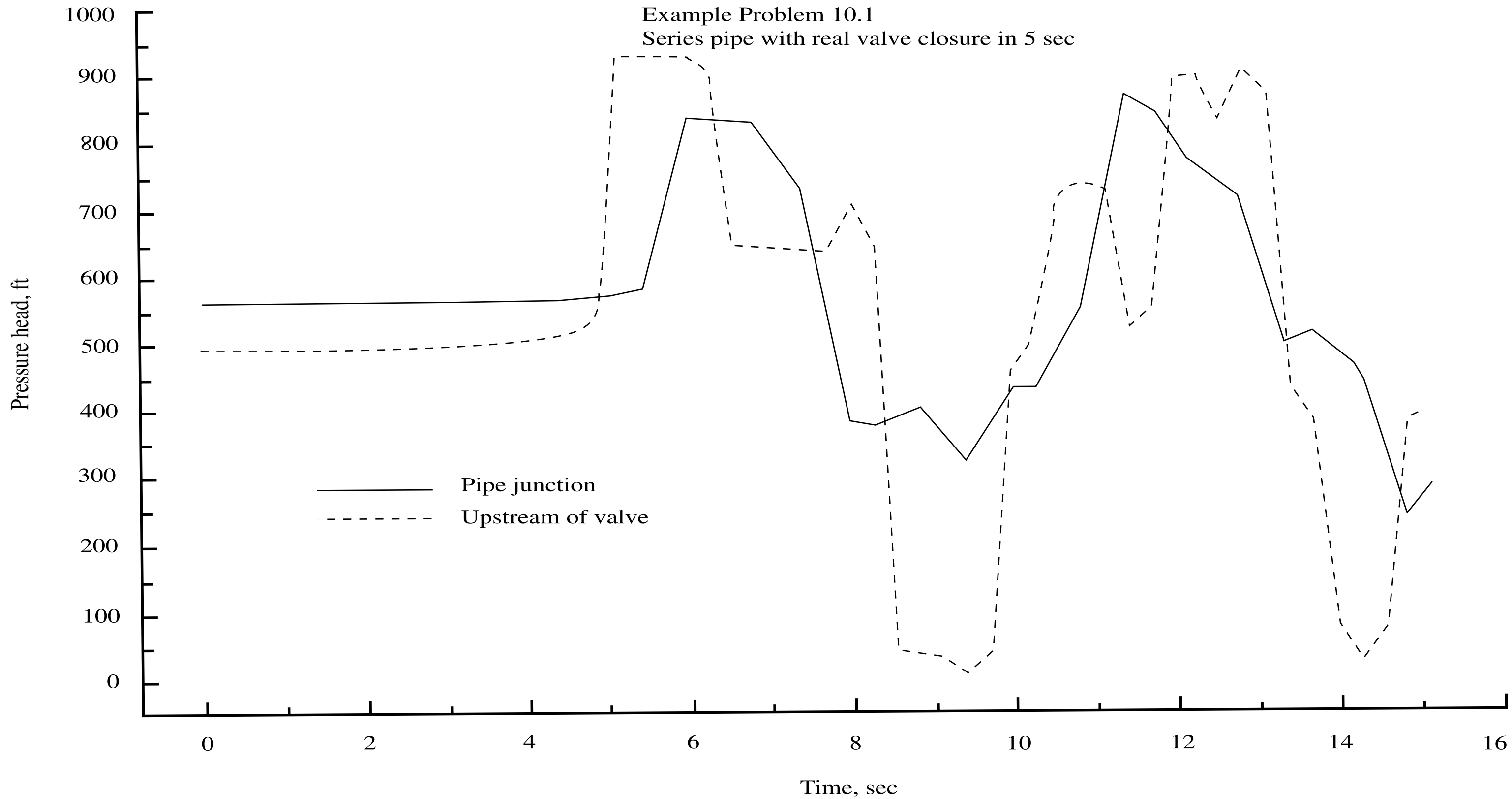


Figure 10.5 The one-in, two-out, three-pipe junction.



Example Problem 10.1
Series pipe with real valve closure in 5 sec
Pressure head, ft
1000
900
800
700
600
500
400
300
200
100
0
Pipe junction
Upstream of valve
0
2
4
6
8
10
12
14
16
Time, sec

For the pipe junction with one inflow and two outflows, the following equations describe the relations between the six unknowns

Pipe 1, C^+ $$V_{P_1} = C_1 - C_2 H_{P_1} \quad (10.10)$$

Pipe 2, C^- $$V_{P_2} = C_3 + C_4 H_{P_2} \quad (10.11)$$

Pipe 3, C^- $$V_{P_3} = C_5 + C_6 H_{P_3} \quad (10.12)$$

Conservation of mass $$V_{P_1} A_1 = V_{P_2} A_2 + V_{P_3} A_3 \quad (10.13)$$

Work-energy, neglecting local loss $$H_{P_1} = H_{P_2} = H_{P_3} \quad (10.14)$$

Here the subscripts indicate the unknown velocities and heads at the pipe junction. Equation 10.14 is actually two independent equations.

Solving this linear set of equations leads to

$$H_{P_1} = H_{P_2} = H_{P_3} = \frac{C_1 A_1 - C_3 A_2 - C_5 A_3}{C_2 A_1 + C_4 A_2 + C_6 A_3} \quad (10.15)$$

Back substitution of these heads into Eqs. 10.10, 10.11, and 10.12 yields the velocities.

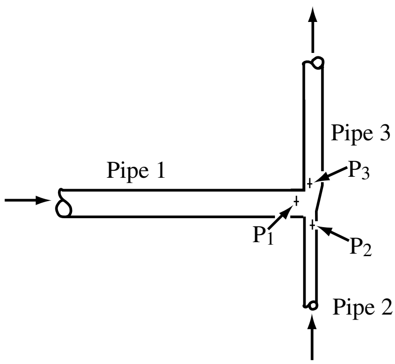


Figure 10.6 The two-in, one-out, three-pipe junction.

If instead we have a three-pipe junction with two inflows and one outflow, as shown in Fig. 10.6, an similar analysis would lead to the following equations for finding the unknown velocities and heads:

$$H_{P_1} = H_{P_2} = H_{P_3} = \frac{C_1 A_1 + C_3 A_2 - C_5 A_3}{C_2 A_1 + C_4 A_2 + C_6 A_3} \quad (10.16)$$

$$V_{P_1} = C_1 - C_2 H_{P_1} \quad (10.17)$$

$$V_{P_2} = C_3 - C_4 H_{P_2} \quad (10.18)$$

$$V_{P_3} = C_5 + C_6 H_{P_3} \quad (10.19)$$

All three-pipe junctions fit into one of the two categories above unless there is an external demand at the junction. Figure 10.7 shows such a situation. The only effect of this external demand is to modify the mass conservation relation. The energy and C$^+$ and C$^-$ equations remain the same. The modified mass conservation equation is

$$V_{P_1} A_1 = V_{P_2} A_2 + V_{P_3} A_3 + Q \tag{10.20}$$

The impact on the solution is the addition of one term in the head equations:

$$H_{P_1} = H_{P_2} = H_{P_3} = \frac{C_1 A_1 - C_3 A_2 - C_5 A_3 - Q}{C_2 A_1 + C_4 A_2 + C_6 A_3} \tag{10.21}$$

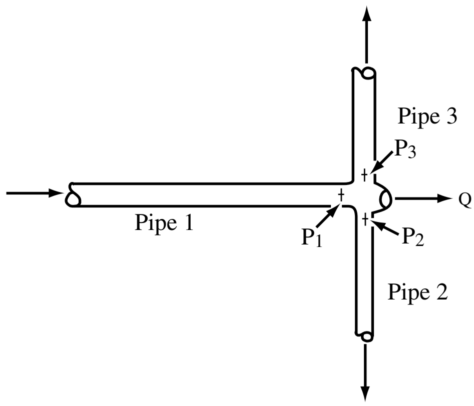


Figure 10.7 The three-pipe junction with a constant demand outflow.

Examination of the above sets of equations for H_P for the various junctions reveals a consistency in form which would allow the reader, with some experience, to write directly the equation sets for H_P by inspection.

10.2.2. FOUR-PIPE JUNCTIONS

Four-pipe junctions are analyzed with the same techniques as were the three-pipe junctions. We will examine only one application and let the reader practice on others.

Figure 10.8 shows a one-in, three-out, four-pipe junction with, in addition, a constant

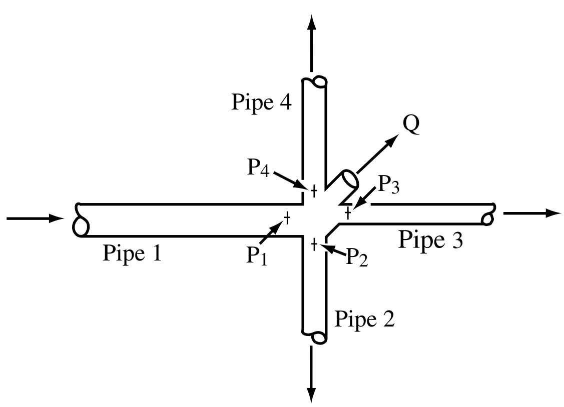


Figure 10.8 The one-in, three-out, four-pipe junction.

demand Q at the junction. The equations of the four characteristic lines are

Pipe 1, C^+ $$V_{P_1} = C_1 - C_2 H_{P_1} \tag{10.22}$$

Pipe 2, C^- $$V_{P_2} = C_3 + C_4 H_{P_2} \tag{10.23}$$

Pipe 3, C^- $$V_{P_3} = C_5 + C_6 H_{P_3} \tag{10.24}$$

Pipe 4, C^- $$V_{P_4} = C_7 + C_8 H_{P_4} \tag{10.25}$$

Conservation of mass $$V_{P_1} A_1 = V_{P_2} A_2 + V_{P_3} A_3 + V_{P_4} A_4 + Q \tag{10.26}$$

Work-energy, neglecting local loss $$H_{P_1} = H_{P_2} = H_{P_3} = H_{P_4} \tag{10.27}$$

Solving these equations for the head values at the junction,

$$H_{P_1} = H_{P_2} = H_{P_3} = H_{P_4} = \frac{C_1 A_1 - C_3 A_2 - C_5 A_3 - C_7 A_4 - Q}{C_2 A_1 + C_4 A_2 + C_6 A_3 + C_8 A_4} \tag{10.28}$$

As before, back substitution into the C^+ and C^- equations will give the velocities.

As we will see when pipe networks are discussed, a computer program can be written which will examine the pipes at each junction and automatically classify them according to number of pipes and junction configuration. The approach to the analysis is to employ the pattern we have observed in the previous examples.

10.3 INTERIOR MAJOR LOSSES

Occasionally a device is located in the interior of a pipeline which causes a significant loss in the system, either by choice (a pressure-reducing valve) or by necessity (e.g., a constriction, a meter, a partially-closed valve). Whatever the cause, if the loss is significant in comparison with other frictional losses, it must be included in the analysis.

Possible approaches to the treatment of this loss are to distribute it uniformly along the pipe by increasing the roughness of the pipe, to lump it into the boundary condition at one of the junctions, or to analyze it at its actual pipe location. Because the latter approach most resembles the true physical situation, we will approach the problem in this way.

We assume the pipe on each side of the loss has a different size, wave speed, and length; we further assume the energy loss across the device is proportional to the square of the velocity in the downstream pipe. The approach is similar to that used for the series pipe junction (see Fig. 10.9):

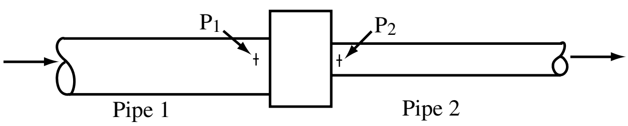


Figure 10.9 A model for interior major losses.

The first three equations apply directly and are rewritten below:

Pipe 1, C^+ $$V_{P_1} = C_1 - C_2 H_{P_1} \tag{10.29}$$

Pipe 2, C^- $$V_{P_2} = C_3 + C_4 H_{P_2} \tag{10.30}$$

Conservation of mass $$V_{P_1} A_1 = V_{P_2} A_2 \tag{10.31}$$

Application of the work-energy equation across the device assumes the loss would be the same as for steady flow at the instantaneous unsteady velocity:

Work-energy $$H_{P_1} = H_{P_2} + K_L \frac{V_{P_2}^2}{2g} \tag{10.32}$$

Combining Eqs. 10.29 through 10.32 provides the following equation for V_{P_2}:

$$V_{P_2}^2 + \frac{2g}{K_L}\left(\frac{1}{C_4} + \frac{A_2}{A_1}\frac{1}{C_2}\right)V_{P_2} - \frac{2g}{K_L}\left(\frac{C_3}{C_4} + \frac{C_1}{C_2}\right) = 0 \tag{10.33}$$

To solve the quadratic equation, we define

$$C_5 = \frac{2g}{K_L}\left(\frac{1}{C_4} + \frac{A_2}{A_1}\frac{1}{C_2}\right) \qquad C_6 = \frac{2g}{K_L}\left(\frac{C_3}{C_4} + \frac{C_1}{C_2}\right) \tag{10.34}$$

so that Eq. 10.33 can be written as

$$V_{P_2}^2 + C_5 V_{P_2} - C_6 = 0 \tag{10.35}$$

The quadratic formula solution is

$$V_{P_2} = \frac{1}{2}\left[-C_5 \pm \sqrt{C_5^2 + 4C_6}\right] \tag{10.36}$$

Because C_2 and C_4 are always positive, C_5 must be positive. Therefore, the (+) sign in front of the radical must be retained, for the velocity would otherwise always be negative. The final equation is

$$V_{P_2} = \frac{C_5}{2}\left[-1 + \sqrt{1 + \frac{4C_6}{C_5^2}}\right] \tag{10.37}$$

The upstream velocity and the two heads can be found by back-substitution into Eqs. 10.29 through 10.31.

One additional problem is created by this type of loss. If V_{P_2} should become negative, the work-energy equation would not be valid in its present form. For reverse flow the proper work-energy equation would be

$$H_{P_1} + K_{L_{rev}} \frac{V_{P_1}^2}{2g} = H_{P_2} \tag{10.38}$$

We must then reconstitute Eq. 10.33, revise the constants C_5 and C_6 and develop anew the equation for V_{P_2}.

10.4 REAL VALVES

Of all unsteady flow situations in pipes, it is likely that those caused by valve movement will be the most common. By constricting the flow, the closing valve creates an increasing head loss in the pipe system which causes the flow to decelerate. Different types of valves create head loss in different ways which are determined by not only the structure of the valve but also the details of the closure sequence.

For steady-state hydraulics the equation for head loss through a valve has traditionally been based on a form that has its roots in dimensional analysis:

$$h_L = K_L \frac{V^2}{2g} \tag{10.39}$$

Here h_L is the head loss, V is the velocity in the pipe (not in the valve), and K_L is the valve loss coefficient (see Chapter 2). Again we assume the steady-flow equation can be used in the unsteady situation to predict the head loss at the instantaneous velocity.

In many instances the user may have information which quantifies the valve loss coefficient for only two or three valve settings. It is then important to create a continuous variation in K_L with valve position so a transient analysis can be performed with some confidence that the results will be reasonably accurate.

Loss coefficients for a given valve at different actuator positions are determined by direct laboratory measurements. That is, K_L is a function of actuator position. If we know how the valve actuator moves with time, then we can determine a numerical value for K_L at that time. We then insert that value into Eq. 10.32 and solve for the values of the other variables. We will now demonstrate the application of this technique for valves at the interior of a pipeline and at the downstream end.

10.4.1. VALVE IN THE INTERIOR OF A PIPELINE

We presume the valve in the interior of the pipeline is scheduled to close (or open) according to some prescribed timetable. The equations describing this internal boundary condition are given by Eqs. 10.29 through 10.32 with equal pipe areas on both sides of the valve. Figure 10.10 illustrates the situation:

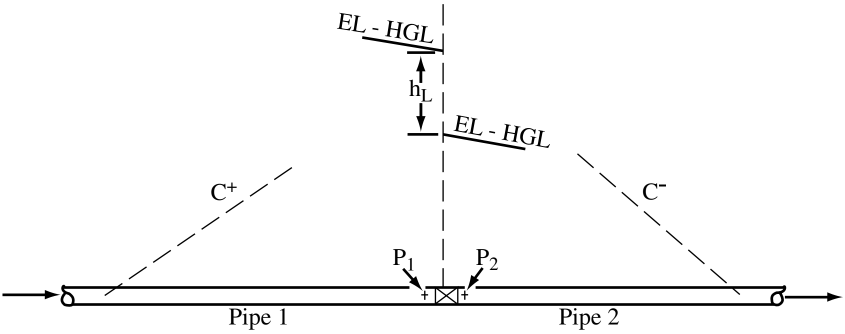


Figure 10.10 Valve in a pipeline of constant diameter.

We assemble the applicable equations in their modified form as

Pipe 1, C^+ $$V_{P_1} = C_3 - C_4 H_{P_1} \tag{10.40}$$

Pipe 2, C^- $$V_{P_2} = C_1 + C_2 H_{P_2} \tag{10.41}$$

Conservation of mass $$V_{P_1} = V_{P_2} \tag{10.42}$$

Work-energy $$H_{P_1} = H_{P_2} + K_L \frac{V_{P_2}^2}{2g} \tag{10.43}$$

The equation obtained by combining Eqs. 10.40 through 10.43 is

$$V_{P_2}^2 + \frac{2g}{K_L}\left(\frac{1}{C_4} + \frac{1}{C_2}\right) V_{P_2} - \frac{2g}{K_L}\left(\frac{C_3}{C_4} + \frac{C_1}{C_2}\right) = 0 \tag{10.44}$$

While keeping K_L separate, definition of the coefficients

$$C_5 = 2g\left(\frac{1}{C_4} + \frac{1}{C_2}\right) \qquad C_6 = 2g\left(\frac{C_3}{C_4} + \frac{C_1}{C_2}\right) \tag{10.45}$$

leads to the velocity expression

$$V_{P_1} = V_{P_2} = \frac{C_5}{2K_L}\left[-1 + \sqrt{1 + \frac{4C_6 K_L}{C_5^2}}\right] \tag{10.46}$$

This equation is correct so long as the flow is in the original downstream direction. If the flow reverses, then we must modify Eq. 10.43 and re-solve the set of equations to obtain

$$V_{P_1} = V_{P_2} = \frac{C_5}{2K_{L_{rev}}}\left[1 - \sqrt{1 - \frac{4C_6 K_{L_{rev}}}{C_5^2}}\right] \tag{10.47}$$

We should recall here that the C^+ and C^- characteristics and the equations associated with them are determined by the original flow direction and need not be redefined as the result of a temporary flow reversal. However, the reverse-flow head loss characteristics of valves are generally different from those for forward flow and are often not available. Under these circumstances the analyst may be forced to use the forward-flow characteristics for flow in both directions. While this assumption may well be acceptable for gate, ball, cone, and plug valves, it is questionable for globe, angle, and butterfly valves.

10.4.2. VALVE AT DOWNSTREAM END OF PIPE AT RESERVOIR

Valves quite often are located at the downstream ends of pipelines, so we will consider this common case to see what modifications of the previous equations are needed to effect a solution. Now we are no longer talking about an interior boundary condition, but the approach is the same. We consider the valve to be positioned just before a reservoir so the pressure head downstream of the valve is fixed at the reservoir elevation (see Fig. 10.11). Other downstream conditions may occur; e.g., a free discharge from the valve into the atmosphere would fix the pressure downstream of the valve at zero gage.

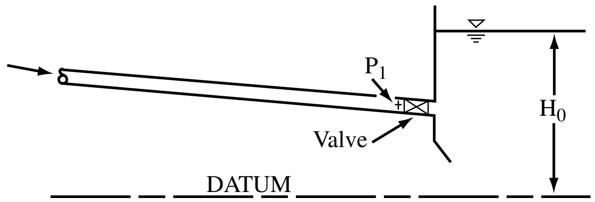


Figure 10.11 Valve and reservoir at the downstream end of a pipeline.

In Fig. 10.11 we now see only two unknowns at the valve, so the required equations are

C+
$$V_{P_1} = C_3 - C_4 H_{P_1} \tag{10.48}$$

Work-energy
$$H_{P_1} = H_0 + K_L \frac{V_{P_1}^2}{2g} \tag{10.49}$$

Combining these two equations to form the quadratic equation for V_{P_1} yields

$$V_{P_1}^2 + \frac{2g}{K_L C_4} V_{P_1} + \frac{2g}{K_L}\left(H_0 - \frac{C_3}{C_4} \right) = 0 \tag{10.50}$$

With the coefficients

$$C_5 = \frac{2g}{C_4} \qquad C_6 = 2g\left(H_0 - \frac{C_3}{C_4} \right) \tag{10.51}$$

the solution to the quadratic equation is

$$V_{P_1} = \frac{C_5}{2K_L}\left[-1 + \sqrt{1 - \frac{4C_6 K_L}{C_5^2}} \right] \tag{10.52}$$

or, for reverse flow,

$$V_{P_1} = \frac{C_5}{2K_{L_{rev}}}\left[1 - \sqrt{1 + \frac{4C_6 K_L}{C_5^2}} \right] \tag{10.53}$$

It is now clear that we can determine the impact of valve movement on pressures and velocities in the pipe system whenever we are able to express the valve loss coefficient as a function of time for the given closure (or opening) schedule. We will now see how to achieve this objective.

10.4.3. EXPRESSING K_L AS A FUNCTION OF TIME

To solve transient problems with closing or opening valves, we must learn how to enter the valve position schedule into the computer program so that the value of K_L can be found at any time. We begin by assuming that values of K_L for more than one valve position or setting are available from the manufacturer. We find it convenient to arrange the computer program so it accepts values of K_L at 11 evenly spaced positions ranging from 100% open to 0% open (closed). In other words, we wish to synthesize a K_L vs. percent-open table from available data. Soon we will see how to do this. Consequently, if the percent-open is known at a particular time from the closure schedule, then the computer program can interpolate the correct value of K_L from the percent-open table. And this value can next be entered into Eq. 10.52 or Eq. 10.53, and the solution can be completed for this time step. Next we examine the details of a technique to generate a table of K_L vs. percent-open.

Usually head loss characteristics of valves are expressed in one of three ways. Two of these employ the K_L or C_v coefficients discussed in Chapter 2. The third method relies on the nondimensional valve-closure function τ, defined as

$$\tau = \sqrt{\frac{K_{L_0}}{K_L}} \tag{10.54}$$

in which K_{L_0} is the loss coefficient when the valve is fully open. This nondimensional form of the loss coefficient has the advantage of varying between 0 and 1 and is preferred by some. Because head loss coefficients are generally provided as K_L's, we will use this form here. Several attempts have been made to present the transient pressures developed by valve closure in graphical form. A typical work by Wood and Jones (1973) briefly reviews these methods and then presents their own comprehensive graphs. As they point out, it is impossible to include all of the effects of friction and system configuration in simple graphical form. For this reason we will bypass the graphical approaches and concentrate on computerizing the representation of any valve in any pipeline configuration.

We will begin by determining the values of K_L needed to complete the K_L vs. percent-open table. In this example we will work with a gate valve (see Street et al., 1996), for which K_L values are provided for only four positions (see Table 10.1). Loss coefficients and other data for different valves are provided in Appendix C. The next step is

Table 10.1

Loss Coefficients for a Gate Valve (Street et al., 1996)

% Open	K_L	$1/K_L$
100	0.19	5.27
75	1.15	0.87
50	5.6	0.18
25	24.0	0.04

to construct a graph of K_L and $1/K_L$ as a function of percent-open by plotting the values from Table 10.1, constructing a smooth curve through the data, and extending it over the full range of percent-open from 100% to 0%. The result is shown in Fig. 10.12.

Clearly a problem exists in the plot of K_L vs. percent-open near valve closure where K_L approaches infinity as velocity goes to zero. To avoid this problem, we will develop the full-range curve only for $1/K_L$. Note in our solution for velocity that it is $1/K_L$ that appears in the equations rather than K_L.

Next we construct a table of $1/K_L$ values for uniform increments of percent-open by reading the values from Fig. 10.12. The results are listed in Table 10.2. Now, if the valve-closure schedule is known (percent-open vs. time), then we will know the percent-open at any given time. We can then interpolate the proper value of K_L from Table 10.2 and proceed with our solution. This requires the computer program to be capable of performing the interpolation process. This is accomplished by fitting a straight line (or curve) between two (or more) data points in the table and interpolating. In such problems linear (straight-line) interpolation is often adequate, and parabolic interpolation is usually sufficient to cover the remaining situations.

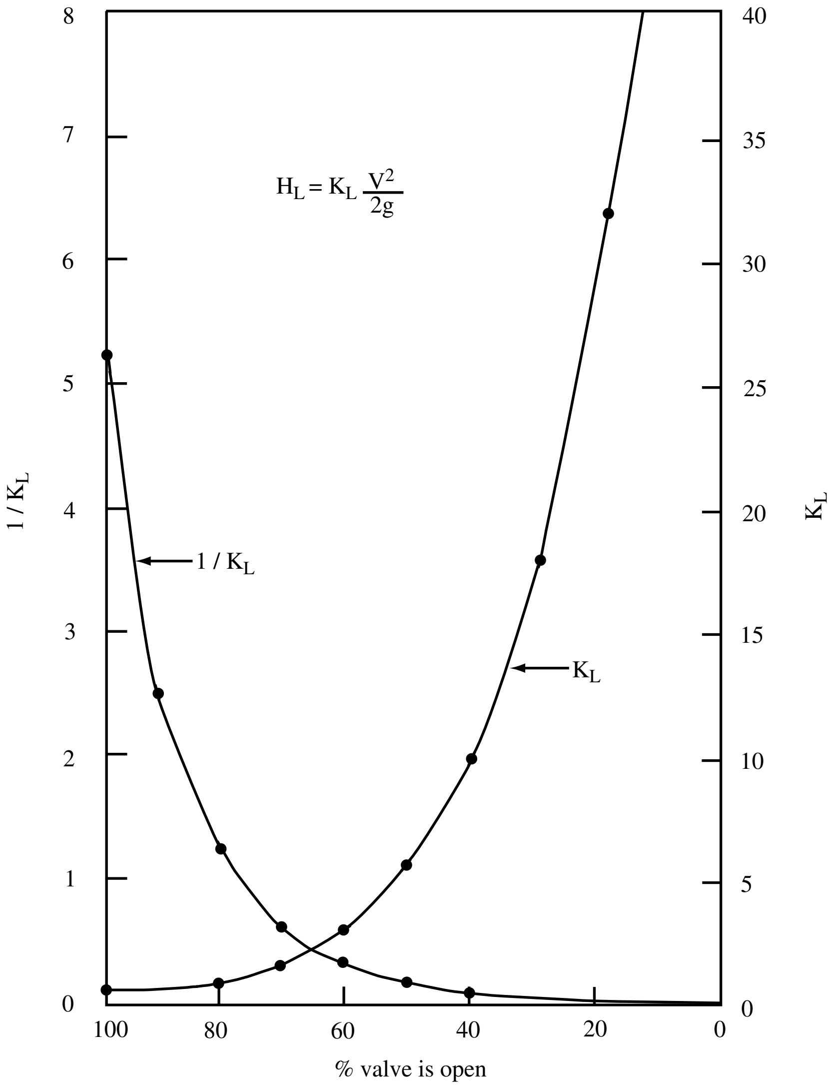


Figure 10.12 K_L and $1/K_L$ as functions of percent-open.

10.4.4. LINEAR INTERPOLATION

Because linear or straight-line interpolation is easiest to understand, we examine it first. For example, if the valve-closure schedule required, at a particular time, the determination of $1/K_L$ at 72.4% open, we may compute it from Table 10.2 with the following interpolation:

$$1/K_L = 0.625 + \frac{72.4 - 70}{80 - 70}(1.25 - 0.625) = 0.775 \qquad (10.55)$$

Now let us see how to program this step. The $1/K_L$ values for each percent-open are read into the program as data. The percent-open values are stored in an array called *PCT()* while the values of $1/K_L$ are stored in an array called *KI()*. The instantaneous value of

Table 10.2
Values of $1/K_L$ for Uniform Increments of Percent-open

Percent open	$1/K_L$
100	5.27
90	2.50
80	1.25
70	0.625
60	0.333
50	0.179
40	0.100
30	0.0556
20	0.0313
10	0.0167
0	0.0

percent-open is *OPEN* (72.4% in the example), and the desired value of $1/K_L$ is called *KLI* (0.775 above). Figure 10.13 presents the computer code which is inserted into the program to perform the interpolation.

```
      DO 32 I=1,10
      ITEST=(OPEN-PCT(I))*0.10
      IF(ITEST.EQ.0) GO TO 33
   32 CONTINUE
   33 FACT=(OPEN-PCT(I))*0.10
      KLI=KI(I)+FACT*(KI(I+1)-KI(I))
```

Figure 10.13. Linear interpolation computer code.

Owing to its simplicity, the linear interpolation procedure should be used whenever possible. However, for functions which are sharply curved or for table data which vary substantially, a higher-order interpolation should be considered.

10.4.5. PARABOLIC INTERPOLATION

The purpose of parabolic interpolation is to obtain a more accurate interpolation than is possible with linear interpolation. While this goal is generally achieved for smoothly varying functions, data sets which are highly curved or which possess points of inflection may not be represented well.

In this case we fit a parabola through subsets of three consecutive data points which cover the range over which interpolation is required (see Fig. 10.14). Once the parabolic equation has been found, the interpolated value is calculated by direct substitution. The value of x in Fig. 10.14 is the percent-open *OPEN* of the valve, and the value of $f(x)$ is *KLI*, the value of $1/K_L$ which is sought. A displaced local coordinate system is placed at x_n, and the parabolic equation is first written in this local coordinate system. The general form of the parabolic equation is

$$\eta = A\xi^2 + B\xi + C \tag{10.56}$$

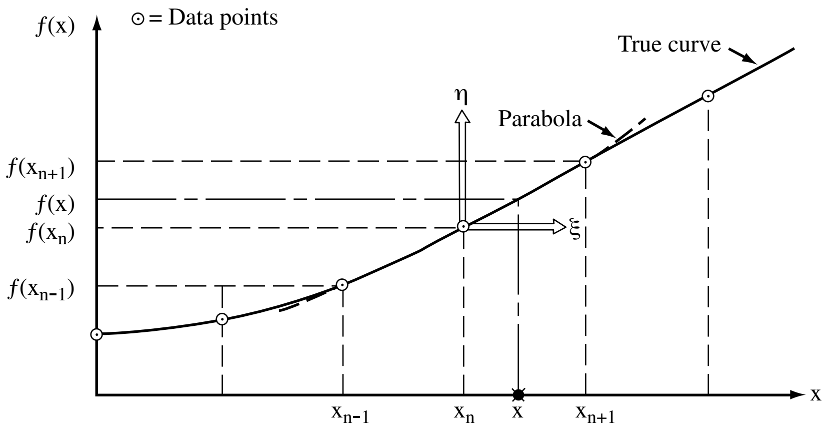


Figure 10.14 Definition sketch for parabolic interpolation.

but here $C = 0$ owing to the choice of the local coordinate system origin. The values of A and B are found by using the known data values at points x_{n-1} and x_{n+1}. Assuming a constant x-spacing Δx for the data points, the equations for A and B become

$$A = \frac{f(x_{n+1}) + f(x_{n-1}) - 2f(x_n)}{2\Delta x^2} \tag{10.57}$$

$$B = \frac{f(x_{n+1}) - f(x_{n-1})}{2\Delta x} \tag{10.58}$$

Recognizing that the two coordinate systems are related by the equations

$$\eta + f(x_n) = f(x) \qquad \xi + x_n = x \tag{10.59}$$

the parabolic equations in the local coordinate system can be transformed back into the original coordinate system, giving

$$f(x) = f(x_n) + \frac{1}{2}\left[f(x_{n+1}) - f(x_{n-1})\right]\frac{x - x_n}{\Delta x} + \frac{1}{2}\left[f(x_{n+1}) + f(x_{n-1}) - 2f(x_n)\right]\left(\frac{x - x_n}{\Delta x}\right)^2 \tag{10.60}$$

While this equation looks bulky, the computer code is straightforward, and the computation is efficient.

If, however, x is in the first segment, then no x_{n-1} exists. In this instance we simply shift the local coordinate system so its origin is at x_{n-1}. For the first interval only we then use the following equation for $f(x)$:

$$f(x) = f(x_1) - \frac{1}{2}\left[f(x_3) + 3f(x_1) - 4f(x_2)\right]\frac{x - x_1}{\Delta x} + \frac{1}{2}\left[f(x_3) + f(x_1) - 2f(x_2)\right]\left(\frac{x - x_1}{\Delta x}\right)^2 \tag{10.61}$$

The computer code segment that carries out this interpolation is shown in Fig. 10.15; it uses the same symbols as the linear interpolation code. While this code could be placed in a subroutine, it is so brief that it seems unnecessary.

```
      IF(OPEN.LT.10.) GO TO 9000
      DO 9001 I=2,10
      ITEST=(OPEN–PCT(I))*0.10
      IF(ITEST.EQ.0) GO TO 9002
 9001 CONTINUE
 9002 FACT=(OPEN–PCT(I))*0.10
      KLI=KI(I)+0.5*FACT*(KI(I+1)-KI(I–1))+
     $0.5*FACT*FACT*(KI(I+1)+KI(I–1)–2.0*KI(I))
      GO TO 9004
 9000 FACT=OPEN*0.10
      KLI=KI(1)-0.5*FACT*(KI(3)+3.0*KI(1)–4.0*KI(2))+
     $0.5*FACT*FACT*(KI(3)+KI(1)–2.0*KI(2))
 9004 CONTINUE
```

Figure 10.15. Parabolic interpolation code.

A word of caution is appropriate when one considers the use of parabolic interpolation. One has no control over the shape of the local parabolic curve and could sometimes experience an odd result. Such is the case if, under certain circumstances, the parabolic code is used with the gate valve head loss data. At the point when the valve is just about closed, it is possible for the locally parabolic curve to dip below the axis and produce a negative value for $1/K_L$. This creates numerous problems in the analysis and is the major reason why programs in this text use linear interpolation for the computation of $1/K_L$.

10.4.6. TRANSIENT VALVE CLOSURE EFFECTS ON PRESSURES

The use of real valves in a transient situation has a more substantial impact on pressures than might be expected from our limited experience with valves that artificially vary the velocity linearly at the valve. This impact is even more pronounced with gate valves; in this case the valve must be nearly closed before it generates enough head loss to decrease the velocity by a significant amount. The result for simple pipe-reservoir systems is that the linear valve closure time must be substantially greater than $2L/a$ to reduce the transient pressure appreciably below that obtained for sudden valve closure. Example Problem 10.2 demonstrates this fact using three different closure schedules.

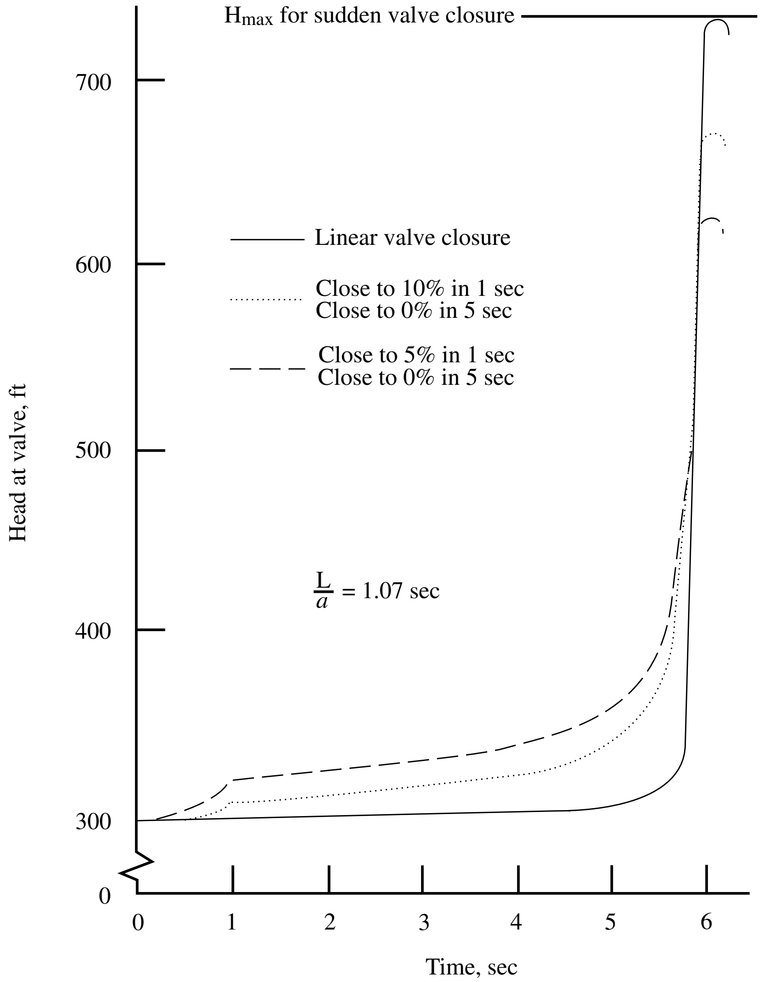
Example Problem 10.2

A pipe-reservoir system has a reservoir at the upstream end and a gate valve and reservoir at the downstream end. The steady-state pressure head at the valve is 300 ft. For sudden valve closure, Eq. 8.8 predicts an increase in pressure head of 431 ft.

Compute the pressure head at the valve for the following three closure schedules, and compare the results with the sudden-closure values. For this system, L/a = 1.07 seconds.

(1) Linear closure in 6 seconds.
(2) Close linearly to 10% open in 1 sec; close the remainder linearly over 5 sec so the valve is completely closed in 6 sec.
(3) Close linearly to 5% open in 1 sec; close the remainder linearly over 5 sec so the valve is completely closed in 6 sec.

The results of the analyses are shown in the following diagram. It is clear that the last 5% or less of the valve closure is critical in this case. Examining the results for case (1), it appears for all practical purposes that the valve in effect does not begin to close until the last 0.2 sec. Even though the valve is closed over approximately three times the critical closure time of $2L/a$ = 2.14 sec, we have achieved almost no reduction in pressure head increase. In fact, the fluid velocity doesn't change much in any of these cases until the valve is over 90% closed. For the gate valve it is the manner in which the last 5-10% of actuator movement is managed that will determine the pressure head increase.



* * *

10.5 PRESSURE-REDUCING VALVES

Pressure reducing valves (PRV's) may routinely be placed in pipe systems, particularly water distribution networks. Because of their common occurrence and the fact that their computer models must be treated somewhat differently numerically than other internal boundary conditions, we will briefly examine their behavior.

10.5.1. QUICK-RESPONSE PRESSURE-REDUCING VALVES

In this case we assume the pressure reducing valve is spring actuated and undamped so that it responds instantaneously to changes in flow conditions. You will recall that the purpose of the PRV is to maintain a specified pressure on the downstream side of the valve within prescribed limits; the pressure there remains essentially unchanged so long as the upstream pressure is greater. However, if the upstream pressure drops so low that the prescribed downstream pressure cannot be maintained, then the PRV causes a major interior head loss with flow in the original direction. If under transient conditions the downstream pressure increases to the point where backflow could occur through the valve, the PRV then acts as a one-way check valve and prevents back flow.

The PRV could act in any one of three different modes. The first set of equations represents the normal mode of operation and assumes the PRV operates as intended. In solving this set of equations for the valve, if we discover that the PRV is not operating in the normal mode, we must then shift to another set of equations which describe one of the other two modes. The equations for each of these three modes follow.

For the normal mode of operation, the equations are similar to the major interior loss equations of Section 10.3. Figure 10.16 defines the variables applicable to this case. The

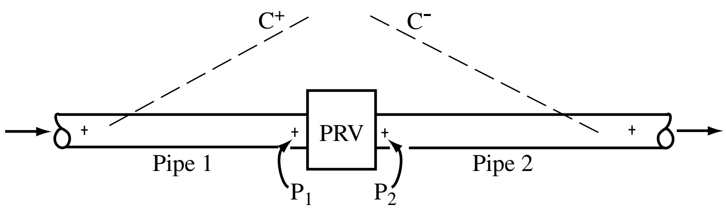


Figure 10.16 Definition sketch for the pressure reducing valve.

resulting equations are

Pipe 1, C^+ $$V_{P_1} = C_1 - C_2 H_{P_1} \tag{10.62}$$

Pipe 2, C^- $$V_{P_2} = C_3 + C_4 H_{P_2} \tag{10.63}$$

Conservation of mass $$V_{P_1} = V_{P_2} \tag{10.64}$$

Work-energy $$H_{P_1} = H_{P_2} + K_{L_{PRV}} \frac{V_{P_2}^2}{2g} \tag{10.65}$$

The last equation must be altered for normal PRV operations when $H_{P_2} = H_{PRV} =$ constant. Consequently for normal operation

$$V_{P_2} = C_3 + C_4 H_{PRV} = V_{P_1} \tag{10.66}$$

and

$$H_{P_1} = \frac{C_1 - V_{P_1}}{C_2} \tag{10.67}$$

Before these calculations can be considered to be correct, we must verify that normal operation is indeed occurring. If, for example, V_{P_2} is negative, normal operation does not occur, and we must set both velocities to zero and solve Eqs. 10.62 and 10.63 for the proper heads. If, however, V_{P_2} is positive, we then must check whether the pressure head drop across the valve is larger than the minimum that is allowable with the valve wide open. This check can be made with Eq. 10.65 by using the wide-open value of $K_{L_{PRV}}$ to compute the drop in head ΔH_{min} for the given velocity and comparing it with the value obtained from Eq. 10.67, $H_{P_1} - H_{PRV} = \Delta H_{act}$. If $\Delta H_{act} \geq \Delta H_{min}$, then the valve is operating normally. If $\Delta H_{act} \leq \Delta H_{\min}$, then the PRV cannot sustain the downstream pressure requirement, even though the flow is still in the original direction. Now the velocities and heads must be computed from Eqs. 10.62 through 10.65 using $K_{L_{PRV}}$ for a fully-open PRV.

10.5.2. SLOWER ACTING PRESSURE-REDUCING OR PRESSURE-SUSTAINING VALVES

Most valves of this type are operated by a pilot system which senses the pressure on the downstream (or upstream) side of the valve and actuates a valve system in the pilot piping which moves a diaphragm to change the valve setting and maintain the required pressure. Because the pilot operation requires fluid to move through the pilot system, there is a discrete or finite response time to sudden and large pressure changes in the pipeline. These valves are designed to respond to much more slowly fluctuating pressures than occur when transients are present. In addition, the time-varying response of these valves is unknown and varies from valve to valve as well as with the magnitude of the transient pressure. In most instances these uncertainties prevent any attempt at a sophisticated analysis; instead the system is simply conservatively assumed to respond instantaneously, as was described in the previous section.

10.6 WAVE TRANSMISSION AND REFLECTION AT PIPE JUNCTIONS*

In many instances it is desirable to be able to estimate what portions of pressure waves are reflected and transmitted at pipe junctions. We already know at reservoirs that none of the pressure wave is transmitted into the reservoir. We will now briefly look into the reflection and transmission properties of series pipe junctions and tee junctions. In all cases we assume that the head lost at the junction is negligible.

10.6.1. SERIES PIPE JUNCTIONS

The equations of mass and linear momentum conservation can be applied to flow at a junction as a pressure head increase ΔH reaches a junction. At that instant ΔH_1 passes through the junction (is transmitted) and ΔH - ΔH_1 is reflected. Figure 10.17 depicts the

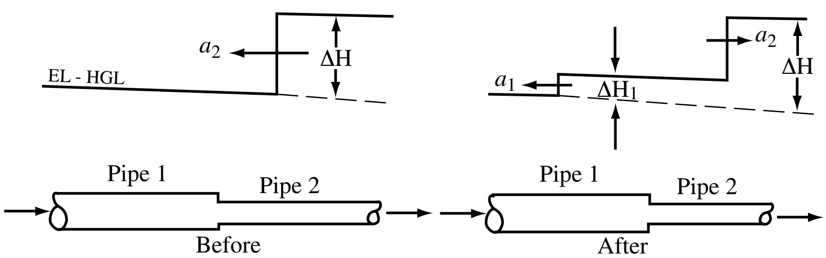


Figure 10.17 Wave transmission and reflection at a series pipe junction.

configuration of the EL-HGL before and after the pressure wave reaches the junction. The analysis produces the following equation for transmission and reflection:

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* This section is adapted from Elementary Fluid Mechanics, by R. L. Street, G. Z. Watters, and J. K. Vennard, Ed. 7, Copyright 1996 by John Wiley & Sons, Inc. Reprinted by permission.

$$\Delta H_1 = \frac{2a_1A_2}{a_2A_1 + a_1A_2}\Delta H \tag{10.68}$$

Here A is the cross-sectional area of the pipes. When the wave speeds a are approximately the same, we obtain

$$\Delta H_1 = \frac{2A_1}{A_1 + A_2}\Delta H \tag{10.69}$$

Example Problem 10.3

A 24-in-diameter pipeline with a wave speed of 3300 ft/s reduces to a 6-in-diameter pipe with a wave speed of 3700 ft/s. The velocity in the 24-in pipe is 1.0 ft/s which corresponds to a velocity in the 6-in pipe of 16 ft/s. The head difference ΔH for sudden flow stoppage in the 6-in pipe is 1838 ft. Find the portion of this wave which is transmitted through the junction.

From Eq. 10.69,

$$\Delta H_1 = \frac{2a_1A_2}{a_2A_1 + a_1A_2}\Delta H = \frac{2(3300)(\pi/4)(6/12)^2(1838)}{3700(\pi/4)(24/12)^2 + 3300(\pi/4)(6/12)^2} = 194 \text{ ft}$$

With only some 10% of the head difference being transmitted upstream, it appears that the upstream pipe acts much like a reservoir.

* * *

10.6.2. TEE JUNCTIONS

A tee junction is shown in Fig. 10.18. Using the same analysis techniques as before

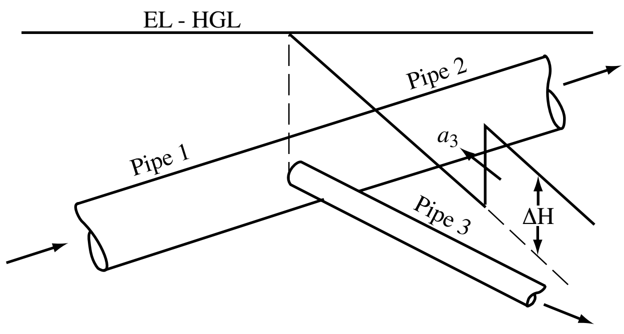


Figure 10.18 Wave transmission and reflection at a tee junction.

leads to the following equations

$$\Delta H_1 = \Delta H_2 = \frac{2a_1a_2A_3}{a_2a_3A_1 + a_1a_3A_2 + a_1a_2A_3}\Delta H \tag{10.70}$$

or, for pipes with similar wave speeds,

$$\Delta H_1 = \Delta H_2 = \frac{2A_3}{A_1 + A_2 + A_3}\Delta H \tag{10.71}$$

Example Problem 10.4

A 24-inch-diameter main line has a 6-in-diameter takeoff (similar to Fig. 10.18) which has a velocity of 10 ft/s. The velocities in the main line are 4.0 ft/s before the takeoff and 3.38 ft/s after the takeoff. A sudden flow stoppage in the 6-in-diameter takeoff causes a head difference $\Delta H = 1150$ ft to occur.

Assuming that the wave speeds in all pipes are similar, compute the portion of ΔH that passes into the 24-in-diameter pipe.

We calculate the transmitted portion of the wave from Eq. 10.71:

$$\Delta H_1 = \Delta H_2 = \frac{2A_3}{A_1 + A_2 + A_3}\Delta H = \frac{2(\pi/4)(6/12)^2(1150)}{(\pi/4)(24/12)^2+(\pi/4)(24/12)^2+(\pi/4)(6/12)^2} = 70 \text{ ft}$$

With a tee connection only 6% of the pressure wave passes into the 24-in-diameter pipe. It is easy to see why transients in pipe networks are absorbed so rapidly.

* * *

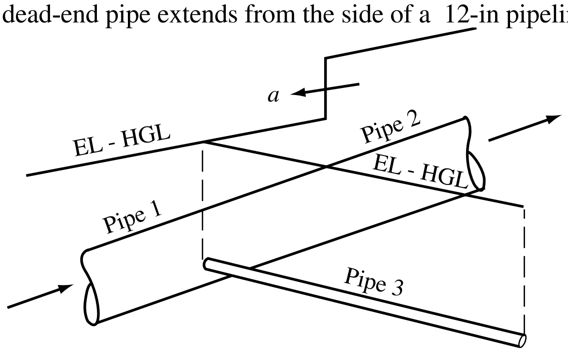
10.6.3. DEAD-END PIPES

If a pipe system contains a member which carries no discharge and terminates in a dead end, e.g., a closed valve, then a unique situation exists which could cause unexpectedly high pressures. This is actually a special case of the tee junction in the previous section. As a high-pressure wave passes the junction from which the dead-end pipe extends, a portion of the wave is transmitted into the dead-end pipe, increasing the pressure there by an increment ΔH_1 and inducing a flow velocity ΔV_1 toward the closed end. When the pressure wave reaches the dead end, the induced velocity is abruptly stopped, thereby increasing the pressure head at the dead end by $2\Delta H_1$.

While pipe system geometry, pipe size, and friction losses all affect the overall pressure increase in varying amounts, the maximum effect at a dead end occurs when the dead-end pipe is very small in comparison with the main pipe. For this condition with small frictional effects, the pressure head increase is at most twice the value of the ΔH that initially passes the junction. The following example demonstrates the dead-end pipe effect for two extreme cases.

Example Problem 10.5

A 3000-ft-long dead-end pipe extends from the side of a 12-in pipeline.



(a) If the dead-end pipe has a diameter of 1.0 in, find the maximum pressure head increase in this pipe if the mainline velocity of 5 ft/s in pipe 2 is suddenly halted. Assume a wave speed of 3000 ft/s for all pipes and neglect friction.

(b) If part (a) were solved with a friction factor of 0.020 in the 1-in pipe, what would be the result?

(c) What is the result if all three pipes are 12 in in diameter and friction is neglected?

(a) From Eq. 8.4 the incremental head increase in the main line is

$$\Delta H = -\frac{a}{g}\Delta V = -\frac{3000}{32.2}(-5) = 466 \text{ ft}$$

From Eq. 10.71 the head increment in the dead-end pipe is

$$\Delta H_3 = \frac{2A_2}{A_1 + A_2 + A_3}\Delta H = \frac{2 \times 12^2}{12^2 + 12^2 + 1^2}(466) = 464 \text{ ft}$$

when the common constants in both numerator and denominator are canceled. The maximum possible head increase would be 2(464) = 928 ft, according to our earlier reasoning. A computer analysis gives an identical 928 ft, thus verifying our earlier conclusion.

(b) The pressure head increment moving up the 12-in pipe would again be 466 ft, and the head increment entering the 1-in pipe would be 464 ft. Although the maximum possible pressure head increment would remain 928 ft, a computer analysis shows that friction effects have reduced the maximum head increment to 770 ft.

(c) The head increment moving into pipe 3 is again computed from Eq. 10.71 as

$$\Delta H_3 = \frac{2A_2}{A_1 + A_2 + A_3}\Delta H = \frac{2 \times 12^2}{12^2 + 12^2 + 12^2}(466) = 311 \text{ ft}$$

The maximum possible head increase is 2(311) = 622 ft. A computer analysis also gives 622 ft as the actual head increase.

* * *

We conclude in the absence of friction that the rule that the head increment doubles is valid. The presence of friction reduces the head increase by an undetermined amount. We note that the neglect of friction when estimating a dead-end pressure increment gives conservative results.

10.7 COLUMN SEPARATION AND RELEASED AIR

It is common knowledge that excessively-high pressures resulting from transients in pipes can cause damage. It is also generally recognized that low pressures could cause the collapse of pipes with thin walls or high external loads. What is not so commonly known or understood is the phenomenon of *column separation* and the consequences of its occurrence.

10.7.1. COLUMN SEPARATION AND RELEASED AIR

When transients in a pipe system cause the pressure to approach the vapor pressure of the liquid, gases in solution begin to come out of solution and dramatically affect the flow behavior. If the drop in pressure is severe enough to cause the local pressure to reach the vapor pressure of the liquid, then the liquid boils (cavitates, vaporizes), forming large pockets of undissolved gases and vapor. This phenomenon is called *column separation.*

One consequence of this occurrence is a substantial change in the wave speed caused by the presence of entrained gases and vapor bubbles which affect the compressibility of the liquid (see Section 8.4). A second consequence is the fact that the liquid "column" is no longer homogeneous and in fact may have large cavities. This means that the analyses we have developed no longer apply directly.

Whenever the pressure at any point in the pipeline drops below the pressure at the pipeline source, the saturation pressure of the dissolved gases may be reached, and these gases will begin to come out of solution. This is one of the reasons for placing air release valves at pipeline summits. The amount of gas that comes out of solution depends on the degree of initial saturation and the severity and extent of the low pressure. If the pressure drops to the fluid vapor pressure for an extended time period, large cavities of vapor and gases may form.

If we look carefully at the consequences of closing a valve, we find a simple example of how column separation can form. Upon sudden valve closure, the pressure head just downstream of the valve attempts to drop by an amount (given by Eq. 8.8) which should be just enough to bring the liquid column to rest. However, if this pressure drop is greater than that required to reach the fluid vapor pressure, a vapor cavity will form because a liquid cannot remain a liquid at a pressure which is lower than its vapor pressure (see Fig. 10.19). Because the pressure drop is limited, there is not a sufficient pressure gradient to stop the flow, so the flow separates at the valve and forms a vapor cavity. Analysis of the ensuing transient can become exceedingly complex, requiring at the least a means of representing the cavity formation, growth and decay over time. Owing to the large difference in density

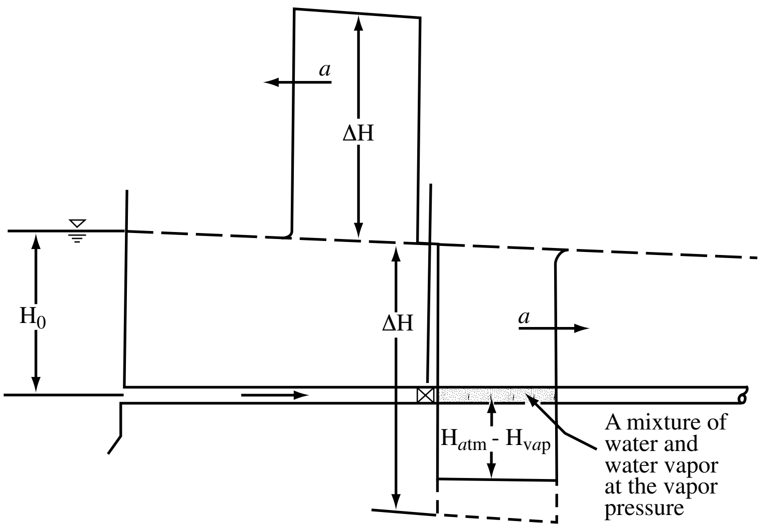


Figure 10.19 Column separation caused by sudden valve closure.

between the liquid and the gases, buoyancy effects encourage a gaseous cavity to lie over the liquid rather than fill the pipe cross section, which calls into question the assumption of the existence of a one-dimensional flow.

10.7.2. ANALYSIS WITH COLUMN SEPARATION AND RELEASED AIR

Tullis et al. (1976) thoroughly discuss the effects of air release at low pressures as well as column separation at vapor pressure. They suggest that volumes of released air may be either uniformly distributed throughout the flow or concentrated in pockets. In the first case Eq. 8.40 can be used to find the reduced wave speed. Because the change in wave speed will cause large interpolations if a rectangular grid is used, they suggest the use of a method whereby the characteristic lines are followed as closely as possible to minimize interpolation errors and maintain numerical stability.

If it is undesirable to use this approach, then the regular wave speed is used, and the air or vapor is assumed to be concentrated in discrete sections along the pipeline with internal boundary conditions imposed at the ends of each cavity. The growth and decline of the cavities is monitored; if they disappear, the regular analysis technique can then resume.

Other investigators who cite experience with the modeling of column separation are Martin et al. (1976), Ewing (1980), and Marsden and Fox (1976). Most studies address the modeling of the vapor cavity, the mechanism of release and re-absorption of air and water vapor, and numerical techniques. Wylie and Streeter (1993) offer a compact summary of the state of the art.

To address the problem of column separation, we must first create a model of the phenomenon. The simplest model of column separation ignores the existence of dissolved gases that might come out of solution at low pressures. Instead it is assumed that the liquid remains intact until the vapor pressure is reached. When that point is reached, it is postulated that the vapor cavity will grow at a constant cavity pressure equal to the vapor pressure. Eventually, when the cavity closes, it is presumed that the vapor re-absorbs so that it disappears at the instant of cavity closure. In effect, the vapor cavity is treated much like a vacuum.

This simple model also requires some assumptions regarding the form of the cavity. In reality, this form is quite complicated and nearly impossible to simulate accurately. Therefore we might as well use the simplest possible model. We assume the walls of the cavity remain normal to the pipe cross section, and the growth or decay of the cavity depends entirely on the relative velocity of the cavity endwalls. This in turn requires an internal boundary condition to be imposed at each node within the cavity where the pressure is fixed at the vapor pressure. Thus, at each node where column separation occurs, there are two velocities, one associated with the upstream face of the cavity and one associated with the downstream face. The relative magnitudes and directions of these velocities determine the growth or decay of the cavity. All of the cavity behavior is concentrated at the computational nodes in the pipeline with the liquid between the nodes intact and retaining the original wave speed. This model is illustrated in Fig. 10.20.

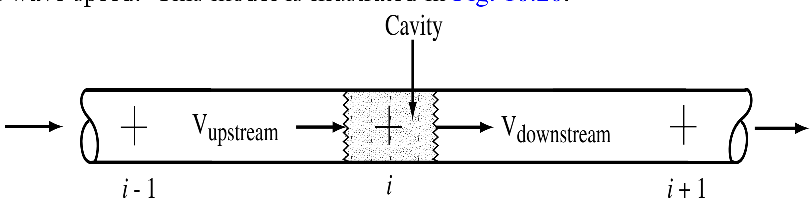


Figure 10.20 Basic model for column separation analysis.

The determination of the consequences of cavity closure is an important prediction of the model. To analyze this feature we apply conservation of momentum to the collision of the two collapsing cavity walls that are moving at different velocities. The result is an equation for the head increase which results from the collision:

$$\Delta H = \frac{a}{2g}\left(V_{upstream} - V_{downstream}\right) \tag{10.72}$$

This head increase ΔH is added to the vapor pressure head at the node to determine the new pressure immediately after cavity closure. PROG8 employs this model of column separation occurring in pumped pipelines.

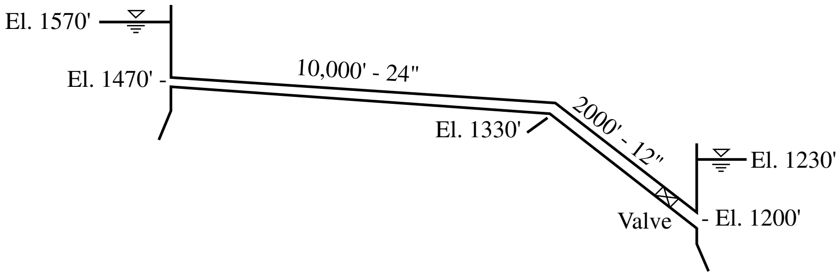
While this model of column separation seems very primitive, it is widely used in practice. Although much research over recent years sought to improve on this basic model, no one has developed a model and analysis which is sufficiently more general and accurate to attract the user community. Commenting on this state of affairs, Wang and Locher (1991) note that this vapor cavity model is a "very simplified formulation of what is really a highly complex problem ... that works surprisingly well in spite of well-grounded theoretical objections to the approach." And in applying this method they caution that it is "essential to understand the formulation of the method, its limitations, and to interpret the results in the light of this knowledge and past experience". Clearly, experience and an understanding of the physical phenomena are crucial ingredients in successfully applying this model.

10.8 PROBLEMS

Note: Use PROG2 for computer analyses in this chapter unless instructed otherwise.

10.1 The gate valve in the pipeline below closes linearly from wide open to completely closed in 30 sec. The diameters shown are inside diameters. The pipe is welded steel with a wall thickness of 0.135 in and Case (b) restraint. Assume the gate valve has the same loss coefficients as shown in Table 10.2.

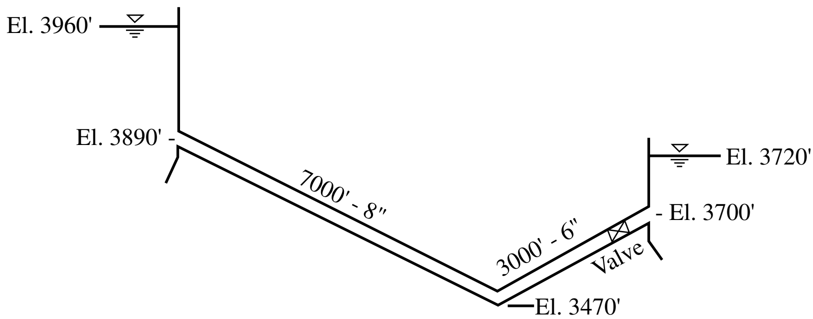
Find the maximum and minimum pressures in the system and where and when they occur. If column separation occurs, identify when and where it first appears.



10.2 The engineer in charge of project design wants you to answer the following questions regarding the proposed pipeline shown below.

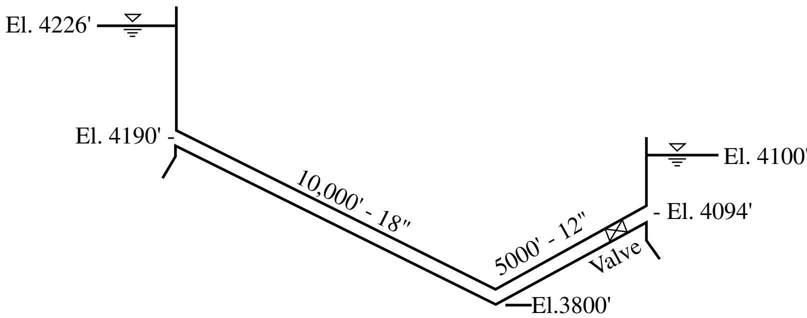
(a) What will be the maximum pressure in the pipeline?
(b) Where and when will it occur?
(c) Will column separation occur?
(d) If so, where and when will it occur?

The pipe diameters shown are inside diameters. The pipe is 14-ga. (0.0747-in) welded steel with Case (b) restraint. Use a Hazen-Williams coefficient of 140 in your calculations. The Pratt butterfly valve, which has the loss characteristics given in Appendix C, closes at a uniform angular rate in 20 sec.



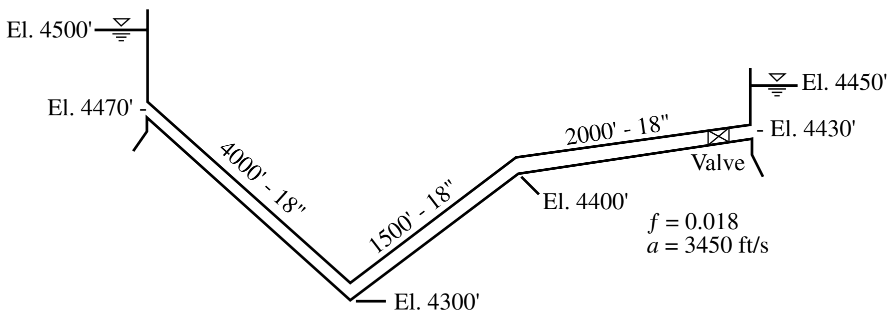
10.3 The reservoir of surface elevation 4226 ft supplies a city water supply through a 15,000-ft pipeline. The pipe is 7-ga (0.1793-in) welded steel with Case (b) restraint; the diameters are inside diameters. The working pressure in the 18-in pipe is 270 lb/in^2 and in the 12-in pipe 420 lb/in^2. The design engineer wants to close the Cla-Val globe valve linearly without exceeding the working pressure in either pipe. Assume the valve loss characteristics in Appendix C for Cla-Val valves follow the GA Industries curve for variation with percent open.

As a consultant, your task is to analyze the transient behavior of this system for valve closure times of 20, 40, and 60 sec and determine whether the working pressure in the pipeline is exceeded in any of these cases. As part of your report, you should find the maximum and minimum pressures in the pipeline and where and when they occur. Also note whether column separation occurs.



10.4 Water flows from the upper reservoir by gravity through a 7500-ft-long steel pipeline. Discharge is controlled by an angle valve at the downstream reservoir. The system is to be shut down as quickly as possible without exceeding the allowable working pressure of 200 lb/in^2 or causing column separation.

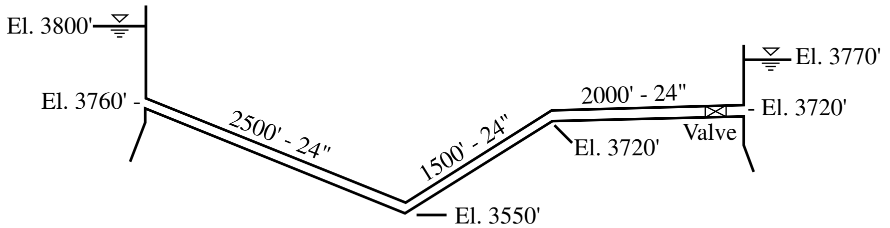
The GA Industries angle valve, with loss characteristics given in Appendix C, is programmed to close at two different rates. Recalling from Example Problem 10.2 how gate valves behave, use this knowledge to adjust the closure stages of the angle valve to minimize the closure time.



10.5 The 24-in (23.65 in inside diameter) pipeline is 7-ga (0.1793-in) welded steel pipe with a working pressure of 200 lb/in^2. Water flows between the two reservoirs shown atop the following page, controlled by a valve at the downstream reservoir. The valve has the following loss coefficients K_L for the different openings:

% Open	100	75	50	37.5	25	12.5
K_L	0.07	0.42	2.20	5.10	12	56

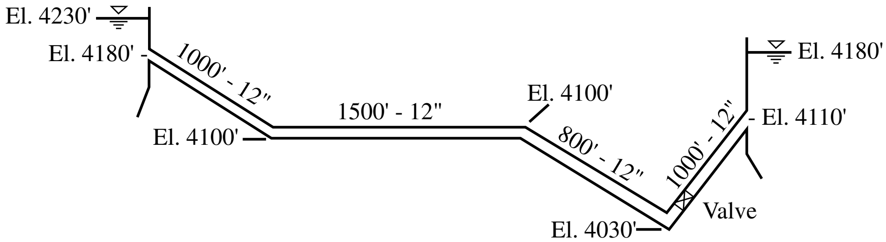
As a consultant to the project engineer, your task is to determine a closure schedule which will close the valve as quickly as possible without exceeding the working pressure or causing column separation. The valve is designed to close at two different rates.



10.6 The pipeline connecting the two reservoirs is 12-inch CL 200 Transite pipe with an inside diameter of 11.56 in and a wall thickness of 1.26 in. The pipe sections are joined with couplings and ring gaskets. Assume a friction factor of 0.014 in your calculations.

The discharge rate is controlled by a GA Industries globe valve at elevation 4030 ft. To determine the valve loss coefficients, use the plot of C_v vs. %-open for the GA Industries valves found in Appendix C.

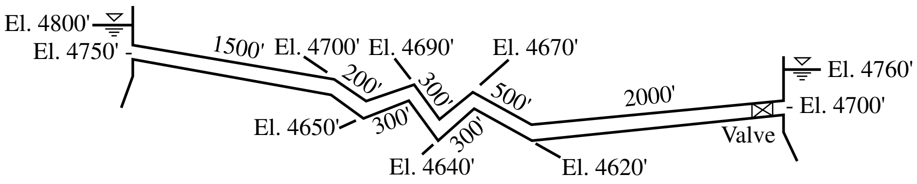
As a consultant, your task is to recommend a fast closure schedule which will not cause pressures in excess of the pipe class rating (200 lb/in^2) or column separation. Use PROG2A for this analysis.



10.7 The pipeline supplying water from the upper reservoir is 12-in Class 200 PVC pipe (wall thickness = 0.61 in, outside diameter = 12.750 in). The pipe is considered hydraulically smooth so use a Hazen-Williams coefficient of 150. The pipe is joined by bell-and-spigot connections, and anchor blocks are installed at all bends and fittings.

Two valves are being considered for use by the design engineer. Valve A is an expensive servo-controlled valve which can cause the velocity at the valve to vary linearly. Valve B is a lower-cost globe valve with a wide-open C_v of 1750. The variation of C_v with %-open is given in Appendix C for GA Industries valves. The globe valve can close at only one rate.

Each valve closes in 30 sec. You are to conduct analyses of the two alternatives and determine the maximum and minimum pressures to be expected in each case. Make a recommendation as to which valve to use.

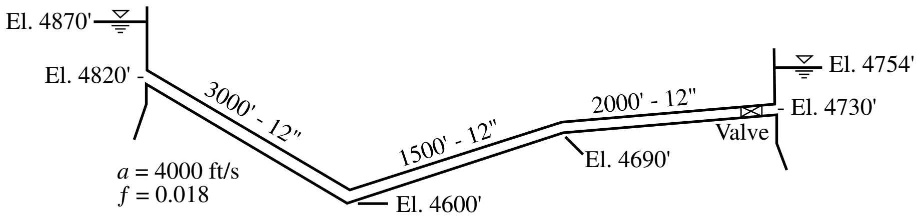


10.8 Water flows from the reservoir at surface elevation 4870 ft shown below at a rate of 6.28 ft^3/s. The system is to be designed to shut down in the least possible time without developing excessively high pressures or column separation. Current plans are to use a gate valve that can be programmed to close at two different rates. The loss characteristics of the valve are given in Table 10.2.

The project design engineer has decided to try the following schedules:

	Stage 1	Stage 2
(a)	90% closed at 5 sec	100% closed at 15 sec
(b)	95% closed at 5 sec	100% closed at 15 sec
(c)	90% closed at 10 sec	100% closed at 20 sec
(d)	95% closed at 5 sec	100% closed at 20 sec
(e)	95% closed at 10 sec	100% closed at 20 sec

Your task is to identify the high and low pressures, their location, and the time they occur.

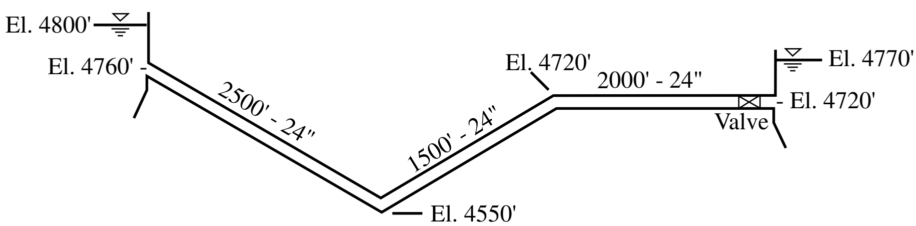


10.9 The pipeline below is constructed of 7-ga. (0.1793-in) welded steel with a working stress of 13,500 lb/in^2, which corresponds to a working pressure of 200 lb/in^2. Water flows from the reservoir at 4800 ft through a 6000-ft pipeline 24 in in diameter (23.65 in inside diameter). The discharge is controlled by a Pratt ball valve at the lower reservoir whose loss characteristics are given in Appendix C.

The system is to be designed to shut down as quickly as possible without developing pressures greater than the working pressure or causing column separation. It is possible to purchase a valve which can be programmed to close at two different uniform angular rates. The project design engineer has decided to try the following five schedules:

	Stage 1	Stage 2
(a)	90% closed at 4 sec	100% closed at 15 sec
(b)	95% closed at 4 sec	100% closed at 15 sec
(c)	98% closed at 4 sec	100% closed at 15 sec
(d)	90% closed at 2 sec	100% closed at 20 sec
(e)	98% closed at 2 sec	100% closed at 20 sec

Your task is to identify the high and low pressures, their location, and the time they occur.



CHAPTER 11

PUMPS IN PIPE SYSTEMS

The designers of liquid conveyance systems are frequently faced with a pump selection problem. While vendors of pumps and pumping appurtenances are generally quite helpful in the selection process, it is better to be well informed on pumps and their operating characteristics, particularly under transient conditions. For this reason Chapter 2 presented some fundamental elements of pump theory and operation; this chapter will address the issue of transients, building on the knowledge of similarity relationships from Chapter 2. We will restrict our coverage to centrifugal, turbine, and axial-flow pumps. Positive displacement pumps are not considered.

11.1 PUMP POWER FAILURE RUNDOWN

The sudden loss of energy to a pump can be caused by an unexpected power failure or simply because an individual has switched off the power. In either case the rotating pump impeller begins to decelerate with the pressure dropping on the discharge side of the pump and rising on the suction side (if it is an inline booster pump configuration). The resultant transient may quickly lead to column separation with ensuing hard-to-predict consequences, including cavity collapse or exceedingly high pressures, perhaps caused by the closure of a check valve. Whatever the cause, it can be very important to be able to simulate this rather common occurrence to determine whether dangerous pressures develop.

As the pump slows down after power failure, its head vs. discharge and torque vs. discharge characteristics change. It is customary to assume as the pump speed changes that the pump characteristics at any speed can be found by using the similarity relations that are presented in Chapter 2 for homologous pumps. While the changes in pump torque are important in the rundown process, we will first concentrate on how the pump head itself varies with discharge.

The pump characteristics at various speeds can be displayed as shown in Fig. 11.1. The solid line labeled N_0 is the pump characteristic curve during steady-state conditions, while the similarly-shaped dashed lines represent that characteristic curve at successively lower speeds. The position of each dashed characteristic line can be calculated from the steady-state line by using the similarity rules from Chapter 2. Noting that the diameter of the slowing pump is a constant, we can incorporate it into the similarity constant to yield a form of the similarity equations which applies directly to pump power failure rundown:

$$\frac{Q}{N} = \text{constant} \tag{11.1}$$

$$\frac{h_p}{N^2} = \text{constant} \tag{11.2}$$

$$\frac{T}{N^2} = \text{constant} \tag{11.3}$$

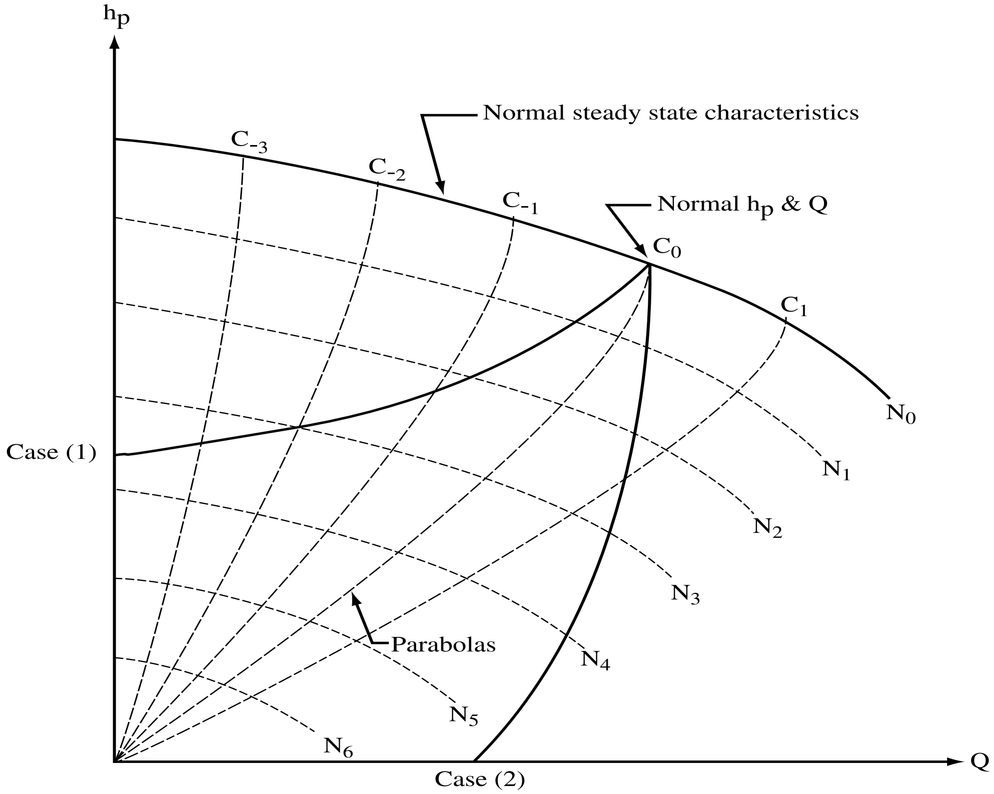


Figure 11.1 Multi-characteristics for a given pump at various speeds.

The curves at successive speeds N_i can be created by selecting a range of values for Q_0 and h_{p_0} at a set of points C_{-3}, C_{-2}, etc. along the original characteristic curve and calculating the corresponding values for each other speed with the following equations:

$$Q_i = Q_0 \frac{N_i}{N_0} \tag{11.4}$$

$$h_{p_i} = h_{p_0} \left(\frac{N_i}{N_0}\right)^2 \tag{11.5}$$

In fact, by using the same Q_0 and h_{p_0}, say at point C_1, and varying N, we can generate a large set of corresponding points, each on a different characteristic curve. These points all lie on a parabola passing through C_1 and the origin. If the same procedure is followed for points C_{-3}, C_{-2}, etc. on the original curve, then a set of pump characteristic lines can be drawn for a set of speeds, and in principle can be drawn for all speeds. Figure 11.1 shows one set of parabolas drawn through the origin.

As the speed of the pump decreases, a path develops on the characteristic diagram of Fig. 11.1 which traces the changes of pump head and discharge as the rundown progresses. However, this trace does not follow a particular parabolic curve; instead the path is determined by the pump and motor rotational inertia and the back pressure exerted by the water in the pipe on the pump impeller. Two typical examples are shown in Fig. 11.1:

Case (1) occurs when the static lift of the pipeline is high, and the line is relatively short. In this case the inertia of the water in the pipeline is relatively small, and gravity helps to decelerate the flow. As a consequence, the discharge through the pump drops to zero rather quickly while a positive head across the pump still exists. Then flow backward through the pump occurs unless a check valve has been installed in the line.

Case (2) occurs when the pipeline is relatively long, and a large portion of the head that has been generated by the pump is needed to overcome the friction loss in the line. When power fails in this case, the large inertia of the moving fluid prevents the flow from decelerating rapidly. The rotation rate of the pump also decreases more slowly, and the head across the pump drops to zero before the discharge does. At this point the flow will either continue to flow forward through the pump, doing work on the pump and causing it to "windmill," or else the flow goes around the pump in a bypass line.

There are actually four possible flow configurations through the pump:

(1) Flow is forward through the pump, and the pump rotates forward.
(2) Flow is in the reverse direction while the pump is still rotating forward (generally of short duration).
(3) Flow is in the reverse direction while the pump also rotates backwards.
(4) Flow is in the forward direction while the pump rotates backwards (also of short duration).

The actual occurrence of any of these situations depends on the inertia of the pump and motor and on the existence of check valves, bypasses and other appurtenances. Unfortunately, the data needed to simulate these conditions are usually only available from manufacturers for the first situation. Even in this situation, the data are not available for Case (2) in Fig. 11.1 when there is a head *loss* through the pump. If the pump is expected to operate in any of the other three modes, then additional information must be sought either through model tests of that pump or by a study of data from tests of similar pumps.

We will analyze a common pump power failure situation, assuming there is a check valve in the pump discharge line. If the pump is a booster pump, we will assume there is a low-loss bypass line around the pump station. This will permit us to complete an analysis while using only information that is commonly available from pump manufacturers.

11.1.1. SETTING UP THE EQUATIONS FOR BOOSTER PUMPS

Along each of the parabolic curves passing through C_{-3}, C_{-2}, etc. in Fig.11.1, the values of Q/N and h_p/N^2 are constant. Hence we can represent all of the pump characteristic behavior in Fig. 11.1 by a single plot of Q/N vs. h_p/N^2. The same reasoning applies to the torque description where a single plot of Q/N vs. T/N^2 will suffice. A typical example of each curve is shown in Fig. 11.2. The curves can be constructed by selecting h_p and Q pairs from the manufacturer's curves, dividing by N and N^2, respectively, and plotting the results. To see how this works, let's look at an example.

Consider a booster pump station in the interior of a line (see Fig. 11.3). There is a check valve on each pump discharge line, and there is a bypass line around the pump station which also has a check valve in it. All pumps are assumed to experience power failure simultaneously. The appropriate equations are the following:

Suction side C^+
$$V_{P_s} = C_1 - C_2 H_{P_s} \tag{11.6}$$

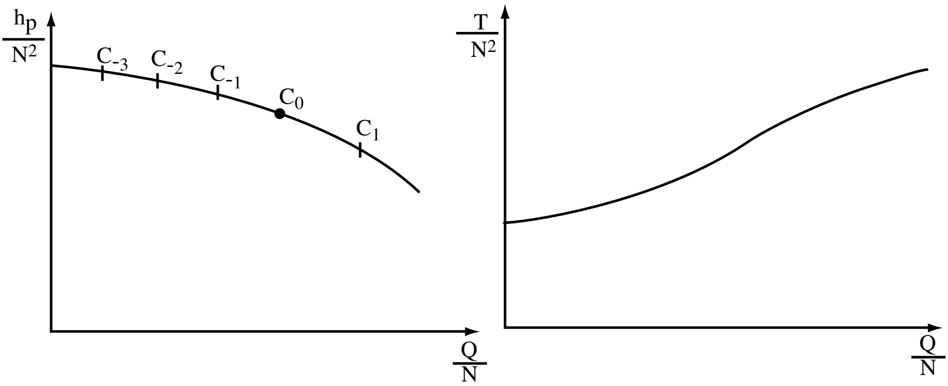


Figure 11.2 Typical h_p/N^2 and T/N^2 curves for a pump.

Discharge side C^- $$V_{P_d} = C_3 + C_4 H_{P_d} \tag{11.7}$$

Conservation of mass $$V_{P_s} A_s = V_{P_d} A_d \tag{11.8}$$

Work-energy $$H_{P_s} + h_p = H_{P_d} \tag{11.9}$$

Pump characteristic $$h_p = f(Q) \text{ or } f_1\left(V_{P_d}\right) \text{ or } f_2\left(V_{P_s}\right) \tag{11.10}$$

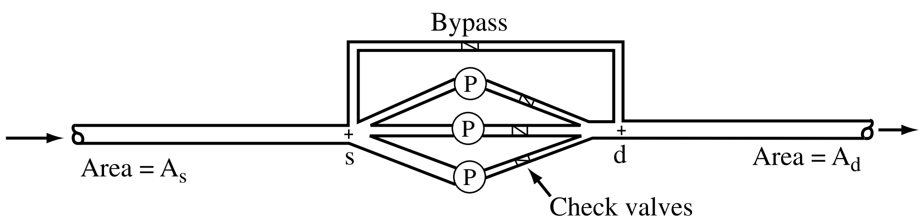


Figure 11.3 A typical parallel pump booster configuration.

This set of equations contains the unknown value of N plus five additional unknowns. If we presume that N can be found before we begin the solution for the other five unknowns, then we can proceed to seek a solution.

In the work-energy equation, Eq. 11.9, we find only the head increase across the pump, as is given by the pump characteristic diagram. If there are significant losses in the pump discharge column, discharge head, check valve or isolation valve, then adjustments to h_p must be made. That is, h_p must be reduced by the amount of the hydraulic losses for a given discharge. Consequently, we must redraw the pump characteristic diagram, corrected for the head losses occurring for each discharge. Because these losses are typically of the form $K_L V^2/2g$, the loss coefficients for the individual local losses can be summed appropriately for the full range of pump discharge and then applied to reduce the pump head. Since h_p is the sum of the head increases across each stage of a multistage pump, the local losses for a given discharge must be divided by the number of stages before computing the head per stage and the pump characteristic diagram is redrawn. This step is necessary because the pump characteristic curve for only one stage is entered into the computer program and is then internally multiplied by the number of stages. One might ask why a head loss term was not included in Eq. 11.9 so one could then proceed in a straightforward manner. The answer can be seen in Section 11.1.3. In that section we linearize the pump characteristic curve to avoid parabolic or higher-order interpolation techniques. To reintroduce a quadratic equation now would defeat this strategy. In short, we adjust the pump characteristic diagram for local losses in order to retain its subsequent linear representation.

It is apparent that we need a representation for Eq. 11.10 that can be combined with the other four equations. In Chapter 9 we modeled the pump curve with a parabolic equation. This approach was restrictive in that it worked well only for characteristic curves which were already nearly parabolic in shape. We will now follow a much more general approach and represent the Q/N vs. h_p/N^2 curve by a series of straight-line segments. At any point the curve is then a straight line valid over a limited range of Q/N. The details of this process will follow after we find the current speed N.

11.1.2. FINDING THE CHANGE IN SPEED

To this point we have assumed that the new speed is known at the time when the new head and velocity values are to be computed. The change in speed is found by calculating the decelerating torque and using rotational dynamics to find ΔN. The rotating portions of the pump — shaft, motor, and impeller — have a rotational moment of inertia I. Normally the motor is by far the largest contributor. However, the rotation of the water-filled impeller and the pump shaft must be included. Values for these elements must be obtained from the manufacturer or estimated by comparison with values for similar pumps whose moments of inertia I are known. In the United States pump manufacturers usually give the value of I as Wr^2 in units of lb-ft^2. This is the value the computer program expects. Should the value of I for a particular motor not be immediately available, then the following formula (Thorley, 1991), adjusted to U.S. units, may be used as an estimate:

$$I = 1818\left(\frac{HP}{N}\right)^{1.48} \tag{11.11}$$

Here I is in lb-ft^2, HP is in horsepower, and N is in rev/min or rpm.

Under steady-state conditions the driving torque of the motor is balanced by the resisting torque exerted by the water on the impeller vanes. When power fails, the driving torque disappears and the resisting torque decelerates the pump. This deceleration is described by

$$T = I\alpha = I\frac{d\omega}{dt} = \frac{2\pi}{60}I\frac{dN}{dt} \tag{11.12}$$

where N is again the speed in rev/min, and I is the total rotational moment of inertia of the rotating parts.

To find the change in speed which occurs over a time increment Δt, we now integrate Eq. 11.12:

$$\int dN = \frac{60}{2\pi I}\int T dt \tag{11.13}$$

Since the functional relation for T is not known, we choose to keep Δt small and let T be constant over Δt at its known value at the previous instant in time. This approximation is a good one because torque normally does not change much over even the full discharge range. The new rotational speed can then be calculated as

$$N(t + \Delta t) = N(t) - \frac{60}{2\pi I}T(t)\Delta t \tag{11.14}$$

The first change in speed will be calculated from the steady-state values of N_0 and T_0. Subsequent torque values are interpolated from the table of Q/N vs. T/N^2 using the just-computed values of Q. We can now proceed with the solution of Eqs. 11.6 through 11.10 for the new head and velocity.

11.1.3. SOLVING THE EQUATIONS

The first step in solving the equations is to represent Eq. 11.10 as a linear function over a finite range of Q/N. Figure 11.4 shows h_p/N^2 vs. Q/N as a sequence of linear segments. The linear equation over the segment is

$$\frac{h_p}{N^2} = N_{st}\left[\left(\frac{\left(h_p/N^2\right)_A - \left(h_p/N^2\right)_B}{(Q/N)_A - (Q/N)_B}\right)\frac{Q}{N} - \left(\frac{\left(h_p/N^2\right)_A - \left(h_p/N^2\right)_B}{(Q/N)_A - (Q/N)_B}\right)\left(\frac{Q}{N}\right)_B + \left(\frac{h_p}{N^2}\right)_B\right] \tag{11.15}$$

in which the values of h_p/N^2 and Q/N are for one stage of the pump, and N_{st} is the number of pump stages. To simplify the algebra, we rewrite Eq. 11.15 as

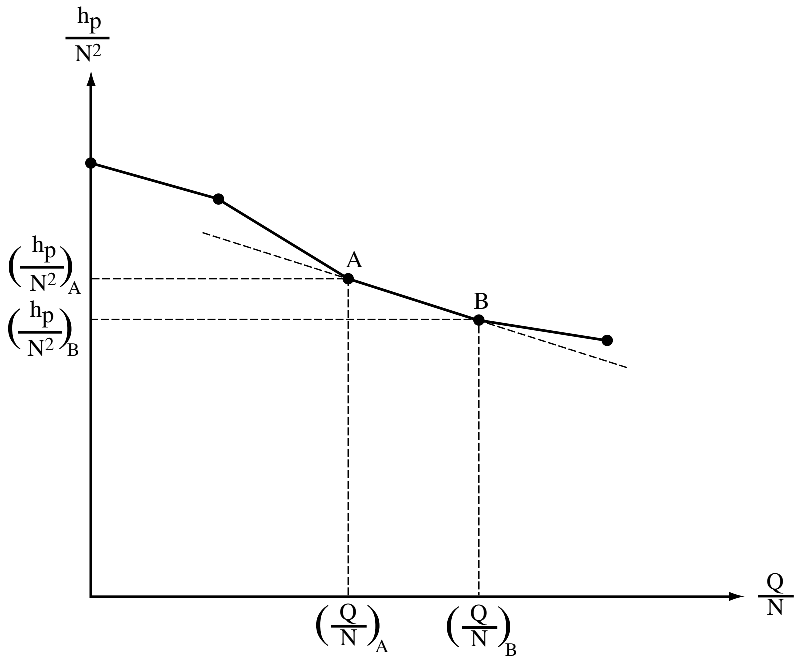


Figure 11.4 Piecewise linear representation of h_p/N^2 vs. Q/N.

$$\frac{h_p}{N^2} = N_{st}\left[C_7\frac{Q}{N} + C_8\right] \tag{11.16}$$

Now the simultaneous solution of the five equations for the pipeline velocity on the discharge side of the pump station produces

$$V_{P_d} = \frac{\frac{C_1}{C_2} + N_{st}N^2C_8 + \frac{C_3}{C_4}}{\frac{1}{C_4} + \frac{A_d}{C_2A_s} - \frac{N_{st}NA_dC_7}{N_{pu}}} \tag{11.17}$$

in which N_{pu} is the number of pumps in parallel. If $V_{P_d} > 0$, Eqs. 11.6 through 11.10 can be used to find the remaining unknowns. However, if $h_p < 0$, then we must open the bypass line by setting $h_p = 0$ and $H_{P_s} = H_{P_d}$; then we must recompute the velocity from the following two equations:

$$V_{P_d} = \frac{C_1C_4 + C_2C_3}{C_2 + C_4\frac{A_d}{A_s}} \tag{11.18}$$

$$V_{P_s} = \frac{A_d}{A_s}V_{P_d} \tag{11.19}$$

Now Eqs. 11.6 and 11.7 can be used to determine H_{P_s} and H_{P_d}.

If the solution of Eq. 11.17 yields a negative velocity, we must set both V_{P_s} and V_{P_d} to zero and use Eqs. 11.6 and 11.7 to compute H_{P_s} and H_{P_d}. Finally, we must compute Q/N and verify that we are indeed within the interval between A and B in Fig. 11.4. If not, we must recompute C_7 and C_8 and repeat the solution process.

Example Problem 11.1

To examine the effect of booster pump power failure, we will place a four-pump station in the interior of a 45,000-ft pipeline. The 30-in diameter pipeline is constructed of welded steel with a friction factor of 0.013 and a wave speed of 3590 ft/s. The line extends between two reservoirs, and the booster station is 15,000 ft downstream from the first reservoir.

The pumps are three-stage Ingersoll-Dresser 15H277 turbine pumps with 11.83-inch impellers having pump characteristics shown in Appendix B. For each pump and motor unit Wr^2 is approximately 475 lb-ft^2. To set up data tables for pump performance, we select six data points along the Q-axis, $Q = 0$, 1000, 2000, 3000, 4000, and 4500 gal/min. We then read the corresponding h_p and bhp values for each Q and enter them into the input data file.

The program determines the steady discharge, so no preliminary hydraulic computations are needed. One need only select an accuracy standard for the iterative process. In this case an accuracy of 0.50 gal/min was chosen.

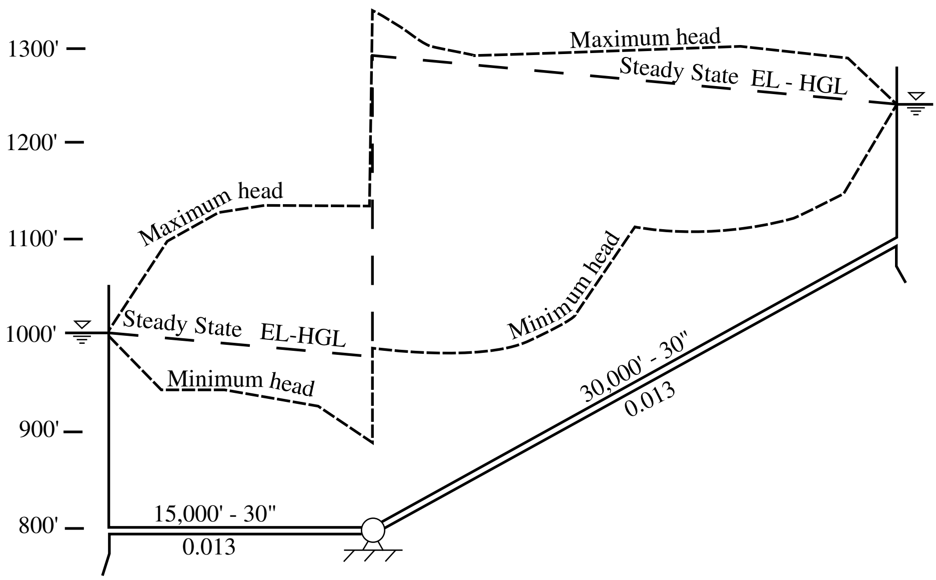
We have also elected to obtain additional output detail at two points. The first is at the suction side of the pump; the second is at the discharge side. This information is read by the PGRAPH subroutine.

The input data file follows:

```
DEMONSTRATION OF PROGRAM NO. 4 - INPUT DATA FILE "EP111.DAT"
BOOSTER PUMP POWER FAILURE, FOUR INGERSOLL-DRESSER 15H277 3-STAGE PUMPS
&SPECS NPIPES=2,NPARTS=5,IOUT=10,HRESUP=1000.,HRESDN=1240.,
       ZEND=1100.,HATM=30.,QTRY=0.,QACC=0.50,TMAX=60.,
       PFILE=T,HVPRNT=T,PPLOT=T,GRAPH=T,RERUN=F/
1  30.  15000.  0.013  3590.  800.
2  30.  30000.  0.013  3590.  800.
&PUMPS NPUMPS=4,NSTAGE=3,IPUMP=1,RPM=1775.,WRSQ=475.,
```

```
        QN=0.,1000.,2000.,3000.,4000.,4500.,
        HNSQ=129.,127.5,121.,103.5,67.5,0.,
        TNSQ=50.,58.,78.,92.,97.,80./
&GRAF NSAVE=2,IOUTSA=1,PIPE=1,2,0,0,NODE=999,1,0,0/
```

The booster pump power failure program PROG4 is used to analyze the problem. The source and executable programs are on the CD. The following plot of extreme pressure values along the pipeline is one of the primary results to come from this analysis. We observe that high heads occur on the suction side of the pump, as well as low heads on the discharge side. No column separation occurs in this case.



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11.1.4. SETTING UP THE EQUATIONS FOR SOURCE PUMPS

We will follow the same general procedure as for booster pumps. There is a check valve on each pump discharge line. There is also a low-friction, essentially frictionless, bypass line with a check valve around the pump station to supply the pipeline if the pump head should drop to zero during the transient. We again assume all pumps fail simul-taneously.

Four equations are needed to model the pump behavior:

Discharge side C^-
$$V_{P_d} = C_3 + C_4 H_{P_d} \tag{11.20}$$

Conservation of mass
$$N_{pu} Q = V_{P_d} A_d \tag{11.21}$$

Work-energy
$$H_{sump} + h_p = H_{P_d} \tag{11.22}$$

Pump characteristic
$$\frac{h_p}{N^2} = N_{st}\left[C_7 \frac{Q}{N} + C_8\right] \tag{11.23}$$

Here N_{pu} is the number of pumps in parallel, and H_{sump} is the pump sump elevation.

When N is found as in the previous case, we have only four unknowns here. With Eqs. 11.20-11.23 we can proceed with a solution. In this case we find

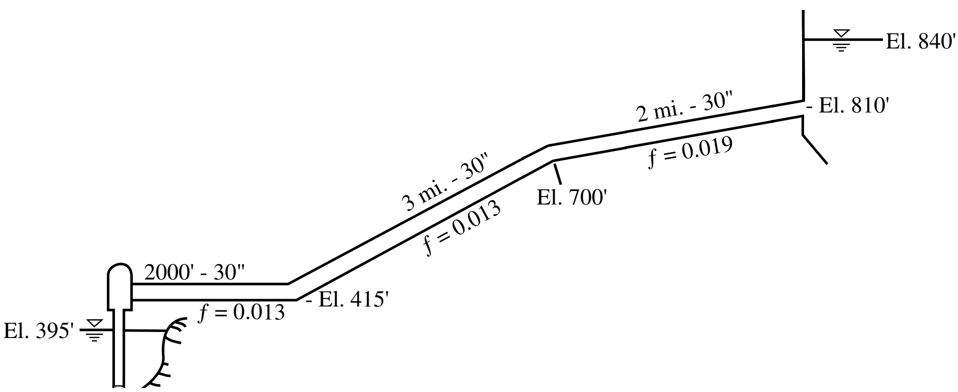
$$H_P = \frac{H_{sump} + \dfrac{N_{st}N}{N_{pu}} C_7 C_3 A_d + N_{st} N^2 C_8}{1 - \dfrac{N_{st}N}{N_{pu}} C_7 C_4 A_d} \tag{11.24}$$

If H_P is less than H_{sump}, then the pump bypass is open, and we must then equate H_P to H_{sump} and use Eq. 11.20 to compute the velocity. If the velocity were found to be negative, then we would set $V_{P_d} = 0$ and compute H_P from Eq. 11.20.

The next example problem applies this analysis to a source pump configuration subjected to a power failure.

Example Problem 11.2

Four pumps in parallel are used to pump approximately 12,000 gal/min from a reservoir at elevation 395 ft to a storage reservoir at elevation 840 ft, as shown below (not to scale). The pump discharge lines have check valves and lead into a manifold which in turn supplies the 30-in welded steel pipeline.



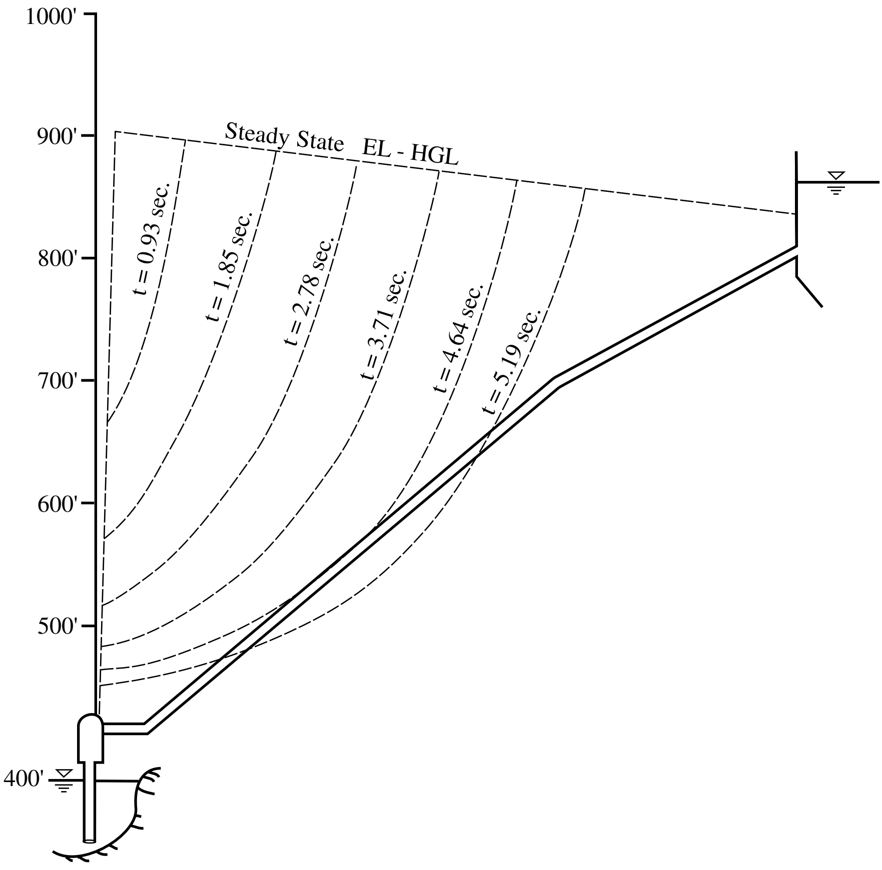
The pipeline extends 2000 ft horizontally from the pump station at an elevation of 415 ft. It then slopes upward for three miles to elevation 700 ft. The remaining two miles of pipe are reinforced concrete and slope gradually upward to enter the storage reservoir at elevation 810 ft. The friction factor and the wave speed for the steel pipe are 0.013 and 3590 ft/s, respectively. For the concrete pipe these values are 0.019 and 3490 ft/s. The pumps are the same Ingersoll-Dresser 15H277 turbine pumps used in Example Problem 11.1, except they now have five stages. Refer to the previous Example Problem for the pump characteristics. The 11.83-in impeller will be used. The total rotary moment of inertia of each pump and motor unit is 475 lb-ft^2.

Find the consequences of pump power failure.

This is a source pump configuration, so PROG3 will be used to determine the effect of pump power failure. The input data file for this program is shown below. This program also uses the subroutine PGRAPH which makes it possible to generate additional tables of output data, printer plots, and data files for external plot programs.

```
DEMONSTRATION OF PROGRAM NO. 3, INPUT DATA FILE "EP112.DAT"
SOURCE PUMP FAILURE, 4 INGERSOLL-DRESSER 15H277 5-STAGE PUMPS
&SPECS NPIPES=3,NPARTS=3,IOUT=5,HRES=840.,HSUMP=395.,
       ZEND=810.,HATM=33.,QACC=0.50,TMAX=10.,DTNEW=0.,
       PFILE=T,HVPRNT=T,PPLOT=T,GRAPH=T,RERUN=F/
1  30.   2000.  0.013  3590.  415.
2  30.  15840.  0.013  3590.  415.
3  30.  10560.  0.019  3490.  700.
&PUMPS NPUMPS=4,NSTAGE=5,RPM=1775.,WRSQ=475.,
       QN=0.,1000.,2000.,3000.,4000.,4500.,
       HNSQ=129.,127.5,121.,103.5,67.5,0.,
       TNSQ=50.,58.,78.,92.,97.,80./
&GRAF NSAVE=3,IOUTSA=1,PIPE=1,2,3,0,NODE=1,1,1,0/
```

The results show that column separation occurs about 5 sec after power failure. At that time the program execution ends because this program is not prepared to analyze vapor cavities. A plot of the EL-HGL for times prior to column separation is presented.



* * *

11.2 PUMP STARTUP

Pressure surges caused by pump startup can be difficult to predict, particularly if air is initially in the system. This air may be in the pump discharge column or located at high spots along the pipeline. Whatever the case, the air-exhaustion process must be simulated in order to approximate the pressures which could occur during startup.

On the other hand, if the pump startup is associated with a recent power failure shutdown, then the restart sequence is important in controlling the pressures. Because the power failure may have caused air to be drawn into the system, we once again must confront the problem of modeling the air removal process from the pipeline.

To simulate this sequence of events, PROG3 has been modified to examine source pump power failure followed by a restart procedure. And PROG8 also can simulate the column separation that may occur during the power failure phase of the sequence and the subsequent air removal process that takes place during the startup phase. Both column separation and air exhaustion are simulated in the simplest manner. The air and vapor cavities are concentrated at the nodes; the pressures at these nodes are set at atmospheric and vapor pressure, respectively, so long as an air bubble or vapor cavity exists. The calculation of pressures that are caused by vapor cavity closure follows the procedure detailed in Section 10.7. Pressures resulting from the elimination of an air bubble through an air valve are treated in the same way. While this simulation procedure is not sophisticated, there are enough uncertainties in the understanding of vapor cavity formation and collapse, the location and movement of vapor cavities and air bubbles, and the extent to which air can be removed from a pipe, that a more thorough analysis is unwarranted.

PROG8 allows air-vacuum valves to be located at pipe junctions along the pipeline which can admit and exhaust air. Vapor cavity formation and collapse are modeled. A bypass line with friction is provided around the pump station to supply water when the pump head drops to zero. The program calculates the steady-flow situation, simulates the power failure rundown and then permits us to explore the behavior of a restart procedure to bring pumps back on line with various ramp times. To simulate power failure without restart, the restart time is simply chosen to be greater than the total execution time for the program. To look at only pump startup, let the program simulate a power failure and then delay the restart until the flow has stabilized. *Always keep in mind that the simulation of vapor cavity behavior and the removal of air from pipelines is a very uncertain process, and the results of such analyses should be viewed very conservatively.*

We will now look further at the pipeline and pumping configuration of Example Problem 11.1 to observe some effects of column separation and pump restart.

Example Problem 11.3
The description in Example Problem 11.2 still applies. In addition, we will restart the pumps at 20-sec intervals, beginning 60 sec after power failure. We will ramp up each pump from 300 rev/min to the full speed of 1775 rev/min in 10 sec. We assume there are air-vacuum valves at the two interior junctions. The loss coefficient K_L for the 24-inch bypass line is 2.5.

The input data file for this analysis follows:

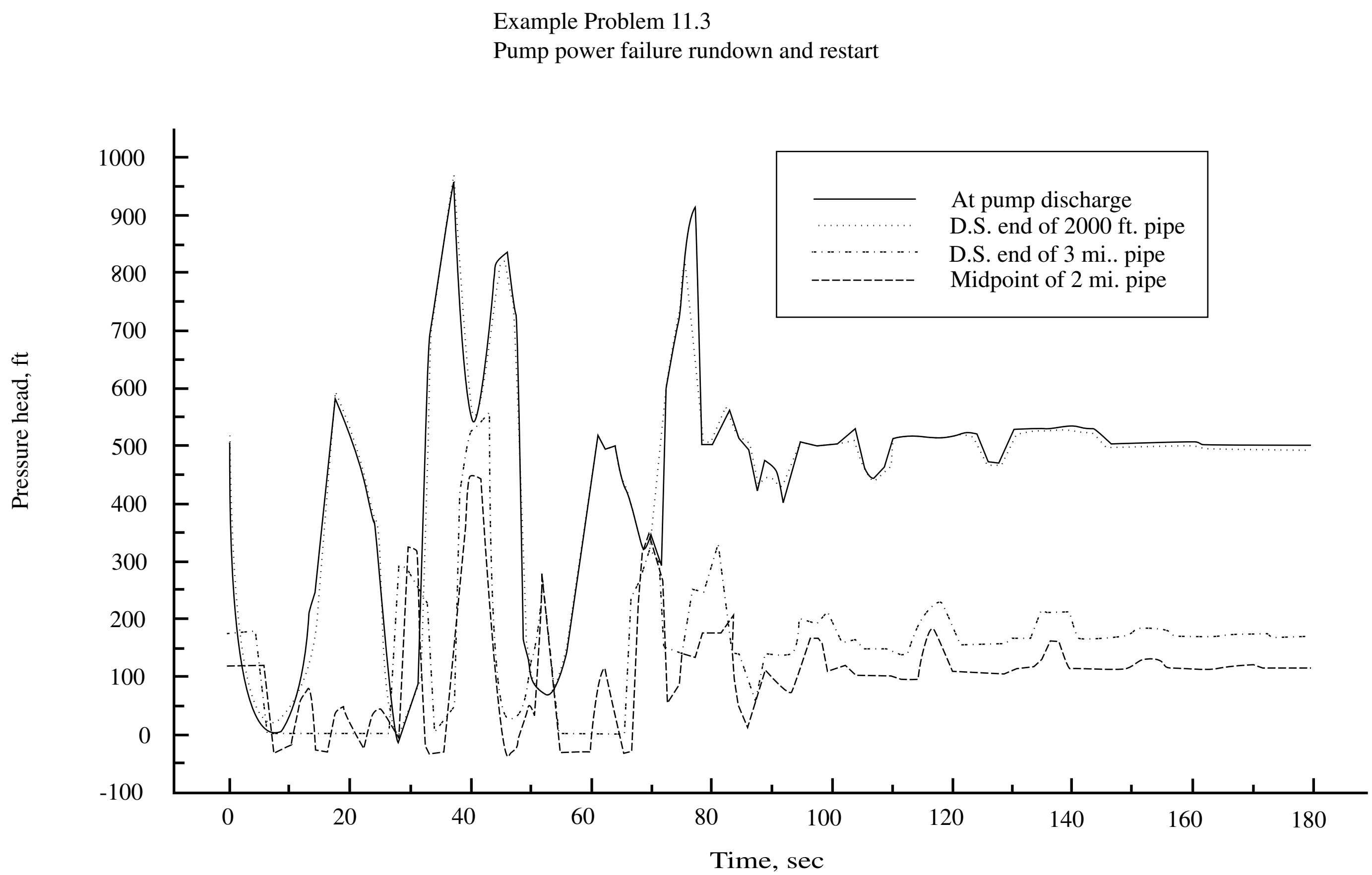
```
DEMONSTRATION OF PROGRAM NO. 8, INPUT DATA FILE "EP113.DAT"
PUMP POWER FAILURE AND RESTART, SAME CONFIGURATION AS EP11.2
&SPECS NPIPES=3,NPARTS=3,HRES=840.,HSUMP=395.,ZEND=810.,
       HATM=33.,QACC=0.50,TMAX=180.,DTNEW=0.,DB=24.,KLB=2.5,
       PFILE=F,HVPRNT=T,PPLOT=F,GRAPH=T,RERUN=F/
1  30.   2000.  0.013  3590.  415.  0
2  30.  15840.  0.013  3590.  415.  1
3  30.  10560.  0.019  3490.  700.  1
&PUMPS NPUMPS=4,NSTAGE=5,RPM=1775.,RPMZ=1775.,WRSQ=475.,
```

```
NSTART=4,QN=0.,1000.,2000.,3000.,4000.,4500.,
       HNSQ=129.,127.5,121.,103.5,67.5,0.,
       TNSQ=50.,58.,78.,92.,97.,80./
&RESTART TSTART(1)=60.,TSTART(2)=80.,TSTART(3)=100.,
       TSTART(4)=120.,TRAMP(1)=10.,TRAMP(2)=10.,TRAMP(3)=10.,
       TRAMP(4)=10.,RPMSTRT(1)=300.,RPMSTRT(2)=300.,
       RPMSTRT(3)=300.,RPMSTRT(4)=300.,RPMEND(1)=1775.,
       RPMEND(2)=1775.,RPMEND(3)=1775.,RPMEND(4)=1775./
&OUTCTRL IOU=1000,    2,    1000,    2,    1000,    1000,
        TIOU= 0.,   60.,    60.5, 179.7,   180.,    400./
&GRAF NSAVE=4,IOUTSA=4,PIPE=1,2,3,3,NODE=1,1,1,7/
```

*

*

*

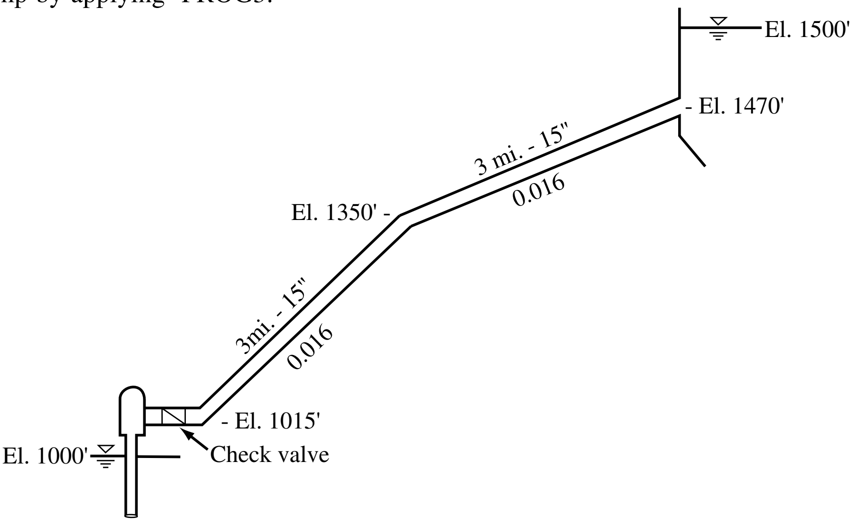


Example Problem 11.3
Pump power failure rundown and restart
At pump discharge
D.S. end of 2000 ft. pipe
D.S. end of 3 mi.. pipe
Midpoint of 2 mi. pipe
Pressure head, ft
1000
900
800
700
600
500
400
300
200
100
0
-100
0
20
40
60
80
100
120
140
160
180
Time, sec

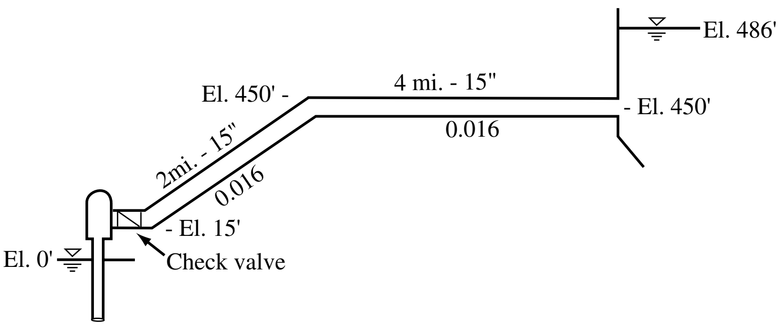
11.3 PROBLEMS

11.1 A seven-stage Ingersoll-Dresser 15H277 pump (see Appendix B) runs at 1775 rev/min with 11.83-in impellers. Water is pumped between two reservoirs with a 500-ft lift. The rotating parts have Wr^2 = 510 lb-ft^2. The wave speed is 3500 ft/s.

Investigate the possibility of column separation occurring after power failure to the pump by applying PROG3.



11.2 The pump of Problem 11.1 is used in the system shown below. If Wr^2 has been reduced to 427 lb-ft^2, determine if, when, and where column separation occurs after power failure to the pump. Use PROG3 for your analysis.



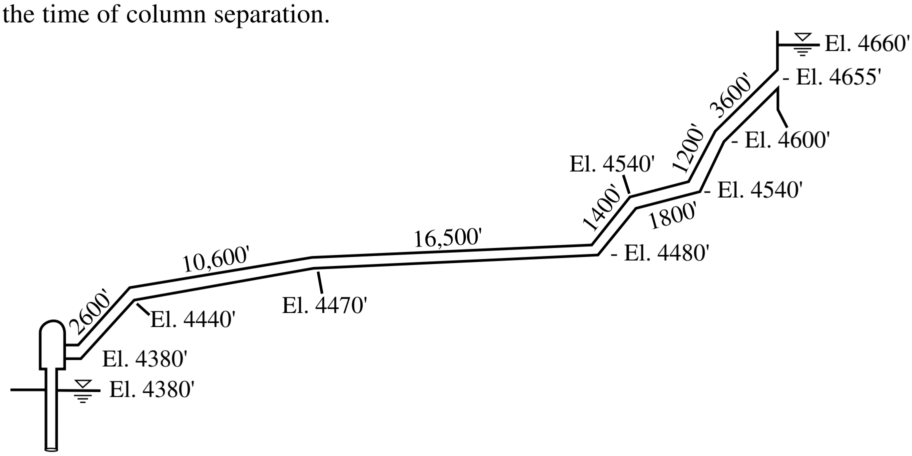
11.3 A 24-in pretensioned reinforced concrete pipe (f = 0.0135) extends 37,700 ft between two reservoirs. The wave speeds for the three separate pressure zones in the pipeline are as follows:

First	2,600 ft:	a = 3550 ft/s
Middle	10,600 ft:	a = 3400 ft/s
Last	24,500 ft:	a = 3270 ft/s

The pump station is equipped with four three-stage pumps turning at 1760 rev/min. For each pump Wr^2 = 138 lb-ft^2. Data from the pump characteristic diagram are tabulated atop the next page.

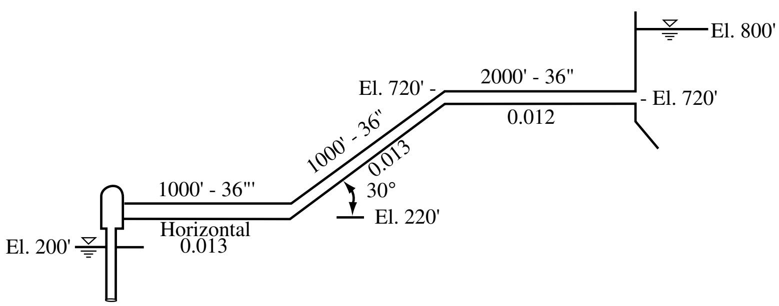
Q (gal/min)	Head/stage (ft)	BHP/stage (hp)
0	213	97
2500	162	130
3000	156	140
3250	151	142
3500	145	145
4000	130	150

Investigate the possibility of column separation following power failure to the pumps by applying PROG3. If it does occur, find the time and location and plot the EL-HGL at the time of column separation.



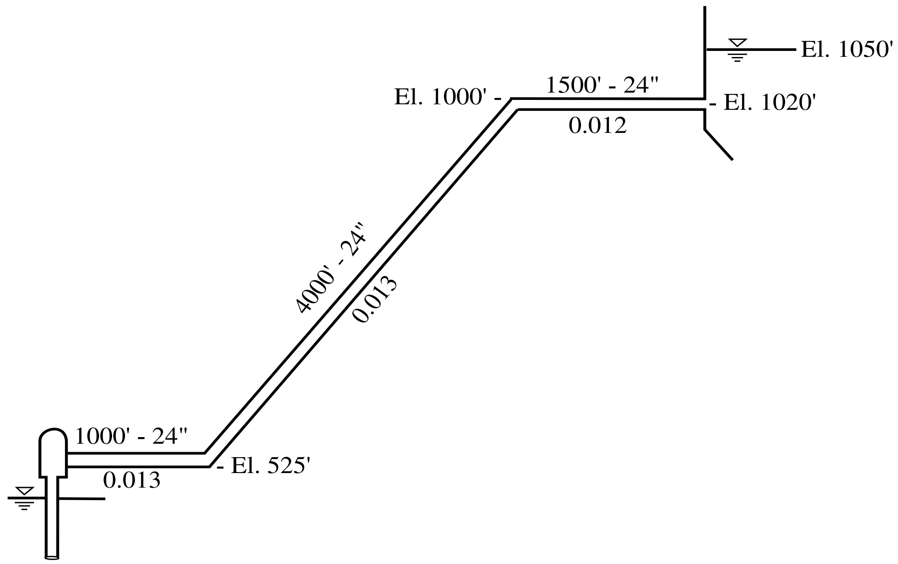
11.4 A proposed project will employ five six-stage 15H277 Ingersoll-Dresser turbine pumps (see Appendix B) to lift approximately 14,000 gal/min of water from the Columbia River gorge to a storage reservoir on a plateau above. The 11.83-in impeller will be used, and the rotating parts have an estimated Wr^2 of 510 lb-ft^2.

The pumps are connected by a manifold to one 36-in line constructed of two materials. The first 2000 ft is welded steel, and the second 2000 ft is asbestos cement. The wave speeds in the pipes are 3190 ft/s and 2860 ft/s, respectively. Use PROG3 to conduct a power failure analysis to determine whether, when, and where column separation occurs.

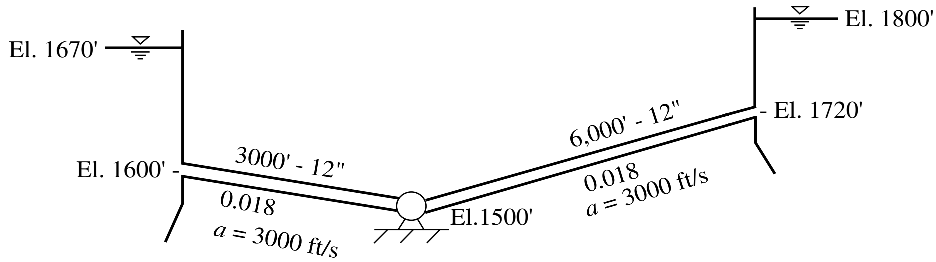


11.5 A project plans to use four five-stage Ingersoll-Dresser 15M185 turbine pumps (see Appendix B) to pump water from the Snake River 6500 ft to a storage reservoir. The approximate pipeline profile is shown below. The four pumps have 11.83-in impellers and are connected via a check-valve and manifold to a 24-in pipeline. For each pump and motor unit it is estimated that Wr^2 = 200 lb-ft^2. The Snake River water level fluctuates between 500 and 520 ft over the pumping season. The deck of the pumping station and the pump manifold are at elevation 525 ft. The portion of the pipeline from the pump station to the top of the incline is welded steel having a wave speed of 3190 ft/s. The remainder of the line is asbestos cement pipe with a wave speed of 2860 ft/s.

Use PROG3 to determine if, when, and where column separation occurs during pump power failure.

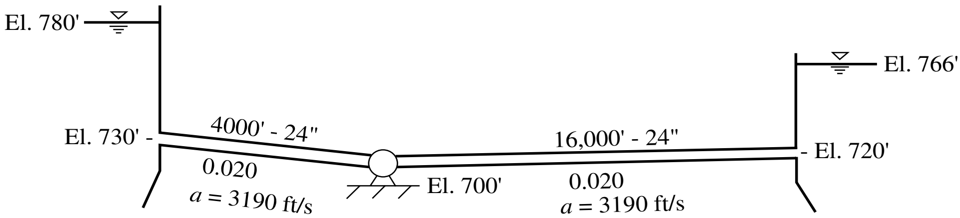


11.6 The pump station in the system below is equipped with three five-stage Ingersoll-Dresser 14JKH pumps operating in parallel (see Appendix B). The pumps use the 10.5-inch impellers and run at 1175 rev/min. Each impeller has $Wr^2 = 2.6$ lb-ft^2; the inertia of the pump shaft can be neglected. For the motor Wr^2 can be estimated with Thorley's formula.



Analyze the system for power failure using PROG4, and determine the maximum and minimum pressures, their location and time of occurrence.

11.7 The booster pump station in the next figure houses two single-stage Ingersoll-Dresser 20KKH turbine pumps. The pumps run at 1180 rev/min and have 15-in impellers. Assume the rotary inertia for the pump and motor unit is 225 lb-ft^2.



Investigate the consequences of power failure for the pipeline using PROG4, and find the extreme pressures occurring and their locations and time of occurrence.

11.8 Use PROG8 to estimate the consequences of column separation in Problem 11.1. There are no air-vacuum valves in the system.

11.9 If an air-vacuum valve exists at the downstream end of the 2-mile pipe in Problem 11.2, use PROG8 to investigate the effects of power failure on the extreme pressures occurring in the system.

11.10 Analysis of Problem 11.3 with PROG3 revealed that column separation occurred. Use PROG8 to estimate the effects of column separation on the maximum and minimum pressures occurring in the pipeline.

11.11 Column separation was found to occur in Problem 11.4 as a result of power failure. Analyze again the system by using PROG8 to estimate the effects of this event. An air-vacuum valve is at the downstream end of the 1000-ft pipe at elevation 720 ft.

11.12 We have established that column separation will occur in Problem 11.5 as a consequence of power failure. Using PROG8, estimate the extreme pressures which will then occur. An air-vacuum valve is at the downstream end of the 4000-ft pipe.

CHAPTER 12

NETWORK TRANSIENTS

12.1 INTRODUCTION

Both rigid column theory and elastic or water hammer analysis will be applied in this chapter to the solution of unsteady flows in pipe networks. Simplified solutions that ignore both inertial and elastic effects in pipe networks were covered in Chapter 6 under the name "extended time simulations."

What conditions require the full consideration of inertial effects, and what situations will make the elastic properties of the liquid and pipe so important that a full water hammer network analysis is necessary? When is an extended time simulation sufficient? There are no precise answers for these questions. The next few paragraphs mention some relevant factors in making such a decision, but in the end professional judgment and personal experience are also factors.

An elastic analysis is required whenever the changes in velocity are sufficiently rapid to cause substantial changes in the flow variables over time intervals that are less than several times the value of L/a for the pipe(s) under investigation. Examples are the rapid closure of a valve, the filling of a pipeline with liquid that moves at high velocity and forces air from the lines, an abrupt change in the operation of pumps, and in general any event that is sufficiently rapid to prevent the fluid throughout the network from gradually accommodating the change. However, the occurrence of a rapid change in a single pipe does not necessarily mean that an elastic analysis of the entire network is warranted. When demands are changing throughout a large distribution system, large pressure changes will alter the system demands so the pressure wave is rapidly absorbed. In this case the need for an elastic analysis may be restricted to that pipe, or possibly to it and a few nearby pipes.

Rigid column theory can be applied to situations in which the demands on an elastic pipe network change rather rapidly but not instantaneously, causing the inertial effects in accelerating the liquid to have a significant effect on the pressure. Examples are found during the morning hours in a large city when additional pumps must accommodate relatively rapid increases in demand, or whenever a major user may shut down rapidly. These changes in demand are not so rapid that elastic effects become significant, yet the effect of accelerating the fluid, owing to the long pipelines that exist between the supply sources and the demand sites, can cause the pressures far downstream in a distribution system to be significantly different than would be the case if only fluid friction were considered.

If both inertial and elastic effects can be ignored, then a quasi-static or extended time simulation would be valid for much of the operation of a water distribution system.

12.2 RIGID-COLUMN UNSTEADY FLOW IN NETWORKS

12.2.1. THE GOVERNING EQUATIONS

In the latter portion of Chapter 7 some unsteady flows in single pipes were studied. That theory assumed the liquid to be incompressible and the pipes to be rigid, thus ignoring the elastic properties of the liquid and the pipe. Here this same rigid column theory will be expanded to multiple-pipe systems. Here we ignore the convective acceleration term $V\partial V/\partial s$ for reasons discussed in Section 8.5.2. In the analysis of steady flows in networks it is also common practice to ignore the difference between the hydraulic grade line and the energy line by assuming they are coincident. This simplification is consistent with the deletion of the convective acceleration term, and it is standard practice in the application of rigid column theory, so long as velocities are low.

Equation 8.59, the equation of motion, can be written as

$$\frac{dV}{dt} = -g\frac{\partial H}{\partial s} - \frac{f}{2D}V|V| \tag{12.1}$$

Since $\partial H/\partial s$ is constant along a pipe, it can be expressed as $(H_j - H_i)/L_k$. The subscripts indicate that pipe k has an upstream node i and a downstream node j. Substituting one expression for the other in Eq. 12.1 gives

$$\frac{dV_k}{dt} = g\frac{H_i - H_j}{L_k} - \frac{f_k V_k|V_k|}{2D_k} \tag{12.2}$$

in which the subscript k has been added to the velocity V, the diameter D, and the friction factor f, to show the equation applies to pipe k in the system. Usually it is more convenient to use the discharge $Q = VA$ as a dependent variable in place of V; then Eq. 12.2 can be written as

$$\frac{dQ_k}{dt} = gA_k\frac{H_i - H_j}{L_k} - \frac{f_k Q_k|Q_k|}{2D_k A_k} \tag{12.3}$$

For unsteady flows Eq. 12.3 relates the time-varying discharge in pipe k, the frictional loss, and the instantaneous heads at the end nodes of the pipe. If $dQ_k/dt = 0$ so the flow is steady, we recover from Eq. 12.3 the Darcy-Weisbach equation itself. Thus Eq. 12.3 is the unsteady-flow analog of the Darcy-Weisbach equation, or an empirical equation such as the Hazen-Williams formula, for the relation between the frictional head loss and the discharge.

The junction continuity equations must also be satisfied for unsteady flows. Therefore, in addition to the equations that can be written by applying Eq. 12.3 to a network, NJ (or NJ - 1 if all external flows are specified) junction continuity equations must be written, one for each node, in the form

$$\sum Q_k - QJ_i = 0 \tag{12.4}$$

Here the summation includes all pipes that join at junction i, and QJ_i is the demand at this junction. In Eq. 12.4 the discharge is positive if it flows into junction i and negative if it flows from the junction.

12.2.2. THREE-PIPE PROBLEM

We begin by describing how Eqs. 12.3 and 12.4 can be used to model the unsteady flow in a small network. For this example we select the three-pipe network in Fig. 12.1. Since all external flows are specified, there are NJ - 1, or 2, junction continuity equations for this network. These continuity equations are

$$\begin{aligned} F_1 &= Q_1 - Q_2 - QJ_2 = 0 \\ F_2 &= Q_2 + Q_3 - QJ_3 = 0 \end{aligned} \tag{12.5}$$

These two equations require the negative demand QJ_1 at node 1 to equal the sum of the

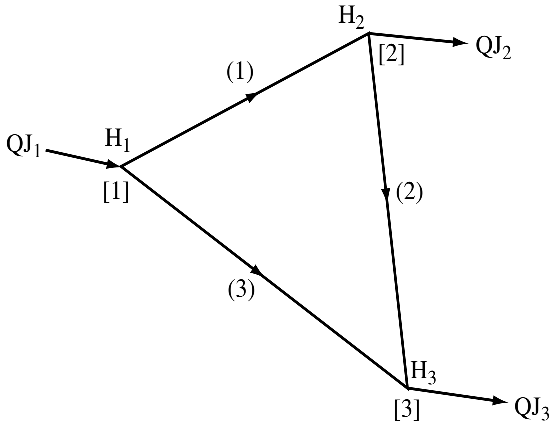


Figure 12.1 Three-pipe network.

positive demands QJ_2 at node 2 and QJ_3 at node 3. In addition, the following three ordinary differential equations apply, one for each pipe:

$$\begin{aligned}
\frac{dQ_1}{dt} &= gA_1 \frac{H_1 - H_2}{L_1} - \frac{f_1 Q_1 |Q_1|}{2D_1 A_1} \\
\frac{dQ_2}{dt} &= gA_2 \frac{H_2 - H_3}{L_2} - \frac{f_2 Q_2 |Q_2|}{2D_2 A_2} \\
\frac{dQ_3}{dt} &= gA_3 \frac{H_1 - H_3}{L_3} - \frac{f_3 Q_3 |Q_3|}{2D_3 A_3}
\end{aligned} \tag{12.6}$$

In this network we assume that the head H_1 is constant since the fluid is supplied from a reservoir. Therefore, if the demands QJ_2 and QJ_3 are specified for all time, then the five variables Q_1, Q_2, Q_3, H_2, and H_3 are the unknown variables in this network.

To determine five unknown variables, we must have five independent equations. In this problem Eqs. 12.5 and 12.6 satisfy this requirement. But how can this system of equations be solved when some of the equations are ordinary differential equations (ODEs) rather than algebraic equations? If a solution can be found, it must then be applied repeatedly as time advances. Thus such a solution is far more than a single steady-flow solution. Instead the solution process must be repeated over and over until the results are known over a sufficiently long time period to satisfy our need for knowledge of the behavior of this system. To simulate the performance of a water main network over a twenty-four-hour period, say at 10 second intervals, would require 8640 incremental solutions. Obviously this is a task for a fast computer if the network is very large. But usually such unsteady-flow solutions are only required for much shorter time intervals, over which rapid changes occur.

With the correct approach, the Newton method will allow us to solve a set of algebraic and ordinary differential equations simultaneously. As a first step in this method, the equations are each equated to zero, as shown:

$$
\begin{aligned}
F_1 &= Q_1 - Q_2 - QJ_2 = 0 \\
F_2 &= Q_2 + Q_3 - QJ_3 = 0 \\
F_3 &= Q_1 - Q_{ODE1} = 0 \\
F_4 &= Q_2 - Q_{ODE2} = 0 \\
F_5 &= Q_3 - Q_{ODE3} = 0
\end{aligned}
\tag{12.7}
$$

In Eqs. 12.7 each of the variables is in general a function of time. At time $t = 0$ we assume all variables are known. These initial values are usually obtained by solving the steady-state network problem. The notation in the last three of Eqs. 12.7 has the following meaning: the Q with a subscript 1, 2, or 3 is the discharge in each pipe. Each Q with the *ODE* subscript is the discharge that is found from the solution of the first, second or third ordinary differential equation at the designated time. When these latter discharges equal the respective pipe flows, then the last three equations are clearly satisfied. However, Q_{ODE1}, Q_{ODE2}, and Q_{ODE3} are obtained by solving the corresponding ODEs numerically over the latest time increment from the last known solution to the new time instant. Thus the last three equations are expressions or functions, just as algebraic equations are expressions. The only difference is that much more algebra is required to evaluate each expression, since an ODE is solved for this purpose. The Newton method is a systematic and effective means of directing the solution process so convergence to the correct solution is obtained in relatively few iterations.

Often we cannot compute formally for every function all of the partial derivatives that are required in the process of solving the ODEs; then the evaluation of the elements of the Jacobian matrix is most conveniently done by using numerical approximations. This numerical evaluation can be accomplished in the same way as it was for algebraic equations, namely by evaluating the equation twice and dividing the difference of these two values by the increment of the unknown Δx_j for which the derivative is sought, or

$$
\frac{\partial F_i}{\partial x_j} = \frac{F_i\left(x_1, x_2, \ldots x_j + \Delta x_j, \ldots x_n\right) - F_i\left(x_1, x_2, \ldots x_j, \ldots x_n\right)}{\Delta x_j} \tag{12.8}
$$

To see the solution process in operation, we now solve the three-pipe network over 8 seconds in 2 second intervals. At $t = 0$ the flows are steady, and for simplicity let (1) the friction factors f for the three pipes retain their steady-state values, and (2) the demand at node 2 be constant in time, so only QJ_3 changes with time. The head H_1 = 100 ft at node 1 is also constant. The values in the following tables define the problem further:

Values at time = 0:

Pipe	***D*** **in.**	***L*** **ft.**	***e*** **in.**	**Node**	**QJ, ft^3/s**
1	8	2000	0.005	1	- 3.0
2	8	2400	0.005	2	1.5
3	8	3000	0.005	3	1.5

Variation of QJ_3 with time:

Time, sec	0	2	4	6	8
QJ$_3$, ft^3/s	1.5	2.0	2.5	3.0	3.5

First the network solution to the steady problem is obtained:

Pipe	Q ft^3/s	h_L ft	f	Node	HGL ft
1	1.652	20.10	0.0193	1	100.00
2	0.152	0.29	0.0270	2	79.90
3	1.348	28.16	0.0196	3	79.61

The program THREPIP.FOR for the solution of this problem can be found on the CD; the reader is encouraged to obtain a listing before reading further. The solution for this unsteady problem is summarized in Table 12.1:

Table 12.1 Unsteady Flow Solution

Step	Time sec	QJ_3 ft^3/s	Q_1 ft^3/s	Q_2 ft^3/s	Q_3 ft^3/s	H_2 ft	H_3 ft
0	0.0	1.500	1.652	0.152	1.348	79.90	79.61
1	2.0	2.000	1.860	0.360	1.640	58.63	35.60
2	4.0	2.500	2.076	0.576	1.924	51.98	26.09
3	6.0	3.000	2.299	0.799	2.201	44.57	14.80
4	8.0	3.500	2.527	1.027	2.473	36.49	1.72

Problem 12.1 (end of chapter) seeks the solution of four analogous steady-flow problems that can be used to study the differences between steady and unsteady network-flow behavior. That comparison will show that inertial effects lower the head at node 3 in much the same way as do steady-state frictional losses. For networks with rapidly-changing nodal demands we conclude that inertial effects must be included in an analysis.

Some study of the listing of THREPIP.FOR to understand the program structure can be valuable. This program calls two standard programs, DVERK from IMSL (see Appendix A) to obtain the solution to the ODEs, and SOLVEQ to solve the linear equation system that is the result of implementing the Newton method. The main program first sets up the problem by obtaining input data from the user, and then it implements the Newton method by defining the Jacobian matrix and the equation vector for each iteration. Then it subtracts the latest linear solution from the previous vector of unknowns to improve the solution. A solution is sought for each time step within the `DO 60` loop, which prompts the user to supply new values for QJ_3. The main program calls subroutine DEFFUN to evaluate the five functions in Eqs. 12.7; the first two functions are the continuity equations, and the other three functions are obtained by calling DVERK. The actual derivatives are computed in subroutine SLOPE, which DVERK calls. After DVERK solves the three ODEs over the latest time increment, subroutine DEFFUN defines functions F(3), F(4), and F(5).

The program assumes constant friction factors. This assumption is questionable in the example because the small discharge in pipe 2 caused the friction factor $f_2 = 0.027$ to be larger than the others. Some program changes would allow the friction factors to be computed from the Colebrook-White Equation. Instead of assigning these factors as input data, the Colebrook-White equation could be solved in subroutine SLOPE to provide a friction factor for each initial discharge.

In the example only the demand at node 3 was a function of time primarily because the effects of inertia on the nodal heads could then be more easily understood. Changes in input data would allow both QJ_2 and QJ_3 to vary with time. The heads H_2 and H_3 might alternatively have been specified as functions of time with the demands QJ_2 and QJ_3 being considered as unknown variables, along with Q_1, Q_2, and Q_3. In this case the set of five governing equations would be unchanged, but the computer program to solve the problem must then to be modified to indicate correctly which variables are known and which are unknown.

When a steady network problem is solved by using the Q-equations or the ΔQ-equations, the nodal heads were then computed later as secondary dependent variables, as discussed in earlier chapters. This sequential process no longer is successful for unsteady network analyses, for the discharges and heads are now coupled. The computations are more extensive because ODEs have replaced algebraic equations, and discharges and heads are wanted at numerous time increments.

12.3 A GENERAL METHOD FOR RIGID-COLUMN UNSTEADY FLOW IN PIPE NETWORKS

12.3.1. THE METHOD

The solution methodology that was applied in the unsteady-flow analysis of the three-pipe network will now be described in general terms so it can be used to analyze any network having NP pipes and NJ nodes. We assume that the network has at least two supply sources. For such networks the number of independent simultaneous equations consists of NJ junction continuity equations and NP ODEs that govern the rigid-column unsteady flow in the pipes. These equations are

NJ junction continuity equations $$\sum Q_k - QJ_i = 0 \qquad (12.9)$$

NP ODEs $$\frac{dQ_k}{dt} = gA_k \frac{H_i - H_j}{L_k} - \frac{f_k Q_k |Q_k|}{2D_k A_k} \qquad (12.10)$$

in which one head is the water surface elevation of a reservoir if this pipe connects the reservoir to the network. This equation system will allow NP + NJ unknowns to be determined. For the analysis problem these unknowns are NP discharges in NP pipes and the heads at NJ nodes. This set of unknowns assumes that the time-dependent nodal demands are specified. But one might alternatively specify heads as functions of time at the nodes, and then the demands at the nodes would replace the heads as the unknowns. In fact it is possible to mix the specification of demands and heads as functions of time. For any node where QJ is specified as a function of time, the head must be unknown at that node, and at any node where the head is specified as a function of time, the demand QJ there must be an unknown.

The continuity equations are linear and are identical to those that would be written for any steady-state analysis. The equations of motion for the fluid in the pipes, i.e. the ODEs, must be appropriately solved over the time increment for which new information is wanted, and these solutions must also relate the discharges and heads to each other at the nodes of the network. To solve this system of algebraic and ordinary differential equations, any iterative method could in principle be used, but we prefer to use Newton's method. When applying the Newton method to the ODEs, functions are created that are simply the difference between the current discharge value in the pipe and the value that is found by solving the ODE for this pipe over the present time increment. In short, we create functions of the form

$$F_i = Q_k - Q_{ODE} = 0 \qquad (12.11)$$

This set of equations must be solved for each new time increment.

Thus the process of obtaining an unsteady solution to a problem in which demands or heads are specified functions of time consists of seven tasks:

1. The time span, over which the unsteady solution is to be obtained, is divided into NT time increments or steps.
2. The discharges in all pipes and the heads at all nodes are assigned initial values that are chosen from a steady state solution that has the same demands, etc. as the unsteady solution has at time zero. (In place of a steady state solution, this initialization may be obtained from the last time-step solution from a prior unsteady-flow solution.)
3. All demands over each time increment must be specified.
4. Over each new time increment define and evaluate the functions (identify the equations to be solved, and substitute the current value of each variable into them if they are algebraic, or solve the ODE using the current values of all variables) and the Jacobian matrix of derivatives of these functions.
5. Solve the resulting linear equation system. The solution of this equation system is then subtracted from the set of unknown values, according to the Newton method.
6. Steps 4 and 5 are repeated iteratively, until the specified convergence criterion has been satisfied.
7. Write the solution for the discharges and the nodal heads for this time increment, and then repeat steps 3 through 7 until the unsteady solution spans the entire time period.

12.3.2. AN EXAMPLE

Figure 12.2 depicts a network with 19 pipes and 12 nodes. The nodal demands sum to 10.3 ft^3/s, and this discharge must come from the two reservoirs. The steady state solution to this network is listed in Tables 12.2. As in the tables, all pipe diameters are in inches and lengths in feet. The largest head loss, 24.3 ft, occurs in pipe 1 that supplies

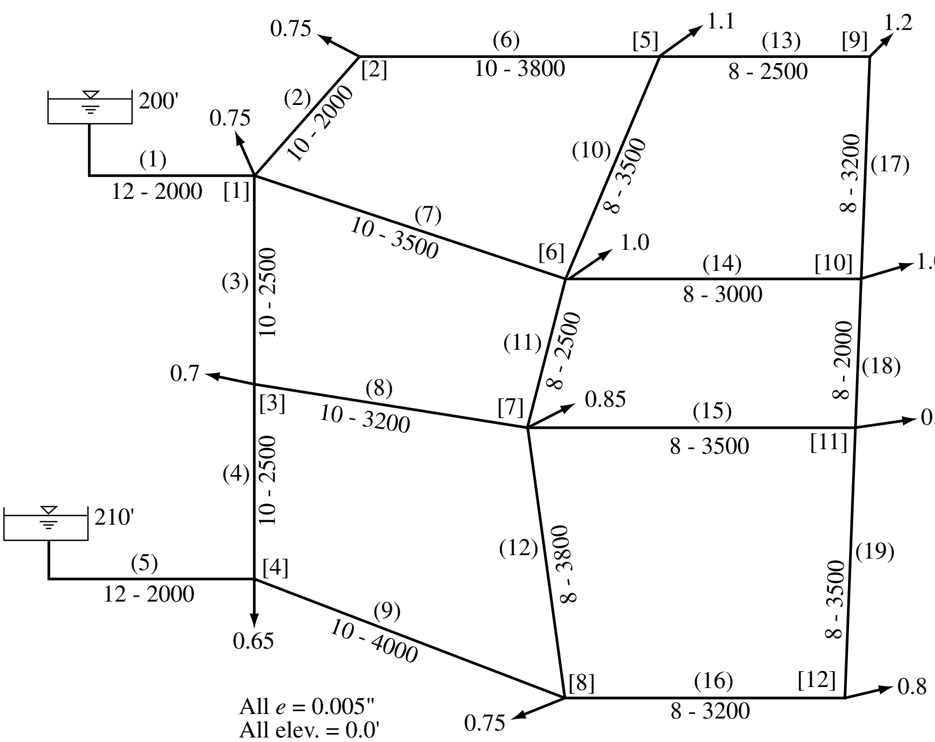


Figure 12.2 Typical pipe network.

the network from a reservoir, and the third largest head loss, 21.7 ft, occurs in the other reservoir supply line, pipe 5. The steady problem was solved by using seven ΔQ-equations around loops. Six of these loops are real loops, and one is a pseudo loop that connects the two reservoirs through a sequence of connected pipes.

Tables 12.2 Steady-flow solution, 19-pipe network.

PIPE DATA

PIPE NO.	NODES FROM	TO	L ft.	DIA. in	e x10^3 in	Q ft^3/s	VEL. ft/s	HEAD LOSS ft.	HLOSS /1000
1	0	1	2000.	12.0	5.0	5.30	6.75	24.26	12.13
2	1	2	2000.	10.0	5.0	2.17	3.98	10.96	5.48
3	1	3	2500.	10.0	5.0	0.41	0.74	0.60	0.24
4	4	3	2500.	10.0	5.0	2.13	3.90	13.19	5.27
5	0	4	2000.	12.0	5.0	5.00	6.37	21.67	10.84
6	2	5	3800.	10.0	5.0	1.42	2.61	9.29	2.44
7	1	6	3500.	10.0	5.0	1.97	3.62	15.95	4.56
8	3	7	3200.	10.0	5.0	1.83	3.36	12.70	3.97
9	4	8	4000.	10.0	5.0	2.22	4.07	22.89	5.72
10	6	5	3500.	8.0	5.0	0.55	1.56	4.30	1.23
11	7	6	2500.	8.0	5.0	0.50	1.44	2.65	1.06
12	8	7	3800.	8.0	5.0	0.43	1.23	3.00	0.79
13	5	9	2500.	8.0	5.0	0.87	2.48	7.34	2.94
14	6	10	3000.	8.0	5.0	0.93	2.66	10.06	3.35
15	7	11	3500.	8.0	5.0	0.91	2.61	11.29	3.23
16	8	12	3200.	8.0	5.0	1.04	2.99	13.32	4.16
17	10	9	3200.	8.0	5.0	0.33	0.95	1.59	0.50
18	11	10	2000.	8.0	5.0	0.40	1.16	1.41	0.70
19	12	11	3500.	8.0	5.0	0.24	0.69	0.97	0.28

AVE. VEL. = 2.80 ft/s, AVE. HL/1000 = 3.625, MAX. VEL. = 6.75 ft/s, MIN. VEL. = 0.69 ft/s

NODE DATA

NODE	DEMAND ft^3/s	gal/min	ELEV. ft.	HEAD ft.	PRESSURE lb/in^2	HGL ELEV. ft.
1	0.75	336.6	0.0	175.74	76.15	175.74
2	0.75	336.6	0.0	164.78	71.40	164.78
3	0.70	314.2	0.0	175.14	75.89	175.14
4	0.65	291.7	0.0	188.33	81.61	188.33
5	1.10	493.7	0.0	155.49	67.38	155.49
6	1.00	448.8	0.0	159.79	69.24	159.79
7	0.85	381.5	0.0	162.44	70.39	162.44
8	0.75	336.6	0.0	165.44	71.69	165.44
9	1.20	538.6	0.0	148.15	64.20	148.15
10	1.00	448.8	0.0	149.74	64.89	149.74
11	0.75	336.6	0.0	151.15	65.50	151.15
12	0.80	359.1	0.0	152.11	65.92	152.11

AVE. HEAD = 162.4 ft., AVE. HGL = 162.4 ft.,
MAX. HEAD = 188.3 ft., MIN. HEAD = 148.2 ft.

The number of simultaneous equations to model this unsteady flow problem is 31, based on rigid column theory. Twelve of these are algebraic junction continuity equations, and the other 19 are ODEs that define the relation between the rates at which the discharges change and the slopes of the HGL in the pipes.

We will seek the solution of the unsteady problem in which all demands but one remain constant; at node 9 the demand gradually increases from 1.2 ft^3/s to 2.5 ft^3/s over 20 seconds, as outlined in the table:

Time, sec	0	5	10	15	20
Demand at node 9, ft^3/s	1.20	1.50	1.75	2.00	2.50

This increase in demand at node 9 of 1.3 ft^3/s over 20 seconds is approximately a doubling of the original demand and is typical of demand increases that might be expected in a network that has an average demand of 10.3 ft^3/s. An increased demand of 1.3 ft^3/s in 20 seconds could occur when a single major water user begins operation at the beginning of the work day. Thus this problem, while having relatively few pipes, can provide a basis for an evaluation of the importance of inertial effects during periods when one or more demands vary rapidly. The input data for the solution of the problem can be found in file EPB12F_2.IN on the CD.

Results for the unsteady-flow solution are tabulated in Tables 12.3a-d after each of four 5-sec time increments. Tables 12.3a-b list the discharge in each pipe after each of the 5-sec intervals. In these tables the results from this solution are also compared with results from four steady-state solutions; in each steady solution the demand at node 9 matches the unsteady-flow demand at this node at the end of the five-second interval. The first tables compare the discharges in the pipes, and Tables 12.3c-d compare the heads at the nodes. We find the steady and unsteady discharges differ only by small amounts.

Table 12.3a Discharge Comparison, ft^3/s.

Pipe	$QJ_9 = 1.50$ ft^3/s Time = 5 sec			$QJ_9 = 1.75$ ft^3/s Time = 10 sec		
	Unsteady	Steady	Difference	Unsteady	Steady	Difference
1	5.49	5.48	0.01	5.65	5.63	0.02
2	2.28	2.26	0.02	2.36	2.34	0.02
3	0.41	0.42	- 0.01	0.41	0.42	- 0.01
4	2.18	2.18	0.00	2.23	2.23	0.00
5	5.11	5.12	- 0.01	5.20	5.22	- 0.02
6	1.53	1.51	0.02	1.61	1.59	0.02
7	2.06	2.05	0.01	2.13	2.12	0.01
8	1.89	1.90	- 0.01	1.94	1.95	- 0.01
9	2.27	2.29	- 0.02	2.32	2.34	- 0.02
10	0.61	0.59	0.02	0.65	0.64	0.01
11	0.54	0.54	0.00	0.57	0.57	0.00
12	0.45	0.45	0.00	0.47	0.47	0.00
13	1.03	1.01	0.02	1.16	1.13	0.03
14	0.99	1.00	- 0.01	1.05	1.05	0.00
15	0.95	0.96	- 0.01	0.99	1.00	- 0.01
16	1.07	1.09	- 0.02	1.10	1.12	- 0.02
17	0.47	0.49	- 0.02	0.59	0.62	- 0.03
18	0.47	0.50	- 0.03	0.54	0.57	- 0.03
19	0.27	0.29	- 0.02	0.30	0.32	- 0.02
Absolute Averages			0.013			0.015

Table 12.3b Discharge Comparison, ft^3/s.

Pipe	$QJ_9 = 2.00\ ft^3/s$ Time = 15 sec			$QJ_9 = 2.50\ ft^3/s$ Time = 20 sec		
	Unsteady	Steady	Difference	Unsteady	Steady	Difference
1	5.80	5.78	0.02	6.12	6.09	0.03
2	2.44	2.42	0.02	2.61	2.57	0.04
3	0.41	0.43	- 0.02	0.42	0.44	- 0.02
4	2.28	2.28	0.00	2.37	2.37	0.00
5	5.30	5.32	- 0.03	5.48	5.51	- 0.03
6	1.69	1.67	0.02	1.86	1.82	0.04
7	2.20	2.19	0.01	2.34	2.32	0.02
8	1.99	2.01	- 0.02	2.09	2.11	- 0.02
9	2.37	2.39	- 0.02	2.46	2.49	- 0.03
10	0.69	0.68	0.01	0.79	0.77	0.02
11	0.60	0.60	0.00	0.66	0.67	- 0.01
12	0.48	0.48	0.00	0.52	0.52	0.00
13	1.28	1.25	0.03	1.54	1.50	0.04
14	1.10	1.11	- 0.01	1.21	1.22	- 0.01
15	1.02	1.04	0.02	1.10	1.11	- 0.01
16	1.14	1.16	- 0.02	1.20	1.22	- 0.02
17	0.71	0.75	- 0.04	0.96	1.00	- 0.04
18	0.61	0.64	- 0.03	0.74	0.78	- 0.04
19	0.34	0.36	- 0.02	0.40	0.42	- 0.02
Absolute Averages			0.019			0.024

Table 12.3c Head Comparison, ft.

Node	$QJ_9 = 1.50\ ft^3/s$ Time = 5 sec			$QJ_9 = 1.75\ ft^3/s$ Time = 10 sec		
	Unsteady	Steady	Difference	Unsteady	Steady	Difference
1	171.70	174.10	- 2.40	170.71	172.69	- 1.98
2	157.76	162.25	- 4.49	156.36	160.06	- 3.70
3	171.16	173.47	- 2.31	170.04	172.05	- 2.01
4	186.18	187.32	- 1.14	185.51	186.46	- 0.95
5	143.17	151.82	- 8.65	141.53	148.60	- 7.07
6	151.41	156.88	- 5.47	149.95	154.37	- 4.42
7	155.91	159.89	- 3.98	154.44	157.73	- 3.29
8	160.35	163.15	- 2.80	158.94	161.22	- 2.28
9	127.13	142.09	- 14.96	124.26	136.55	- 12.29
10	137.25	145.37	- 8.12	134.95	141.61	- 6.66
11	141.56	147.43	- 5.87	139.48	144.29	- 4.81
12	144.74	148.75	- 4.01	142.71	145.91	- 3.20
Averages			- 5.35			- 4.39

The largest difference in discharges occurs at 20 sec when the average absolute difference in discharges is 0.024 ft^3/s, with individual differences no larger than 0.04 ft^3/s. In Tables 12.3c-d where head solutions are compared, the difference should probably be examined in comparison with the frictional losses. From the steady solutions for this network we find the head losses in pipes 13 and 17 that supply node 9 are 7.34 and 1.59 ft, respectively. The largest difference in heads between the unsteady and steady solutions is 23.47 ft, which is approximately five times the average frictional head loss in the pipes that supply node 9. It is therefore obvious that an extended time simulation would be quite inadequate in determining the transient pressure distribution, especially near the location (node 9) of a rapid change in demand over a short time period (20 sec).

Table 12.3d Head Comparison, ft.

Node	$QJ_9 = 2.00$ ft^3/s Time = 15 sec			$QJ_9 = 2.50$ ft^3/s Time = 20 sec		
	Unsteady	Steady	Difference	Unsteady	Steady	Difference
1	169.28	171.25	- 1.97	164.38	168.25	- 3.87
2	154.12	157.80	- 3.68	145.85	153.06	- 7.21
3	168.58	170.59	- 2.01	163.62	167.57	- 3.95
4	184.64	185.59	- 0.95	181.91	183.79	- 1.88
5	138.24	145.25	- 7.01	124.37	138.14	- 13.77
6	147.39	151.80	- 4.41	137.76	146.44	- 8.68
7	152.24	155.52	- 3.28	144.46	150.96	- 6.50
8	157.00	159.26	- 2.26	150.69	155.24	- 4.55
9	118.43	130.57	- 12.14	93.84	117.31	- 23.47
10	131.09	137.75	- 6.66	116.50	129.65	- 13.14
11	136.28	141.09	- 4.81	124.90	134.51	- 9.61
12	139.88	143.04	- 3.16	130.71	137.19	- 6.48
Averages			- 4.36			- 8.59

12.4 SEVERAL PUMPS SUPPLYING A PIPE LINE

A common network problem concerns several pumps that deliver discharges to a single pipeline. As an example, Fig. 12.3 shows two pumps. In general the number of pipes containing pumps is NPUMPS, and the number of pipes is NP = NPUMPS + 1 in the system. Often the supply source water surface elevations are identical for all pumps, but

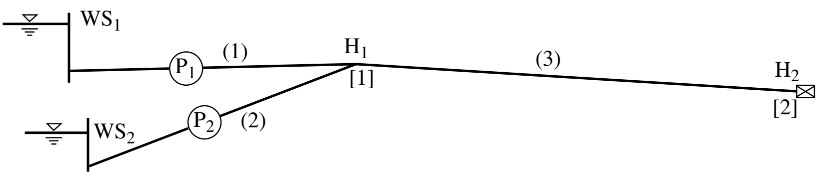


Figure 12.3 Multiple Pumps.

for generality each supply water surface elevation is denoted by WS_i, $i = 1, 2 \ldots$ NPUMPS, and each pipe containing a pump is also indexed by i. There will always be two nodes in such systems, node 1 where the pipes containing the pumps join with the downstream pipe, and node 2 at the other end of this pipe. The downstream pipe will be numbered NP; at its downstream end there will be a valve to control the discharge, or in its place there may be some other type of boundary condition. To allow several possibilities, we assume at node 2 that either the discharge or the head is specified as a function of time.

The equations to model the network behavior are therefore (1) one junction continuity equation at node 1 that indicates that the sum of the discharges in the pipes containing the pumps must equal the discharge in pipe NP, and (2) an ordinary differential equation for each of NP pipes. Thus the number of unknowns that can be found is NE = NP + 1. In the example we have 4 equations and 4 unknowns. If the discharge in pipe 3 is specified, then these unknowns will be Q_1, Q_2, H_1, and H_2. But if H_2 is specified, then the unknowns are Q_1, Q_2, Q_3, and H_1. The equations can be written as

$$
\begin{aligned}
F_1 &= \sum Q_i - Q_{NP} = 0 \qquad i = 1, \ldots, NPUMPS \\
F_{i+1} &= Q_i - Q_{ODE\,i} = 0 \qquad i = 1, \ldots, NP
\end{aligned}
\tag{12.12}
$$

in which Q_{ODEi} is obtained by solving the unsteady ODE with the discharge as the dependent variable. For the pipes containing the pumps this ODE will be

$$\frac{dQ_i}{dt} = gA_i \frac{WS_i + h_{pi} - H_1}{L_i} - \frac{\left(f_i L_i / D_i + K_e\right) Q_i |Q_i|}{2 D_i A_i} \tag{12.13}$$

in which the pump head can be given by the usual second-order polynomial equation. If this representation is used for h_p, then

$$h_{pi} = (A_i Q_i + B_i) Q_i + C_i \tag{12.14}$$

For the last pipe, numbered NP, the ODE is

$$\frac{dQ_{NP}}{dt} = gA_{NP} \frac{H_1 - H_2}{L_{NP}} - \frac{\left(f_{NP} L_{NP} / D_{NP}\right) Q_{NP} |Q_{NP}|}{2 D_{NP} A_{NP}} \tag{12.15}$$

In our 2-pump, 3-pipe system the equation system becomes

$$F_1 = Q_1 + Q_2 - Q_3 = 0 \tag{12.16a}$$

$$F_2 = Q_1 - Q_{ODE1} = 0, \qquad \frac{dQ_1}{dt} = gA_1 \frac{WS_1 + h_{p1} - H_1}{L_1} - \frac{\left(f_1 L_1 / D_1 + K_e\right) Q_1 |Q_1|}{2 D_1 A_1} \tag{12.16b}$$

$$F_3 = Q_2 - Q_{ODE2} = 0, \qquad \frac{dQ_2}{dt} = gA_2 \frac{WS_2 + h_{p2} - H_1}{L_2} - \frac{\left(f_2 L_2 / D_2 + K_e\right) Q_2 |Q_2|}{2 D_2 A_2} \tag{12.16c}$$

$$F_4 = Q_3 - Q_{ODE3} = 0, \qquad \frac{dQ_3}{dt} = gA_3 \frac{H_1 - H_2}{L_3} - \frac{\left(f_3 L_3 / D_3\right) Q_3 |Q_3|}{2 D_3 A_3} \tag{12.16d}$$

The program PUMPPAR on the CD solves problems of this type. As the listing shows, this program consists of the main program, a subroutine FUNCT which inserts equation values into array F when it is called, and a subroutine DQT that evaluates the derivative dQ/dt when it is called by the ODE solver. The program permits either the discharge or the head at the downstream node to be given as a function of time.

Example Problem 12.1

The flows from two pumps (with operating characteristics defined by tabular data which follow) are combined into a single pipe line, as diagrammed in Fig. 12.3. The supply water surface elevations are $WS_1 = 80$ m and $WS_2 = 70$ m; assume also $\nu = 1.31\times10^{-6}$ m^2/s and $e = 0.5$ mm. Additional pipe data are listed in the following table:

Pipe	1	2	3
L, m	500	400	2000
D, mm	400	400	500

Determine the unsteady discharges in the pipes if (a) the discharge in pipe 3 is given by

Time sec	0.5	1.0	2.0	3.0	4.0	5.0	7.5	10	15	20	25	30	35
Q_3 m^3/s	0.65	0.60	0.55	0.50	0.45	0.40	0.35	0.30	0.25	0.20	0.15	0.10	0.05

and (b) if the head at node 2 is given by

Time sec	0.5	1.0	2.0	3.0	4.0	5.0	7.5	10	15	20	25	30	35
H_2 m	70	80	90	100	100	100	90	80	70	60	60	60	60

Assume prior to time zero that the head at the downstream end of pipe 3 is 60 ft.

Pump 1		Pump 2	
Q, m^3/s	h_p, m	Q, m^3/s	h_p, m
0.25	25	0.25	30
0.35	23	0.35	28
0.45	20	0.45	25

We must first solve the steady-flow problem to provide the initial condition for the unsteady problem. This solution can be obtained from NETWK, which is $Q_1 = 0.334$ m^3/s, $Q_2 = 0.357$ m^3/s, $H_1 = 91.5$ m, and $H_2 = 60$ m. Fitting second-order polynomials to the pump data yields $h_{p1} = 25.625 + 10Q_1 - 50Q_1^2$, and $h_{p2} = 30.625 + 10Q_2 - 50Q_2^2$. The input data to solve part (a) of this problem using program PUMPPAR follows:

```
3 2 0.000001 0.0001 0.0005 9.81 1.31E-6
0.4 500
0.4 400
0.5 2000
- 50 10 25.625 80 0.334
- 50 10 30.625 70 0.357
91.5 60
1  (to indicate Q3 will be specified)
13 .5 .65 1 .6 2 .55 3 .5 4 .45 5 .4 7.5 .35 10 .3 15 .25 20 .2 25 .15 30 .1 35 .05
```

The solution for part (a) can be found in file EPB12_1.OU1 on the CD; we encourage the reader to run the program and compare results with this file.

For part (b) the input file is unchanged, except for the last two lines: a 2 will be given to indicate that H_2 is specified. Then the last two lines of the input file are

```
2
13 .5 70 1 80 2 90 3 100 4 100 5 100 7.5 90 10 80 15 70 20 60 25 60 30 60 35 60
```

The solution for part (b) can be found in file EPB12_1.OU2 on the CD.

* * *

12.5 AIR CHAMBERS, SURGE TANKS AND STANDPIPES

Air chambers, surge tanks and simple standpipes are all devices, basically tanks of various kinds, that are used in piping systems to protect the lines from extreme pressure surges which may be caused when a velocity quickly changes. When any appurtenances are a part of a network, the equations to describe them must be added to the set of continuity equations and unsteady equations of motion, and this enlarged equation system must then be solved. Two example problems which follow will illustrate the inclusion of these devices into the equation systems for the networks.

If the upper portion of the tank contains air under pressure, the tank is called an air chamber; if the top is open to the atmosphere, it is called a surge tank. As the water surface level rises in the closed air chamber, the air above the water is compressed. Consequently the head at node i that is connected to the air chamber has two components, the elevation of the water surface plus an additional water pressure head to represent the air pressure in excess of atmospheric pressure, or $H_i = \mathrm{WS}_i + \Delta p_{air}/\gamma_w$; here Δp_{air} is the air pressure above atmospheric pressure. (See also Section 13.2.6) In a tank of constant volume from which no air escapes, the air mass in the chamber is constant. This air undergoes a process that is likely to be somewhere between isothermal (constant temperature) and adiabatic (no energy transfer), and if a very small time interval is involved in the compression process, then the process will be essentially adiabatic so p/ρ^k = constant. Since the pressure in this equation is the absolute pressure, we compute Δp_{air} as $(\Delta p_{air} + p_{atm})/\rho^k = p_o/\rho_o{}^k$ = Constant (assuming the initial conditions p_o and ρ_o can be used to compute the constant for the adiabatic process), or

$$\Delta p_{air} = p_o\left(\frac{\rho}{\rho_o}\right)^k - p_{atm} \tag{12.17}$$

Since the density ρ is the ratio of mass M to volume $\forall$ we have $\rho = (M/\forall)_{air}$ with the air volume_

$$\forall_{air} = (\forall_{air})_o - \int Q\,dt \tag{12.18}$$

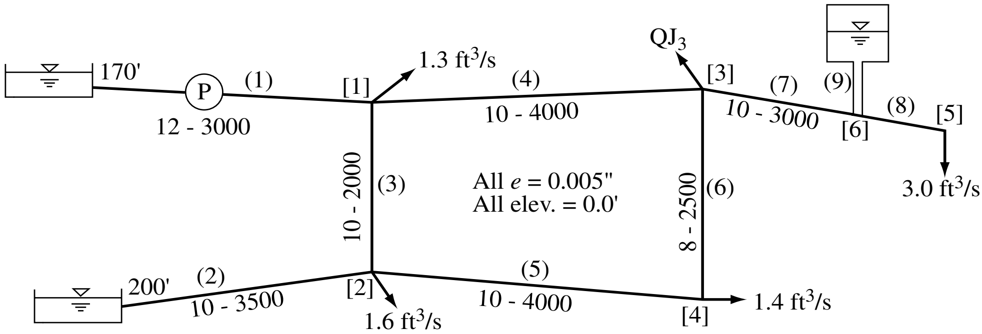
The air volume at any instant is the original air volume minus the increase in water volume $\int Qdt$ as the water flows into the air chamber. The magnitude of the initial air mass is found by first determining the pressure of the air; then its density is computed from the perfect gas law and other initial conditions as $\rho_o = p_o/(RT_o)$, followed by multiplying this density by the initial air volume in the tank, or

$$M = \rho_o(\forall_{air})_o = \rho_o(\forall_{total} - \forall_{water})_o \tag{12.19}$$

Many types of surge tanks exist, and they vary considerably in complexity. Some of the simplest tanks are basically a vertical standpipe, and these pipes may or may not contain an orifice constriction at the base where the device is connected to the network. We turn to two examples to see how air chambers and tanks are incorporated into a network model.

Example Problem 12.2

This network has an air chamber at the midpoint of pipe 7. To accommodate the air chamber, the original pipe 7 is divided into pipes 7 and 8, and the pipe from the connection point to the air chamber is called pipe 9. The tank volume is 100 ft^3 with a cross-sectional area of 10 ft^2, and initially half of the tank is filled with water so the initial water level $x_o = 5$ ft. The pipe connecting the surge tank to the network is 100 ft long and has a 6-in diameter. Data which describe the pump characteristic curve are presented in the table below the figure. At node 5 is a butterfly valve with discharge coefficient $c_D = c_o\exp(cD)$, in which D is the degree of opening (0^o is completely



Q, ft³/s	H, ft
5	50
6	45
7	39

closed and 90^o is fully open). Valve tests indicate that the ratio of c_D between $D = 80^o$ and $D = 10^o$ is 53.3. From an initial valve setting of 40^o at time $t = 0$, the valve is closed linearly at 7^o/sec for 5 sec and thereafter remains at the 5^o position. The demand (discharge) at node 3 is a linear function of the pressure head at this node; thus $QJ_3 = 0$ when $H_3 = 0$, and $QJ_3 = 1.5$ ft^3/s when H_3 is at its steady state value. We seek to simulate the unsteady flow in the network for 10 sec, according to rigid-column theory, to find the discharge in each pipe and the head at each node as a function of time.

The solution process begins with the determination of the initial flow state; NETWK can be used to generate these steady-flow data. There are 17 unknowns in this problem; they are the 9 discharges in the pipes, the nodal heads at the original 5 nodes, the head at node 6 where the air-chamber connector pipe is linked to pipe 7, the head H_7 in the air chamber itself, and finally the unknown demand QJ_3 at node 3. The program SURGNET on the CD will solve this type of problem. Examine its listing. In this program the array of unknowns X( ) is indexed in the following sequence: $1 = Q_1$, $2 = Q_2 \ldots 9 = Q_9$, $10 = H_1$, $11 = H_2$, $12 = H_3$, $13 = H_4$, $14 = H_5$, $15 = H_6$, $16 = H_7$, $17 = QJ_3$. The set of 17 equations to be solved follows.

$$F_i = \sum Q_j - QJ_i = 0 \quad i = 1, 2, 3, 4, 6$$

The sums in these junction continuity equations range over the pipes that are connected to node i. The demand vs. head relation at node 3 must express the nodal demand there as being linearly proportional to the pressure head, as the problem specifies. Thus we write

$$F_5 = QJ_3 - c_3(H_3 - Elev_3) = X(17) - 0.0123H_3 = 0$$

in which the constant $c_3 = 1.5/121.76 = 0.0123$; at steady state the demand is 1.5 ft^3/s, and the associated steady state head, from the input data list which follows, is 121.76 ft. In a similar way the continuity equation for node 5 with the butterfly valve is written

$$F_7 = Q_8 - c_o e^{cD}(H_5 - Elev_5)^{1/2} = Q_8 - 0.0324e^{0.0568D}H_5^{1/2}$$

In these two equations $Elev_3 = Elev_5 = 0.0$, as the diagram indicates. The next 9 equations are the ODEs that describe the rigid-column pipe transients, equation numbers $i = 8, 9, \ldots, 15, 16$; pipe numbers $k = 1, 2, \ldots, 9$:

$$F_i = Q_{ODE\,k} - Q_k = 0 \qquad \frac{dQ_k}{dt} = gA_k \frac{H_u - H_d}{L_k} - \frac{f_k Q_k |Q_k|}{2D_k A_k}$$

In these equations H_u is the upstream head, including any additional head from a pump, and H_d is the downstream head. The final equation models the air chamber:

$$F_{17} = H_7 - (x - x_o) - \left[p_o \left(\frac{M}{\rho_o \left\{ (\forall_{air})_o - \int Q_7 dt \right\}} \right)^k - p_{atm} \right] / \gamma_w = 0$$

The input data to SURGNET consists of

2 3 0.0000001 0.001 1 10 0.000417 32.2 1.217e-5 5 60 10 100 50

which is entered from the keyboard, and the input file SURGNET.DAT that can be found on the CD. The remaining data are

```
3000 1.0   5.9
3500 0.833 2.9
2000 0.833 1.3
4000 0.833 3.3
4000 0.833 2.6
2500 0.667 1.2
1500 0.833 3.0
1500 0.833 3.0
100  0.5   0.0
1.5 170.71
1.6 166.55
1.5 121.76
1.4 135.47
3.0  91.15
170 200 -0.5 0.5 60
```

The resulting solution can be found on the CD in file EPB12_2.OUT. That file contains a sequential listing of the 17 unknown variables at intervals of one second.

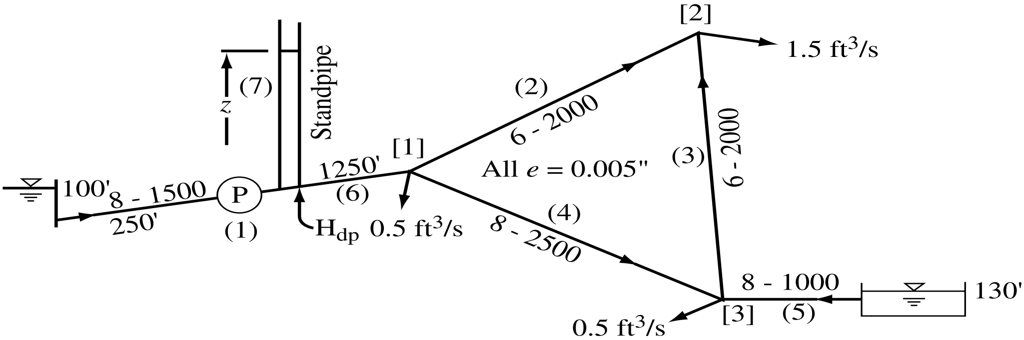
* * *

Example Problem 12.3

A five-pipe network, shown atop the next page, contains a standpipe downstream from the pump in pipe 1. The orifice diameter at the base of this standpipe is 2 in (assuming a contraction coefficient of unity), and the standpipe has a diameter of 1 ft. Initially the system is in steady-state operation, when the demand at node 2 is reduced to zero in 7 seconds according to the following schedule:

Time, sec	1.0	2.0	3.0	4.0	5.0	6.0	7.0
QJ_2, ft^3/s	1.25	1.0	0.8	0.6	0.4	0.2	0.0

Applying rigid-column theory, develop the equation system that describes this network, and then simulate the performance of the network over a 10-sec time period.



Pump data

Q, ft³/s	1.5	2.0	2.5
h_p, ft	50	48	45

Without the standpipe there are three interior nodes in the network, a total of five pipe discharges, and a need for eight equations to describe the network operation. To model the addition of the standpipe we must divide pipe 1 at the standpipe location into pipes 1 and 6, as the figure shows. The standpipe itself becomes pipe 7. To describe this modified system we now need 4 junction continuity equations and 7 rigid-column unsteady flow ODEs for the 7 pipes. In addition, the water surface elevation z in the standpipe becomes another variable. If the nodal demands are all regarded as known, then the list of unknowns becomes Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, Q_7, z, H_1, H_2, H_3, and H_{dp}. This last variable is the head just downstream from pump at the new standpipe node. The full equation set takes the following form:

$$F_1 = Q_6 - Q_2 - Q_4 - QJ_1 = 0$$

$$F_2 = Q_2 + Q_3 - QJ_2 = 0$$

$$F_3 = Q_5 + Q_4 - Q_3 - QJ_3 = 0$$

$$F_4 = Q_1 - Q_6 - Q_7 = 0$$

$$F_5 = Q_1 - Q_{ODE1} = 0, \qquad \frac{dQ_1}{dt} = gA_1 \frac{WS_1 + h_p - H_{dp}}{L_1} - \frac{f_1 |Q_1| Q_1}{2 D_1 A_1}$$

$$F_6 = Q_2 - Q_{ODE2} = 0, \qquad \frac{dQ_2}{dt} = gA_2 \frac{H_1 - H_2}{L_2} - \frac{f_2 |Q_2| Q_2}{2 D_2 A_2}$$

$$F_7 = Q_3 - Q_{ODE3} = 0, \qquad \frac{dQ_3}{dt} = gA_3 \frac{H_3 - H_2}{L_3} - \frac{f_3 |Q_3| Q_3}{2 D_3 A_3}$$

$$F_8 = Q_4 - Q_{ODE4} = 0, \qquad \frac{dQ_4}{dt} = gA_4 \frac{H_1 - H_3}{L_4} - \frac{f_4 |Q_4| Q_4}{2 D_4 A_4}$$

$$F_9 = Q_5 - Q_{ODE5} = 0, \qquad \frac{dQ_5}{dt} = gA_5 \frac{WS_2 - H_3}{L_5} - \frac{f_5 |Q_5| Q_5}{2 D_5 A_5}$$

$$F_{10} = Q_6 - Q_{ODE6} = 0, \qquad \frac{dQ_6}{dt} = gA_6 \frac{H_{dp} - H_1}{L_6} - \frac{f_6 |Q_6| Q_6}{2 D_6 A_6}$$

$$F_{11} = Q_7 - Q_{ODE7} = 0, \qquad \frac{dQ_7}{dt} = gA_7 \frac{H_{dp} - z}{z} - \left[\frac{f_7}{2 D_7 A_7} + \left(\frac{A_7}{A_o} \right)^2 \right] |Q_7| Q_7$$

$$F_{12} = z - z_{ODE} = 0, \qquad \frac{dz}{dt} = \frac{Q_7}{A_7}$$

In equation F_{11} $A_7 = \pi D_7^2/4$ is the cross-sectional area of the standpipe, and A_o is the area of the orifice at the base of the standpipe. In the equation called F_5 the substitution for pump head $h_p = AQ_1^2 + BQ_1 + C$ must be made; otherwise the equation $F_{13} = h_p - AQ_1^2 - BQ_1 - C = 0$ must be added to the equation set, and h_p must be added to the list of unknowns.

The solution relies on the program PIPSTAND on the CD, which is written specifically to solve this problem. It calls on DVERK to solve eight ODEs that are part of the combined system of algebraic and differential equations that describe the behavior of the 12 unknowns in this system. The subroutine DEFFUN returns values for the 12 equations F_1, F_2, ... F_{12}, when it is called, and the main program then obtains the solution by applying the Newton method. The array X contains the unknowns listed in the sequence given in the earlier paragraph. The input to PIPSTAND to solve this problem consists of

```
2  3  0.000001  0.001  1  10  0.000417  32.2  1.217e-5  0.1667
```

from the keyboard, and the following data from an input file (PIPSTAND.IN on the CD):

```
250 0.667 1.81
2000 0.5 0.78
2000 0.5 0.72
2500 0.667 0.53
1000 0.667 0.69
1250 0.667 1.81
1.0 1.0 0.0
0.5 130.97
1.5 110.49
0.5 128.08
100.0 130.0 -2.0 3.0 50.0 145.88
```

The discharges in the pipes, given as the last item on the first seven lines of this file, were obtained with the nodal heads from the steady-state solution of the original network before pipes 6 and 7 were added. (The length of the standpipe, pipe 7, is given as 1.0 since in the program GA(7) = G*AA/L(7), and the varying length z is the length of the fluid in this pipe.) The nodal heads are the second values on the next three lines for the three original nodes. The last line contains the two water surface elevations, the three coefficients that describe the polynomial pump curve, and finally the head (145.88) at the standpipe location. This value is obtained from the steady-state solution by apportioning the head loss in pipe 1 along its length.

The solution, with the output re-organized into two tables for easier review (or as given by program PIPSTAND), follows.

Pipe Discharges

Time sec	QJ_2 ft^3/s	Q_1 ft^3/s	Q_2 ft^3/s	Q_3 ft^3/s	Q_4 ft^3/s	Q_5 ft^3/s	Q_6 ft^3/s	Q_7 ft^3/s
1.0	1.25	1.719	0.659	0.591	0.542	0.549	1.701	0.018
2.0	1.0	1.617	0.540	0.460	0.557	0.403	1.597	0.021
3.0	0.8	1.539	0.446	0.354	0.572	0.282	1.518	0.021
4.0	0.6	1.464	0.354	0.246	0.588	0.158	1.442	0.022
5.0	0.4	1.393	0.263	0.137	0.607	0.030	1.370	0.023
6.0	0.2	1.323	0.173	0.027	0.626	- 0.100	1.300	0.024
7.0	0.0	1.257	0.086	- 0.086	0.647	- 0.233	1.233	0.024
8.0	0.0	1.273	0.095	- 0.095	0.659	- 0.254	1.254	0.019
9.0	0.0	1.293	0.105	- 0.105	0.670	- 0.275	1.275	0.018
10.0	0.0	1.311	0.114	- 0.114	0.680	- 0.294	1.294	0.018

Heads

Time sec	QJ_2 ft^3/s	z ft	H_1 ft	H_2 ft	H_3 ft	H_{dp} ft
1.0	1.25	145.90	146.48	167.38	140.99	148.29
2.0	1.0	145.93	148.41	173.89	142.03	149.18
3.0	0.8	145.96	146.88	168.08	140.30	149.23
4.0	0.6	145.98	147.96	171.53	140.83	149.60
5.0	0.4	146.01	148.93	174.25	141.27	149.93
6.0	0.2	146.04	149.76	176.22	141.59	150.22
7.0	0.0	146.07	150.53	177.56	141.95	150.49
8.0	0.0	146.10	139.13	135.67	132.21	148.74
9.0	0.0	146.12	138.91	135.52	132.12	148.59
10.0	0.0	146.14	138.77	135.41	132.05	148.53

* * *

12.6 A FULLY TRANSIENT NETWORK ANALYSIS

12.6.1. THE INITIAL STEADY STATE SOLUTION

A steady-state solution for a network must be available before a transient (water hammer) analysis of the network can be conducted. Herein these steady-state solutions will be obtained from NETWK, since it can produce a file with the information that is needed by the transient network analysis program TRANSNET. The use of TRANSNET will be described and illustrated in subsequent pages. By setting the option NETPLT=4 (as is done when NETWK is asked to write an input file for UNSTPIP), a file will be written that will enable TRANSNET to know the physical configuration of the network and a suitable set of initial conditions. The next subsection describes the use of program TRANSNET. In this section the use of NETWK to obtain the steady-state solution is described.

To ensure that the file written by NETWK for TRANSNET will contain the correct information, it is necessary (in addition to setting the option NETPLT=4) to set two additional options, NODESP=1 and PCHAR3=0; the first of these assigns node numbers to reservoirs and source pumps, and the second option allows more than three pairs of (Q, h_p) data to be used to define pump characteristics. Program TRANSNET defines pump curves by piecewise linear segments, as illustrated in Section 11.1, and requires six pairs of points. Therefore each pump curve must be defined by supplying six pairs of (Q, h_p) values; the first pair should contain $Q = 0$, and the last pair should contain $h_p = 0$ so the entire pump curve is defined. Booster pump stations, pressure reduction valves, back pressure valves, check valves and some similar appurtenances are not currently accommodated by program TRANSNET; when used to write a file for TRANSNET, the commands to NETWK should be limited to PIPES, NODES, PIPE-, RESER, PUMPS, RUN and END. Furthermore, with current dimensions a maximum of four pipes may join at a node. Under the NODES command the listing of nodes for source pumps should follow real nodes, and nodes assigned to reservoirs should follow source pump nodes.

Since the file written for TRANSNET is different from a file written for UNSTPIP, NETWK will ask which unsteady program will use the file when the option NETPLT=4 is set. Upon selecting 2 for TRANSNET and providing a file name, the user will be asked for the following additional information that is needed for a transient network analysis:

(a) the wave speed in the pipes of the network, and

(b) the following two lines of information for each pump station:

On line one, (1) the number of stages (pumps in series), (2) the number of pumps in parallel, (3) the rotational speed in rev/min for which the pump curve is defined, and (4) the rotational moment of inertia in units of force times length squared (lb-ft^2 for ES units, kN-m^2 for SI units) for each pump stage.

On line two, a list of six values for power (horsepower for ES units, kW for SI units), corresponding to the six (Q, h_p) data pairs that define that pump's characteristic curve.

This additional information that is needed by TRANSNET can be provided in any of three ways: (1) after the RUN (or END) command in the same file provided to NETWK to obtain the steady-state solution, (2) in a separate file, or (3) from the keyboard during execution of NETWK. NETWK will prompt the user to learn which option is to be used to enter the additional information. For example, if 3 is specified, meaning that the keyboard will be used, then a prompt will tell the user what is expected next.

If this information is to be in a file, then the data must be sequenced:

First: One line that contains the wave speeds, formatted as pipe numbers and their wave speeds in pairs, or a range of pipe numbers and the wave speed for this range. For example,

1 3000 2-8 2800 9 2500

assigns a wave speed of 3000 to pipe 1, a wave speed of 2800 to pipes 2 through 8, and a wave speed of 2500 to pipe 9. The pipes with numbers that are not included in the list will be given the wave speed that was assigned to the previous pipe in the list; if there are 12 pipes in the example, then pipes 10, 11 and 12 will be assigned a wave speed of 2500. NETWK contains the default wave speed of 3000 ft/s; if pipes 1 through 5, for example, are not assigned a wave speed, then their wave speed will default to 3000. There is considerable flexibility in providing this wave speed data, since any pipes without assigned wave speeds are given the wave speed of the previous pipe. A comma can be used in place of a blank in any of the lists, and blanks may follow commas. However, both end values in a range of pipes must be specified.

Second: Two lines for each pump, in the same order as the pumps are listed in the input data file, following the instructions for item (b) above.

The program NETWK allows composite pump curves for pumps in series or pumps in parallel to be given with the data pairs rather than using the SERIES and/or PARALLEL commands, and the number of pumps in series or parallel need not be integer values, thus allowing non-identical pumps to be lumped at a pumping station. The user can apply this same treatment of pumps when preparing a file for TRANSNET. However, if data are presented for a composite curve in place of a single pump stage, then one must place a minus sign before the number of stages and/or the number of pumps in parallel; then the discharges and heads will be adjusted before the file for TRANSNET is written. In other words, the final input file for TRANSNET must contain the discharge, head and rotational moment of inertia for one stage of a single pump.

This input is illustrated in the next section as three example network problems are described, analyzed by using NETWK to obtain steady-state solutions, and then transient analyses are performed by using the file written by NETWK to drive TRANSNET.

12.6.2. TRANSNET

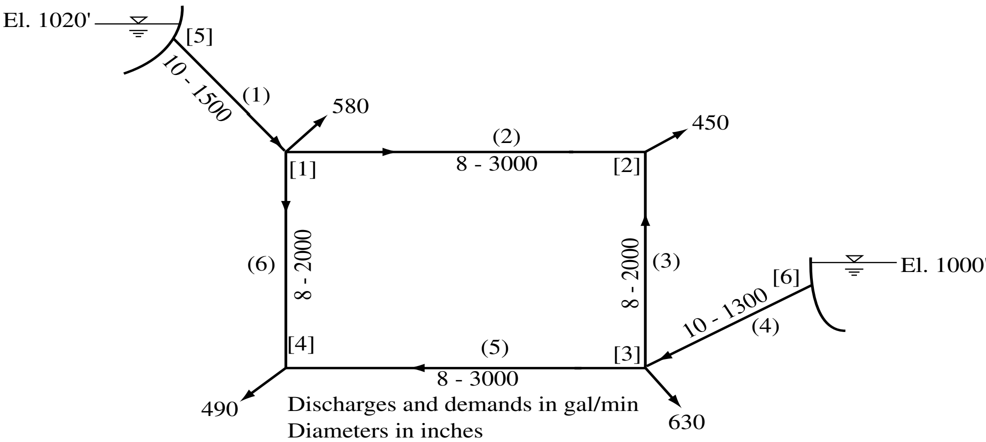
The transient program TRANSNET permits the user to investigate the effects of several events that can cause rapid, possibly severe, transients. The equations that are solved for each pipe of the network are the equations that have been developed in Chapters 7-9 and applied to individual pipes and smaller systems in Chapters 8-11. Power failure at any number of pump stations can be simulated. The effects of sudden valve closure at either end of any number of pipes can be investigated. Staged valve closure at the downstream end of any pipe can be specified. The consequences of sudden demand changes at any number of junctions can also be studied.

Most of the data that are required to initiate the study of a transient is stored in the data file which is created during the execution of NETWK with the option NETPLT=4, as described in the previous section. The only additional data is the specification of the transient-causing event. If the assumed flow direction in any pipe is found to be incorrect, NETWK corrects this direction before writing the file for TRANSNET; the investigator should also change these flow directions on a diagram of the network. The example problems will illustrate these procedures.

A description of the input data parameters which determine the transient behavior is included at the beginning of the source listing of TRANSNET; before reading further, a listing of TRANSNET should be obtained from the CD so it can be studied.

Example Problem 12.4

This network is supplied by gravity flow from two elevated reservoirs. Pipe and node numbers are shown in the diagram with the diameters and lengths of the pipes. Nodes 1 through 4 have ground elevations of 860 ft, while nodes 5 and 6 have ground elevations of 980 ft and 960 ft, respectively. The equivalent sand roughness of all pipes is $e = 0.002$ in. The demand is increased from 450 gal/min to 900 gal/min at node 2 to meet a sudden need for more water for fire suppression. Determine the effect of this increase in demand on the heads at nodes 1, 2, and 4.



In this analysis the first step is to determine the steady-state solution which will define the initial conditions for the ensuing transient. Two alternative input data files have been prepared for NETWK to use in obtaining this steady-state solution. The option OUTPU1=4 tells NETWK to provide the values of friction factors, rather than *e*'s, in the PIPE DATA table. One input file is listed below, and both input files are available on the CD under the names EPB12_4.IN and EPB12_4.IN1.

```
Example Problem 12.4
/*
$SPECIF NETPLT=4,NFLOW=1,OUTPU1=4
 COEFRO=0.002,NODESP=1 $END
PIPE-
1 10.0 1500. 5 980. 1 580. 860.
2  8.0 3000. 1 2 450. 860.
3  8.0 2000. 3 2
4 10.0 1300. 6 960. 3 630. 860.
5  8.0 3000. 3 4 490. 860.
6  8.0 2000. 1 4
RESER
5 1020
6 1000
RUN
1 3000/  Assigns a wave speed of 3000 to all pipes.
```

Since additional data are provided after the RUN command, NETWK will use a prompt to learn where the wave speed information is to be found; the user should select 1, meaning "in the same file after the other data." The steady-state solution found by NETWK is listed in the following two tables:

PIPE DATA

PIPE NO.	NODES FROM	NODES TO	L ft.	DIA. in	f	Q gal/min	VEL. ft/s	HEAD LOSS ft.	HLOSS/ 1000
1	5	1	1500	10.0	0.01598	1447	5.91	15.61	10.41
2	1	2	3000	8.0	0.01877	389	2.48	8.08	2.69
3	3	2	2000	8.0	0.02693	61.2	0.39	0.19	0.10
4	6	3	1300	10.0	0.01750	703	2.87	3.49	2.69
5	3	4	3000	8.0	0.04171	11.7	0.07	0.02	0.01
6	1	4	2000	8.0	0.01820	478	3.05	7.90	3.95

NODE DATA

NODE	DEMAND ft^3/s	DEMAND gal/min	ELEV. ft.	HEAD ft.	PRESSURE lb/in^2	HGL ELEV. ft.
1	1.29	580	860.	144.39	62.57	1004.4
2	1.00	450	860.	136.31	59.07	996.3
3	1.40	630	860.	136.51	59.15	996.5
4	1.09	490	860.	136.49	59.15	996.5
5	- 3.22	- 1447	980.	40.00	17.33	1020.0
6	- 1.57	- 703	960.	40.00	17.33	1000.0

Now the execution of the program TRANSNET can proceed. Two files are needed as input to TRANSNET, the file written by NETWK and a file that describes the transient analysis to be done. The file written by NETWK is currently unformatted, so it can not easily be examined. The reader should have the experience of having NETWK produce this file, but the CD also contains this unformatted information in file EPB12_4.OU1. The following file contains the transient analysis data (on the CD under the name EPT12_4.DAT):

```
DEMONSTRATION OF PROGRAM TRANSNET - INPUT DATA FILE "EPB12_4.DAT"
DEMAND AT JUNCTION 2 IS INCREASED FROM 450 TO 900 GAL/MIN
&SPECS NPARTS=4,IOUT=1000,NQNEW=1,HATM=28.,TMAX=60.,GRAPH=T,
        NODEQ(1)=2,QNEW(1)=900./
&GRAF NSAVE=3,IOUTSA=2,PIPE=1,2,6,0,NODE=999,999,999,0/
```

The output file written by TRANSNET (also on the CD in file EPB12_4.OUT with the plot file EPB12_4.PLT) follows:

```
                    ******************************
                    * NETWORK TRANSIENT ANALYSIS *
                    ******************************

  DEMONSTRATION OF PROGRAM TRANSNET - INPUT DATA FILE "EPB12_4.DAT"
  DEMAND AT JUNCTION 2 IS INCREASED FROM 450 TO 900 GAL/MIN

        IOUT  = 1000
      NPARTS =     4
       NPIPES =    6

       HATM  = 28.0 FT

       TMAX  =  60.0 SEC
       DELT  =  0.10 SEC
```

TRANSIENT CONDITIONS IMPOSED

DEMAND DISCHARGES SUDDENLY CHANGED AT NODE 2 TO 900.0 GAL/MIN

PIPE INPUT DATA

PIPE	DIAMETER in	LENGTH ft	WAVE SPEED ft/s	PIPEZ ft	f	VELOCITY ft/s
1	10.00	1500.	3000.	980.	0.0160	5.89
2	8.00	3000.	3000.	860.	0.0188	2.47
3	8.00	2000.	3000.	860.	0.0267	0.41
4	10.00	1200.	3000.	980.	0.0175	2.90
5	8.00	3000.	3000.	860.	0.0389	0.10
6	8.00	2000.	3000.	860.	0.0182	3.03

PIPE	DELTA sec	PARTS	SINE	L/A sec	INTERPOLATION
1	0.124	4	- 0.0800	0.50	0.202
2	0.249	10	0.0000	1.00	0.003
3	0.167	6	0.0000	0.67	0.103
4	0.100	4	- 0.1000	0.40	0.003
5	0.250	10	0.0000	1.00	0.003
6	0.166	6	0.0000	0.67	0.103

NODE INPUT DATA

NODE	HGL ft	GRND EL ft	PIPES 1	PIPES 2	PIPES 3	PIPES 4	DEMAND ft^3/s
1	1004.51	860.0	- 1	2	6	0	1.29
2	996.52	860.0	- 2	- 3	0	0	2.00
3	996.73	860.0	3	- 4	5	0	1.40
4	996.70	860.0	- 5	- 6	0	0	1.09
5	1020.00	980.0	1	0	0	0	- 3.21
6	1000.00	980.0	4	0	0	0	- 1.58

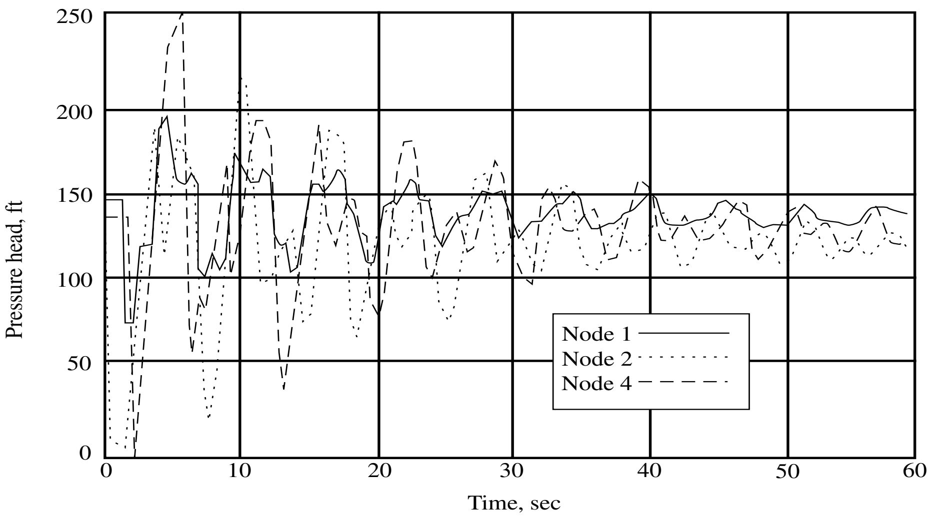
****
* TABLE OF EXTREME VALUES *
****

	X	MAX HEAD ft	TIME sec	MIN HEAD ft	TIME sec	MAX H ft	MIN H ft
PIPE 1							
	0.000	40.0	59.9	40.0	59.9	1020.	1020.
	1.000	200.0	3.9	71.0	1.8	1060.	931.
PIPE 2							
	0.000	200.0	3.9	71.0	1.8	1060.	931.
	1.000	233.9	9.7	- 1.8	1.2	1094.	858.
PIPE 3							
	0.000	190.8	4.2	61.5	1.4	1051.	921.
	1.000	233.9	9.7	- 1.8	1.2	1094.	858.
PIPE 4							
	0.000	20.0	59.9	20.0	59.9	1000.	1000.
	1.000	190.8	4.2	61.5	1.4	1051.	921.
PIPE 5							
	0.000	190.8	4.2	61.5	1.4	1051.	921.
	1.000	259.8	4.6	- 9.5	2.4	1120.	850.
PIPE 6							
	0.000	200.0	3.9	71.0	1.8	1060.	931.
	1.000	259.8	4.6	- 9.5	2.4	1120.	850.

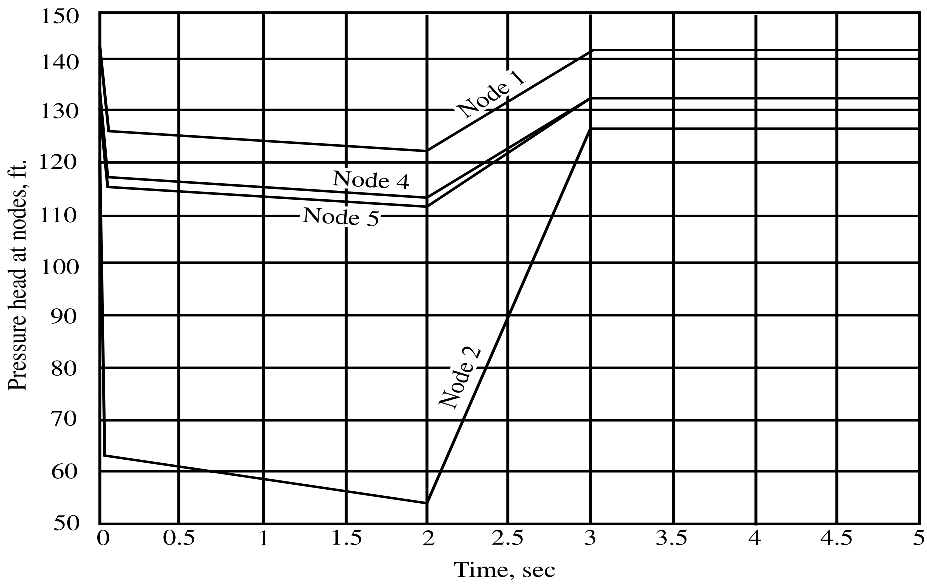
MAXIMUM HEAD = 268.1 FT IN PIPE 5 AT X = 0.800 AT TIME = 4.59 SEC
MINIMUM HEAD = - 12.5 FT IN PIPE 4 AT X = 0.250 AT TIME = 1.10 SEC

Since plot information was requested by setting GRAPH=T (for true), another output file is written by TRANSNET that contains data for the transient pressure heads at nodes 1, 2, and 4. The plot of these data, shown below, indicates how the pressure waves decay with time. From the extreme value table we note that neither the highest nor lowest pressures occur at the node where the demand changes; instead they occur near node 4. Can the reader explain why this occurs?

An unsteady solution that ignores elastic effects will now also be obtained. The input file is again provided to NETWK, but now select 1 when asked whether a file for 1. UNSTPIP or 2. TRANSNET should be written. Even when elastic effects are ignored, the demand clearly can not be increased instantly from 450 to 900 gal/min, so let us assume the increase occurs linearly over 2 sec. We encourage the reader to prepare the input for UNSTPIP and obtain this solution. The following graph presents some of the solution. We see that the pressure head at node 2 becomes smallest, at 53.7 ft, after 2 sec when the demand at this node has just become 900 gal/min and the increase in demand ceases. Note also from this solution, after the demand becomes constant, how rapidly the pressure heads approach the new steady-state conditions with a pressure head at node 2 of 127.8 ft, which is 8.5 ft less than the head for QJ_2 = 450 gal/min. One could solve again this problem (we encourage the reader to do so) with QJ_2 = 900 gal/min



and find after only 5 sec that the nodal pressure heads are essentially identical to these new values. If the demand were to increase more rapidly, then the pressure at node 2 would decrease still further. If one tries to double the demand in 1.5 or 1.0 sec, negative pressures will occur at node 2, which is obviously not physically possible. A more

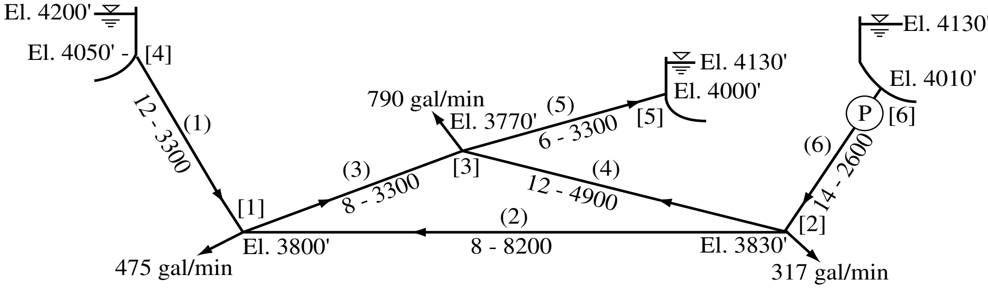


realistic problem description would very rapidly reduce the pressure head at node 2 to zero; then one could determine from the solution the time that is required to increase the demand at this node to 900 gal/min. Therefore rigid-column unsteady solutions should not be used to study rapid changes; that approach is more appropriate to the study of gradual changes that could occur continually throughout a network.

* * *

Example Problem 12.5

Here we see how a pump is incorporated into a network. A source pump (the operating characteristic data are listed in the input file for NETWK) is located at one of the three reservoirs. The Hazen-Williams roughness coefficient is $C_{HW} = 120$ for all pipes. This network experiences a transient that is caused by the sudden closure of a valve at the downstream end of pipe 5. Obtain a transient analysis of this network if the wave speed is 2850 ft/s for all pipes.



To begin the solution process, we have chosen to supply the following input data file (EPB12_5.IN or EPB12_5.IN1 on the CD) to NETWK to obtain the initial condition for the ensuing transient analysis:

```
Example Problem 12.5
/*
$SPECIF NETPLT=4, NFLOW=1, NPGPM=1, OUTPU1=4, NODESP=1, COEFRO=120
 PCHAR3=0 $END
PIPE-
1 12. 3300. 4 4050. 1 475. 3800.
2 8. 8200. 2 317. 3830. 1
3 8. 3300. 1 3 790. 3770.
4 12. 4900. 2 3
5 6. 3300. 3 5 4000.
6 14. 2600. 6 4010. 2
RESER
4 4200
5 4130
PUMPS
6 0 118 2000 92 3000 82 4000 67 4500 52 5300 0 4130/
RUN
1 2850
1 1 1180 50
57 68 77 80 76 60
```

In this input file the option PCHAR3=0 is set so that six (Q, h_p) pairs can be entered to define the pump curve. Additional data needed by TRANSNET are provided after the RUN command. The first line following RUN indicates that all pipes have a wave speed of 2850 ft/s; the second line indicates one pump stage at this station, one pump in parallel with a rotational speed of 1180 rev/min and a rotational moment of inertia of 50 lb-ft^2; the third line lists the six horsepower values corresponding to the six discharge-head pairs provided under the PUMPS command.

The steady-state solution from NETWK is described in the following two tables (and listed in file EPB12_5.OUT on the CD):

PIPE DATA

PIPE NO.	NODES FROM	TO	L ft.	DIA. in	C_{HW}	Q gal/min	VEL. ft/s	HEAD LOSS ft.	HLOSS /1000
1	4	1	3300	12.0	120	340.1	0.96	1.32	0.40
2	2	1	8200	8.0	120	273.0	1.74	15.70	1.91
3	1	3	3300	8.0	120	138.1	0.88	1.79	0.54
4	2	3	4900	12.0	120	1110.0	3.15	17.49	3.57
*5	3	5	3300	6.0	120	458.1	5.20	66.89	20.27
6	6	2	2600	14.0	120	1700.0	3.54	9.64	3.71

NODE DATA

NODE	DEMAND ft^3/s	DEMAND gal/min	ELEV. ft.	HEAD ft.	PRESSURE lb/in^2	HGL ELEV. ft.
1	1.06	475	3800.	398.68	172.76	4198.68
2	0.71	317	3830.	384.38	166.57	4214.38
3	1.76	790	3770.	426.89	184.99	4196.89
6	- 3.79	- 1700	4010.	214.03	92.74	4224.03
4	- 0.76	- 340	4050.	150.00	65.00	4200.00
5	1.02	458	4000.	130.00	56.33	4130.00

The supplemental input file (EPB12_5.DAT on the CD) for TRANSNET can take the following form:

```
DEMONSTRATION OF PROGRAM TRANSNET - INPUT DATA FILE "EPB12_5.DAT"
NETWORK EXAMPLE 12.5 -  SUDDENLY-CLOSED VALVE AT THE DS END OF PIPE  5
&SPECS NPARTS=4, IOUT=100, NSHUT=1, HATM=30., TMAX=20.0, ALLOUT=T,
          HVPRNT=T, ISHUT(1)=-5/
&GRAF  NSAVE=4,  IOUTSA=1,  PIPE=5,5,1,6,NODE=999,1,999,999/
```

Together with the file written by NETWK, this file can be executed by TRANSNET to produce the transient solution. Only part of the output file (see EPB12_5.OU1) is presented here:

```
*******************************
* NETWORK TRANSIENT ANALYSIS *
*******************************

DEMONSTRATION OF PROGRAM NO. 6 -INPUT DATA FILE "EPB12_5.DAT"
NETWORK EXAMPLE 12.5 - SUDDENLY-CLOSED VALVE AT THE DS END OF PIPE 5

     IOUT  =  100
   NPARTS =      4
    NPIPES =      6

    HATM  =  30.0  FT

    TMAX  =  20.00  SEC
     DELT  =   0.227 SEC
```

TRANSIENT CONDITIONS IMPOSED

SUDDENLY CLOSED VALVE AT DOWNSTREAM END OF PIPE 5

PIPE INPUT DATA

PIPE	DIAMETER in	LENGTH ft	WAVE SPEED ft/s	PIPEZ ft	C_{HW}	VELOCITY ft/s
1	12.00	3300.	2850.	4050.	120.	0.97
2	8.00	8200.	2850.	3830.	120.	1.74
3	8.00	3300.	2850.	3800.	120.	0.88
4	12.00	4900.	2850.	3830.	120.	3.15
5	6.00	3300.	2850.	3770.	120.	5.20
6	14.00	2600.	2850.	4010.	120.	3.54

PIPE	DELTA sec	PARTS	SINE	L/A sec	INTERPOLATION
1	0.289	5	- 0.07576	1.16	0.019
2	0.718	12	- 0.00366	2.88	0.052
3	0.289	5	- 0.00909	1.16	0.019
4	0.428	7	- 0.01224	1.72	0.075
5	0.288	5	0.06970	1.16	0.019
6	0.227	4	- 0.06923	0.91	0.004

NODE INPUT DATA

NODE	HGL ft	GRND EL ft	PIPES 1	PIPES 2	PIPES 3	PIPES 4	DEMAND ft^3/s
1	4198.68	3800.0	- 1	- 2	3	0	1.06
2	4214.38	3830.0	2	4	- 6	0	0.71
3	4196.89	3770.0	- 3	- 4	5	0	1.76
6	4224.03	4010.0	6	0	0	0	- 3.79
4	4200.00	4050.0	1	0	0	0	- 0.76
5	4130.00	4000.0	- 5	0	0	0	1.02

PUMP INFORMATION

LINE = 6
PUMPS = 1
STAGES = 1
RPM = 1180. RPM
SUMP ELEV = 4130. FT
WRSQ = 50. LB-FTSQ

Q gal/min	HEAD/STAGE ft	HP/STAGE HP
0.0	118.0	57.0
2000.8	92.0	68.0
3001.1	82.0	77.0
4001.5	67.0	80.0
4501.7	52.0	76.0
5302.0	0.0	60.0

PRESSURE HEADS, HGL'S AND VELOCITIES AS FUNCTIONS OF TIME

	X/L	HEAD ft	HGL ft	VEL ft/s	X/L	HEAD ft	HGL ft	VEL ft/s
TIME = 0.000 SEC								
PIPE 1	0.00	150.	4200.	0.97	0.20	200.	4200.	0.97
	0.40	249.	4199.	0.97	0.60	299.	4199.	0.97
	0.80	349.	4199.	0.97	1.00	399.	4199.	0.97
PIPE 2	0.00	384.	4214.	1.74	0.08	386.	4213.	1.74
	0.17	387.	4212.	1.74	0.25	388.	4210.	1.74
	0.33	389.	4209.	1.74	0.42	390.	4208.	1.74
	0.50	391.	4206.	1.74	0.58	393.	4205.	1.74
	0.67	394.	4204.	1.74	0.75	395.	4202.	1.74
	0.83	396.	4201.	1.74	0.92	397.	4200.	1.74
	1.00	398.	4198.	1.74				
PIPE 3	0.00	399.	4199.	0.88	0.20	404.	4198.	0.88
	0.40	410.	4198.	0.88	0.60	416.	4198.	0.88
	0.80	421.	4197.	0.88	1.00	427.	4197.	0.88
PIPE 4	0.00	384.	4214.	3.15	0.14	390.	4212.	3.15
	0.29	396.	4209.	3.15	0.43	403.	4207.	3.15
	0.57	409.	4204.	3.15	0.71	415.	4202.	3.15
	0.86	421.	4199.	3.15	1.00	427.	4197.	3.15
PIPE 5	0.00	427.	4197.	5.20	0.20	367.	4183.	5.20
	0.40	308.	4170.	5.20	0.60	248.	4156.	5.20
	0.80	189.	4143.	5.20	1.00	129.	4129.	5.20
PIPE 6	0.00	214.	4224.	3.54	0.25	257.	4222.	3.54
	0.50	299.	4219.	3.54	0.75	342.	4217.	3.54
	1.00	384.	4214.	3.54				

PIPE 6 PUMP SPEED = 1180.0 RPM PUMP DISCHARGE = 1700.6 GAL/MIN EACH
PUMP HEAD = 94.0 FT

COLUMN SEPARATION HAS OCCURRED AT 7.73 SEC IN PIPE 5 AT LOCATION 1.00

	X/L	HEAD ft	HGL ft	VEL ft/s	X/L	HEAD ft	HGL ft	VEL ft/s
TIME = 7.726 SEC								
PIPE 1	0.00	150.	4200.	- 0.25	0.20	201.	4201.	- 0.25
	0.40	268.	4218.	- 0.40	0.60	332.	4232.	- 0.47
	0.80	375.	4225.	- 0.25	1.00	407.	4207.	0.04
PIPE 2	0.00	379.	4209.	1.63	0.08	335.	4163.	1.48
	0.17	299.	4124.	1.17	0.25	272.	4094.	1.10
	0.33	240.	4060.	1.12	0.42	228.	4046.	1.34
	0.50	223.	4038.	1.49	0.58	223.	4035.	1.59
	0.67	228.	4038.	1.69	0.75	267.	4074.	1.58
	0.83	316.	4121.	1.71	0.92	367.	4169.	2.15
	1.00	407.	4207.	2.61				
PIPE 3	0.00	407.	4207.	- 0.33	0.20	396.	4190.	- 0.22
	0.40	396.	4184.	- 0.23	0.60	406.	4188.	- 0.23
	0.80	415.	4191.	- 0.22	1.00	422.	4192.	- 0.24
PIPE 4	0.00	379.	4209.	2.60	0.14	397.	4218.	2.66
	0.29	402.	4215.	2.69	0.43	392.	4197.	2.46
	0.57	390.	4186.	2.19	0.71	404.	4191.	2.13
	0.86	414.	4193.	2.10	1.00	422.	4192.	2.07
PIPE 5	0.00	422.	4192.	- 1.11	0.20	373.	4189.	- 1.14
	0.40	254.	4116.	- 0.56	0.60	146.	4054.	- 0.35
	0.80	41.	3995.	- 0.28	1.00	- 39.	3961.	0.00
PIPE 6	0.00	219.	4229.	3.07	0.25	260.	4225.	3.07
	0.50	301.	4221.	3.07	0.75	340.	4215.	3.09
	1.00	379.	4209.	3.10				

PIPE 6 PUMP SPEED = 1180.0 RPM PUMP DISCHARGE = 1470.6 GAL/MIN EACH
PUMP HEAD = 98.9 FT

****
* TABLE OF EXTREME VALUES *
****

	X/L	MAX HEAD ft	TIME sec	MIN HEAD ft	TIME sec	MAX HGL ft	MIN HGL ft
PIPE 1							
	0.00	150.0	7.7	150.0	7.7	4200.	4200.
	0.20	256.1	3.6	176.8	5.9	4256.	4177.
	0.40	308.9	3.9	224.3	6.1	4259.	4174.
	0.60	359.7	4.1	275.5	5.9	4260.	4175.
	0.80	412.0	4.3	327.6	5.7	4262.	4178.
	1.00	463.1	4.5	378.6	5.5	4263.	4179.
PIPE 2							
	0.00	476.5	4.5	337.4	6.8	4307.	4167.
	0.08	476.8	4.8	317.7	7.5	4304.	4145.
	0.17	522.0	5.0	283.6	7.3	4347.	4109.
	0.25	532.8	5.2	267.0	7.5	4355.	4090.
	0.33	535.1	5.5	240.2	7.7	4355.	4060.
	0.42	537.1	5.7	228.2	7.7	4355.	4046.
	0.50	539.0	5.9	223.3	7.7	4354.	4038.
	0.58	538.2	5.7	222.7	7.7	4351.	4035.
	0.67	535.3	5.5	227.6	7.7	4345.	4038.
	0.75	512.4	5.2	266.5	7.7	4320.	4074.
	0.83	460.0	5.0	316.2	7.7	4265.	4121.
	0.92	461.6	4.8	367.0	7.7	4264.	4169.
	1.00	463.1	4.5	378.6	5.5	4263.	4179.
PIPE 3							
	0.00	463.1	4.5	378.6	5.5	4263.	4179.
	0.20	527.4	2.5	360.4	5.7	4321.	4154.
	0.40	537.3	2.7	353.8	5.9	4325.	4142.
	0.60	545.4	3.0	357.6	6.1	4327.	4140.
	0.80	555.1	3.2	367.4	5.9	4331.	4143.
	1.00	563.4	3.4	380.8	6.6	4333.	4151.
PIPE 4							
	0.00	476.5	4.5	337.4	6.8	4307.	4167.
	0.14	498.4	3.0	334.9	5.5	4320.	4156.
	0.29	521.5	3.2	329.6	5.5	4334.	4142.
	0.43	531.0	3.4	333.1	5.7	4335.	4137.
	0.57	540.3	3.6	338.6	5.9	4336.	4134.
	0.71	549.4	3.9	345.7	6.1	4337.	4133.
	0.86	556.4	3.6	374.3	6.6	4335.	4153.
	1.00	563.4	3.4	380.8	6.6	4333.	4151.
PIPE 5							
	0.00	563.4	3.4	380.8	6.6	4333.	4151.
	0.20	799.4	1.4	258.8	3.6	4615.	4075.
	0.40	761.6	1.6	200.1	3.9	4624.	4062.
	0.60	722.2	1.8	146.3	7.7	4630.	4054.
	0.80	682.8	2.0	40.5	7.7	4637.	3995.
	1.00	643.4	2.3	- 39.1	7.7	4643.	3961.
PIPE 6							
	0.00	227.4	5.9	214.0	0.0	4237.	4224.
	0.25	329.6	3.9	225.0	6.1	4295.	4190.
	0.50	386.3	4.1	256.1	6.4	4306.	4176.
	0.75	431.6	4.3	296.1	6.6	4307.	4171.
	1.00	476.5	4.5	337.4	6.8	4307.	4167.

MAXIMUM HEAD = 799.4 FT IN PIPE 5 AT X = 0.20 AT TIME = 1.36 SEC
MINIMUM HEAD = - 39.1 FT IN PIPE 5 AT X = 1.00 AT TIME = 7.73 SEC

The network experiences column separation in pipe 5 after 7.7 sec. The pump is still producing flow, although at a reduced discharge.

*   *   *

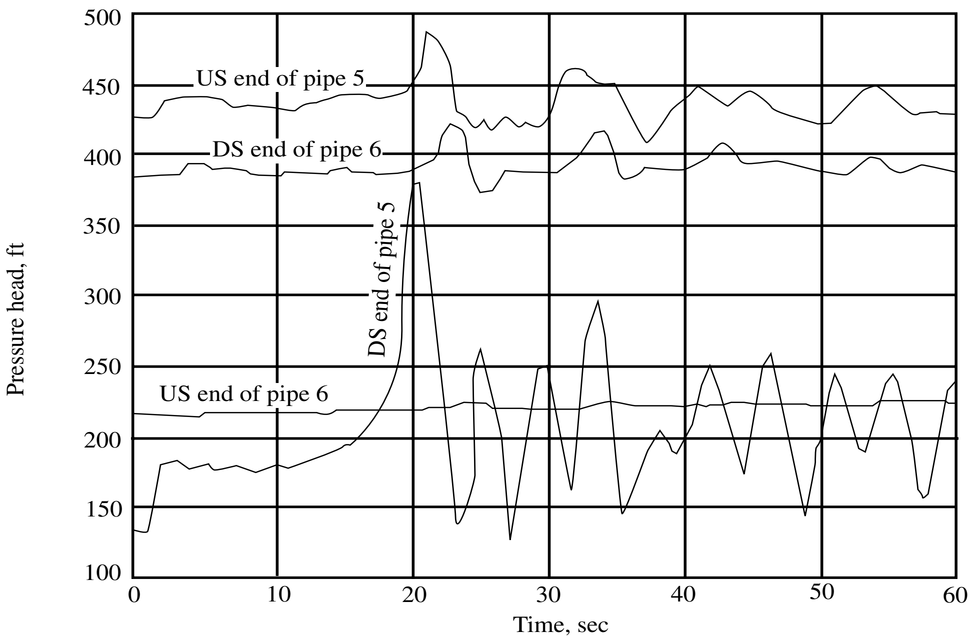
Example Problem 12.6

Reconsider Example Problem 12.5, replacing the sudden valve closure with a gate valve that closes in 20 sec at two different rates. The first stage is to 95 % closure in 1 sec, with the remainder of the closure in 19 sec. Use the gate valve loss coefficient data listed in Table 10.2 in Section 10.4.4.

In this problem the same initial condition applies as in the previous problem; however, a different file is needed to tell TRANSNET what to do. The revised file consists of the following instructions:

```
DEMONSTRATION OF PROGRAM TRANSNET -INPUT DATA FILE "EPB12_6.DAT"
NETWORK OF EXAMPLE 12.6 - GRADUALLY-CLOSED VALVE AT THE DS END OF PIPE 5
&SPECS NPARTS=4, IOUT=1000, IVALVE=5, HATM=30., TMAX=60., GRAPH=T,
            PC1=5., TC1=1., TC2=20., KLI=0.,0.0167,0.0313,0.0556,0.100,0.179,
           0.333,0.625,1.25,2.50,5.27/
&GRAF NSAVE=4, IOUTSA=2, PIPE=5,5,6,6, NODE=999,1,999,1/
```

The output from TRANSNET will be found in file EPB12_6.PLT on the CD. The output plot, followed by the information for the plot file, is presented next:



****
* PLOT FILE INFORMATION *
****

PLOT DATA IS SAVED ON FILE: prb12_6.plt
TMAX =   59.99 SEC
NUMBER OF PRESSURE HEAD VALUES IN FILE = 133
PHMAX = 487.3 FT
PHMIN = 124.4 FT

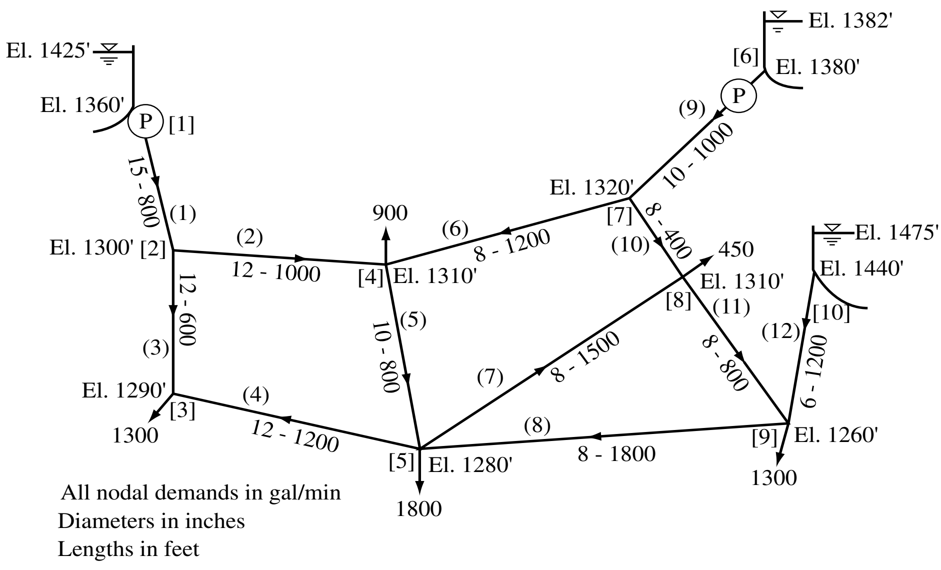
PIPE 5 NODE 6
PIPE 5 NODE 1
PIPE 6 NODE 5
PIPE 6 NODE 1

* * *

Example Problem 12.7

This example examines a larger network which receives water by gravity from one reservoir and additional water that is pumped from two other reservoirs. The demands at all nodes are shown on the diagram, and the roughness is $e = 0.01$ in for all pipes.

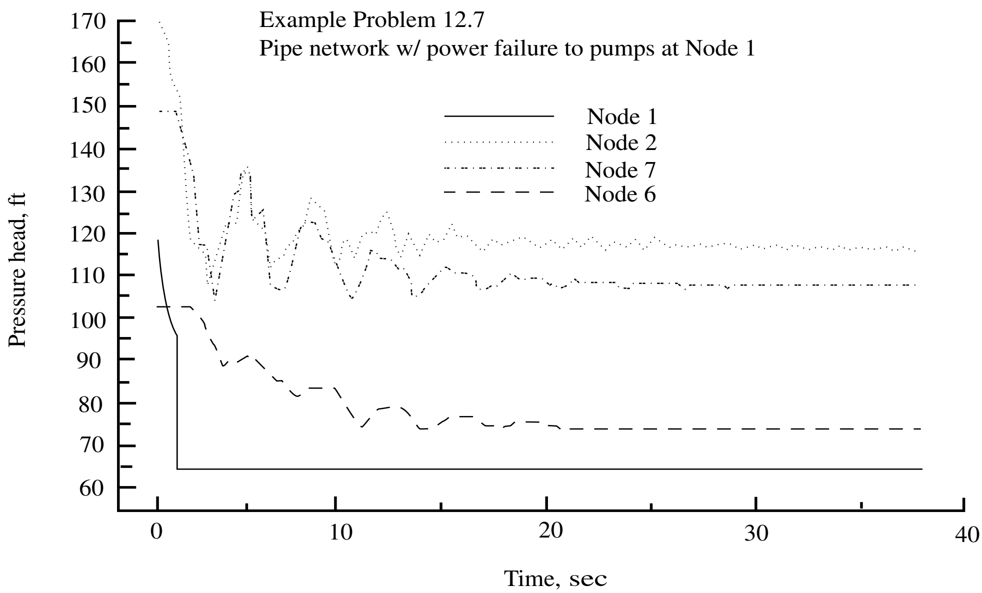
Investigate the effects on the network of power failure at the pump station in line 1. The wave speed is $a = 3000$ ft/s for all pipes. Data for the pump characteristics will be found in the input data file EPB12_7.IN.



The solution of this example problem follows the now-familiar routine. The input data to create the steady-state solution of the problem, to serve as the initial condition for the transient problem, can be found in file EPB12_7.IN on the CD; the steady-state solution is stored in file EPB12_7.OU1. In the input file we will see that the pipes and nodes have been entered in random order. Pipes and nodes need not be numbered sequentially. In addition, composite curves for the two parallel pumps, each with two stages in pipe 9 at node 6, have been developed and entered in this file; to communicate this information to

NETWK, a minus sign must precede the 2 after the RUN statement, thus indicating that there are two stages and two parallel pumps at this station. Moreover, the discharges from the second pump have been multiplied by 6, and the - 6 in the data file indicates that there are actually six parallel pumps at station 2, but that this fact is accounted for in the pump data that have been prepared.

The data for TRANSNET to solve this problem are provided in file EPB12_7.DAT on the CD, the file written by NETWK is EPB12_7.OU2, and the transient output will be found on the CD in file EPB12_7.OU3. A plot of the output data for four nodes is presented in the figure which follows.



* * *

12.7 PROBLEMS

12.1 Obtain steady-state solutions for the three-pipe network in Section 12.2.2. for demands QJ_3 of 2.0, 2.5, 3.0, and 3.5 ft^3/s, and compare the resulting heads at nodes 2 and 3 from each solution with the corresponding heads from the unsteady solution. The differences in these heads can be interpreted as the amount of head that is required to accelerate the fluid columns. Also compare the other discharges.

12.2 For the 6-pipe, 4-node network described below:

(a) Write the 10 equations that will allow the pipe discharges and nodal heads to be determined for any number of time increments with the specification of either the demands or heads at selected nodes. Assume $WS_1 = 100$ ft, $WS_2 = 95$ ft.

(b) Obtain the steady-flow solution for this network for the demands listed in the demand table, which will serve as the initial condition ($t = 0$) for the unsteady solution.

(c) Obtain the unsteady solution, according to rigid-column theory, over an 8-sec time period using 2-sec increments. Assume the demand at node 2 varies in the way listed in the table for QJ_2.

(d) Prepare a set of four steady-flow solutions as in (b), but replace QJ_2 for $t = 0$ with QJ_2 for $t = 2$ sec, $t = 4$ sec, $t = 6$ sec and $t = 8$ sec, respectively.

(e) Compare the pipe discharges and nodal heads from the steady-state solutions, part (d), with those from the unsteady solution, part (c).

Pipe Properties

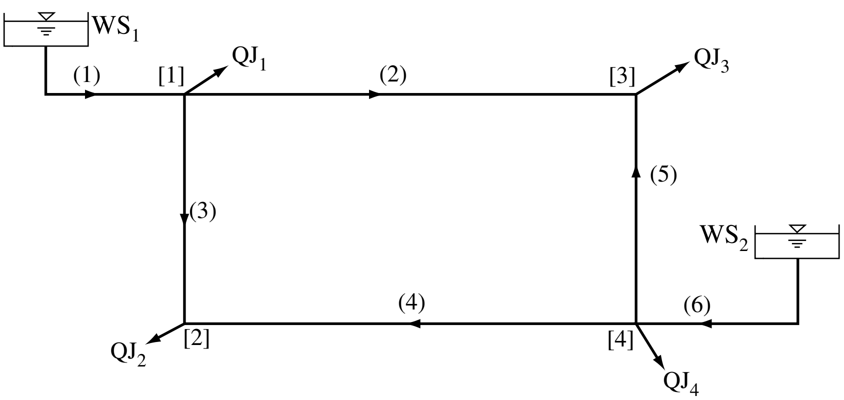
Pipe	Dia. in	Length ft
1	10	2000
2	8	2000
3	8	1500
4	8	2000
5	8	1500
6	10	2000

Demands

Node i	QJ_i ft^3/s
1	1.5
2	1.0
3	1.5
4	2.0

QJ_2 Schedule

Time sec	QJ_2 ft^3/s
0	1.0
2	1.5
4	2.0
6	2.5
8	3.0



12.3 Repeat parts (b) through (e) of problem 12.2 with this change: the Demand Schedule for QJ_2 begins at 3.0 ft^3/s at time $t = 0$, and it then decreases in 0.5 ft^3/s increments to 1.0 ft^3/s after 8 sec.

12.4 A water distribution system is shown below. In it the demands at nodes 2 and 5 change with time, but the demands at the other three nodes remain constant. For a long time there have been no changes in the demands, with the demands at nodes 2 and 5 being zero and 0.5 ft^3/s, respectively. Beginning at time $t = 0$, the demands at these two nodes change as shown in the demand table.

(a) The initial steady-flow condition must be determined before the unsteady problem can be solved. State the ΔQ-equations that can be solved to provide this initial condition.

(b) How many equations must be solved simultaneously over each time step of the unsteady problem using rigid column theory? How many of these are algebraic equations, and how many are differential equations? List the unknown variables.

(c) Write the equations that govern this unsteady problem, and describe how they are to be solved.

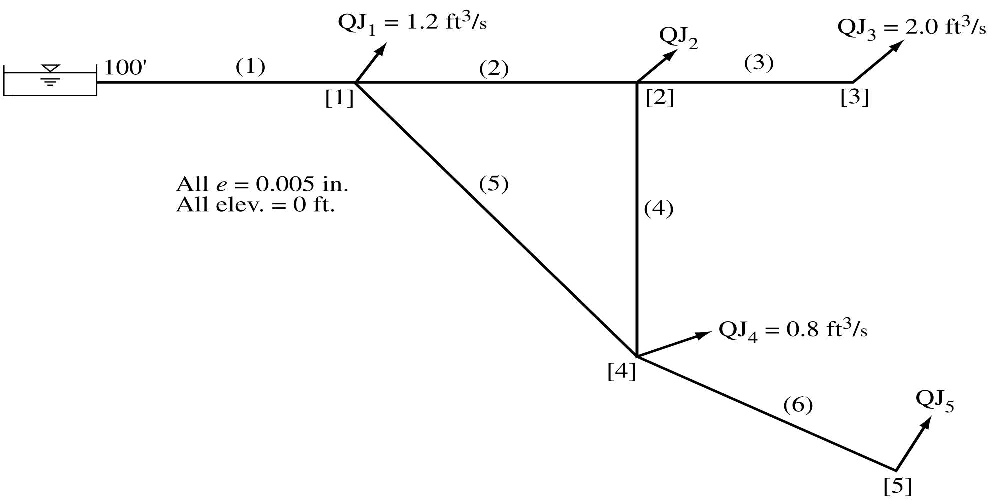
(d) Solve the unsteady problem with QJ_2 and QJ_3 varying as listed in the table.

Pipe Data

Pipe	Dia. in	Length ft
1	14	200
2	12	180
3	10	190
4	10	180
5	10	230
6	8	200

Demand Table

Time sec	QJ_2 ft^3/s	QJ_3 ft^3/s
0.0	0.0	0.5
2.0	0.5	1.0
5.0	0.7	1.2
7.5	0.8	1.0
10.0	0.8	0.8



12.5 A 6-pipe network supplied by a source pump and a reservoir is shown below; the diagram presents pipe data and the initial steady-flow nodal demands. Do the following:

(a) Prepare input data for NETWK to obtain a steady-state solution for this network.

(b) Assuming that the demands at the four nodes are to change in time, apply rigid-column theory to write the equation system that describes the unsteady-flow network problem. In performing this task, identify the unknowns and develop an appropriate number of independent equations to determine them.

(c) Prepare an input data file for the program UNSTPIPD to solve this problem if QJ_4 varies with time, as listed in the Demand Schedule.

Pump Curve

Q ft^3/s	h_p ft
2.0	35
3.0	32
4.0	28

Steady Discharges

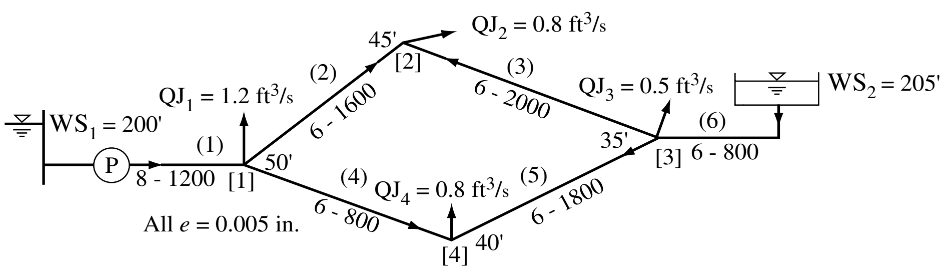
Pipe	Q ft^3/s
1	2.485
2	0.659
3	0.141
4	0.626
5	0.174
6	0.815

Steady Heads

Node	H ft
1	207.11
2	195.24
3	196.09
4	194.98

Demand Schedule

Time sec	QJ_4 ft^3/s
1.0	0.9
2.0	1.0
3.0	1.1
4.0	1.2
5.0	1.3
6.0	1.4



12.6 The network in the next diagram receives its water supply from a pump. The tank at the other end stores water during periods of low demand and supplies water during periods of larger demand. The pump characteristic curve is described by $h_p = -0.4Q^2 + Q + 75$. The demands have been constant for a long time. At time $t = 0$ the demands at nodes 3 and 4 begin to increase at a rate $dQ/dt = 0.05$ ft^3/s^2 for 20 sec and then become constant again.

(a) Describe how to obtain the initial condition for this unsteady problem, including the preparation of the input data file for NETWK to obtain a solution.

(b) Write the equations that must be solved simultaneously, using rigid-column theory, to obtain the unsteady solution at several subsequent times: 0.5, 1.0, 2.0, and 5.0 sec. For each of these times indicate which variables are unknown and which are known.

(c) For the changes shown in the Demand Schedule, solve this problem by modifying program UNSTPIP to include a pump and then applying the new program.

Pipe Data

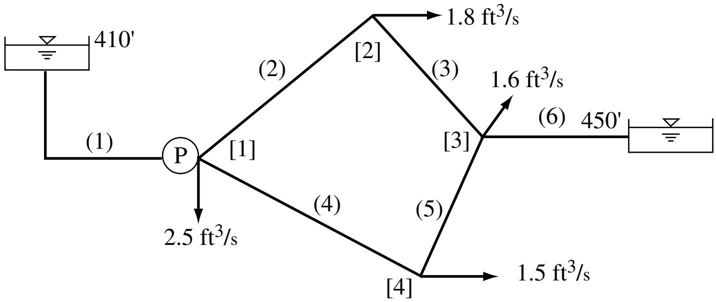
Pipe	Length ft	Dia. in
1	1000	14
2	1500	12
3	1600	12
4	1800	10
5	1400	8
6	1500	8

Nodal Data

Node	QJ ft^3/s	Elev. ft
1	2.50	350
2	1.80	360
3	1.60	340
4	1.50	335

Demand Schedule

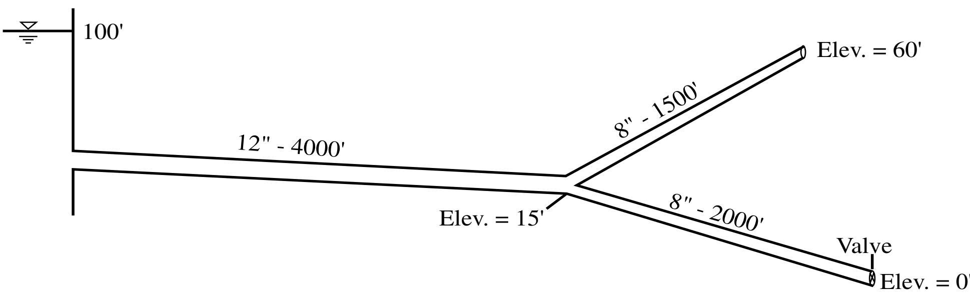
Time sec	QJ_2 ft^3/s	QJ_3 ft^3/s
0.0	1.60	1.50
0.5	1.70	1.60
1.0	1.80	1.75
2.0	1.80	2.00
5.0	1.80	2.30



12.7 A 12-in-diameter 4000 ft long pipe branches into two 8-in-diameter pipes. One is 1500 ft long which discharges freely into the air at an elevation of 60 ft. The other is 2000 ft long with a butterfly valve having a loss coefficient $K = 8000e^{8(x - 1)}$ at the downstream end, in which $x = 0$ for a fully open valve, and $x = 1$ when the valve is 98% closed. The 12-in pipe is supplied by a reservoir with a water surface elevation that is 100 ft above the elevation of the valve. Do the following:

(a) Write the equation system whose solution will provide the initial condition for a transient if the valve is fully open at $t = 0$.

(b) Write the rigid-column equation system to describe the unsteady motion if the valve is 98% closed after 5 sec. Obtain this solution and provide a plot of discharge vs. time for each pipe and also a plot of head vs. time at the junction. Use Δt = 0.5 sec for about 30 time steps. Assume e = 0.004 in for all pipes, and ν = $1.41\text{x}10^{-5}$ ft²/s.

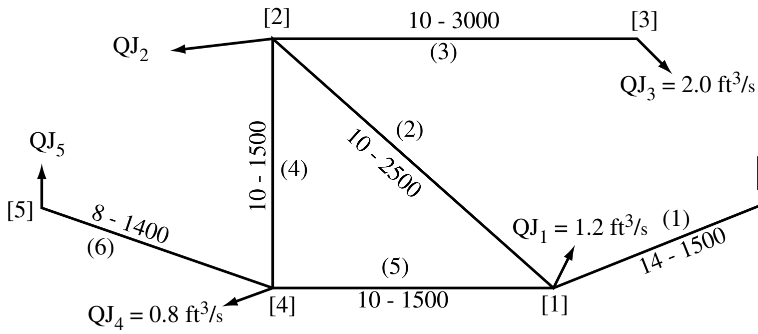


12.8 In the following network the constant demand at node 1 is QJ_1 = 1.5 ft³/s, but the demands vary at nodes 2 and 5, as are listed in the last portion of the file written by NETWK, shown below. Do the following:

(a) Indicate the appropriate unknown variables for an unsteady-flow solution based on rigid column theory.

(b) Write the system of equations that must be solved for these unknowns for each time step of the solution.

The pipe diameters and lengths, both in ft, are given in the file which follows. For all pipes e = 0.005 in = 0.000417 ft, and all nodal elevations are zero ft.



File written by NETWK with option NETPLT = 4:

```
      6    6    0    0   32.2  0.14100E-04    0.00100    3
      1    6 5555    1    1   1500.0  1.167  0.000417  6.30
      2    1    1    2    2   2500.0  0.833  0.000417  2.30
      3    2    2    3    3   3000.0  0.833  0.000417  2.00
      4    2    2    4    4   1500.0  0.833  0.000417 -1.00
      5    1    1    4    4   1500.0  0.833  0.000417  2.80
      6    4    4    5    5   1400.0  0.667  0.000417  1.00
         1.20         0.00         88.40
         1.30         0.00         73.11
         2.00         0.00         59.07
         0.80         0.00         75.00
         1.00         0.00         69.62
       - 6.30       100.00        100.00
   4 0
   2 2 2 1.5 5 1.2
   6 2 2 1.7 5 1.5
   10 2 2 1.5 5 2.0
   20 2 2 1. 5 2.5
```

12.9 Modify the program STANDPIP.FOR so it calls the ODE solver ODESOL rather than the subroutine DVERK.

12.10 Modify the program STANDPIP.FOR so the pipeline will lie at a slope S_o from the horizontal. Then obtain a solution to the following problem with this program:

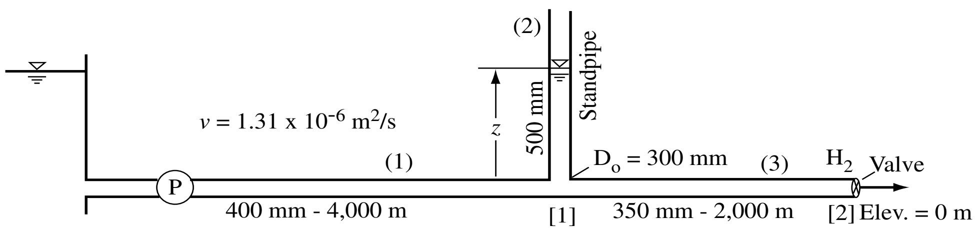
The pipeline has a 24-in diameter, a total length of 5000 ft, with a 36-in-diameter standpipe located 500 ft upstream from a butterfly valve. The diameter of the orifice opening at the base of the standpipe is 18 in. The pipeline is supplied by a constant-head reservoir at its upstream end with a water surface elevation of 40 ft. The delivery pipe slopes upward at 5 ft per 1000 ft and has a roughness $e = 0.003$ in. Assume $\nu = 1.41\text{x}10^{-5}$ ft^2/s. Laboratory tests of the butterfly valve indicate its flow coefficient C_v plots as a straight line on semi-log paper, with the valve opening in degrees (the linear scale), so that $C_v = 420$ at a 10^o opening and $C_v = 42{,}000$ at a 90^o opening, with C_v defined by Eq. C.1 in Appendix C. Initially the pipeline contains a steady discharge of 8 ft^3/s = 3590 gal/min. Starting at time $t = 0$, the valve angle changes linearly with time from its initial steady-flow position to a 3^o position in 10 sec.

12.11 Repeat problem 12.10, assuming there is an opening into the standpipe that creates a constant minor loss coefficient $K = 2$ in the expression $h_L = KV^2/(2g)$ for the head loss at the entrance to the standpipe, with V being the velocity in the standpipe. (In this problem one could also study how the maximum head at the valve and the water height in the standpipe are related to the magnitude of the loss coefficient K.)

12.12 A standpipe is located 4000 m downstream from a reservoir; the reservoir water surface is 30 m above the 400-mm-diameter horizontal supply pipe. The standpipe has a 500 mm diameter, and the orifice from the main pipe has a 300 mm diameter. Downstream 2000 m from the standpipe is a butterfly value, which is initially wide open but which can be almost entirely closed in 10 sec. The loss coefficient for this butterfly valve is $K = 10000e^{9(x-1)}$. Assume the pipe roughness is $e = 0.2$ mm, and use $\nu = 1.31 \times 10^{-6}$ m^2/s for the kinematic viscosity of water. The pump in the upstream pipe has operating characteristics given by the data in the table below. Apply rigid-column theory to analyze the system for 35 sec of operation when the valve is closed over a ten-second interval (x varies linearly from 0 to 1 in 10 sec).

Pump Characteristic Data

Q, m^3/s	0.20	0.25	0.30
h_p, m	31.0	29.6	29.0



12.13 The flow from three pumps is delivered to one pipe line, as shown below. All pipes have a roughness $e = 0.005$ in; assume $\nu = 1.41 \times 10^{-5}$ ft^2/s for water.

(a) If the discharge for a long time is 7.5 ft^3/s and the reservoir water surface elevations are $WS_1 = 50$ ft, $WS_2 = 45$ ft, and $WS_3 = 40$ ft, what are the steady discharges in pipes 1-3 and the heads at the two nodes? Assume the elevation of nodes 1 and 2 is 0.0 ft. The pump operating characteristics are given in the tables below.

(b) Determine the unsteady, rigid-column discharges and heads if the pressure at the downstream end decreases linearly from the steady value to 0 lb/in^2 in 5 sec, and then during the next 10 sec increases linearly to 40 lb/in^2. Follow the transient over 28 sec.

(c) If the discharge in the downstream pipe decreases linearly from 7.5 ft^3/s to 0 in 5 sec, determine with rigid-column theory the velocities and nodal heads over the following 30 sec.

Pump 1

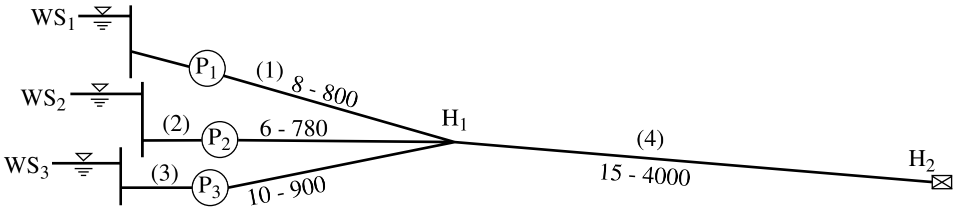
Q ft^3/s	h_p ft
2.0	40
3.0	37
4.0	32

Pump 2

Q ft^3/s	h_p ft
0.8	45
1.2	43
1.6	39

Pump 3

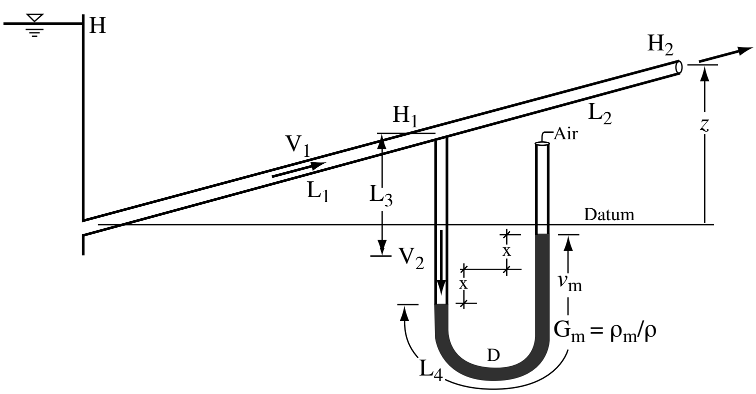
Q ft^3/s	h_p ft
2.5	50
3.0	48
3.5	44



12.14 A U-tube (This is **not** a manometer that contains water on both sides of the manometer fluid and is used to measure pressure differences) taps a pipe at a distance L_1 downstream from a reservoir of constant head H, as shown in the sketch. At a distance L_2 further downstream is a valve that controls the discharge through the pipe. The discharge coefficient c_v in the relation $Q = c_v(H_2 - z)^{1/2}$ or $V_1 = c_v(H_2 - z)^{1/2}/A$ is known for the valve; in particular, it is known as a function of position during the valve closure process, and since the position of the valve is known as a function of time, c_v is known as a function of time.

Formulate the unsteady flow problem in the pipe and U-tube using rigid-column theory, i.e., write the system of equations that govern the velocity $V_1(t)$ in the pipe, the deflection $x(t) = dV_2/dt$ of the manometer fluid in the U-tube, as well as $H_1(t)$ and $H_2(t)$ for a problem in which the following variables are known: H, the specific gravity of the manometer fluid G_m, the diameter d of the pipe, the diameter D of the tube, L_1, L_2, $L = L_1 + L_2$, the distance L_3 from the pipe to the manometer fluid when $x = 0$, the length L_4 of the manometer fluid in the U-tube, and $c_v(t)$.

Time, sec	1.0	2.0	3.0	4.0	5.0
Coefficient c_v	0.50	0.10	0.05	0.025	0.02



12.15 The sketch on the next page shows a 5-pipe network. During periods of low demand water is pumped into the upper reservoir through pipe 5, but during periods of larger demand the pump is turned off, and the valve in the bypass line around the pump is opened so the upper reservoir can supply part of the demand. The low demands are shown by the outward arrows at the nodes, and the larger demands are shown thereafter in parentheses. Assume the elevation of nodes 1, 2, and 3 is 300 ft, that the pipe roughness $e = 0.002$ in, and the kinematic viscosity of the water is $\nu = 1.41\times10^{-5}$ ft^2/s.

Do the following:

(a) Prepare the input to NETWK to determine the steady flows during the period of low demand.

(b) List the changes that should be made to this input file to analyze the network performance in response to the larger demands.

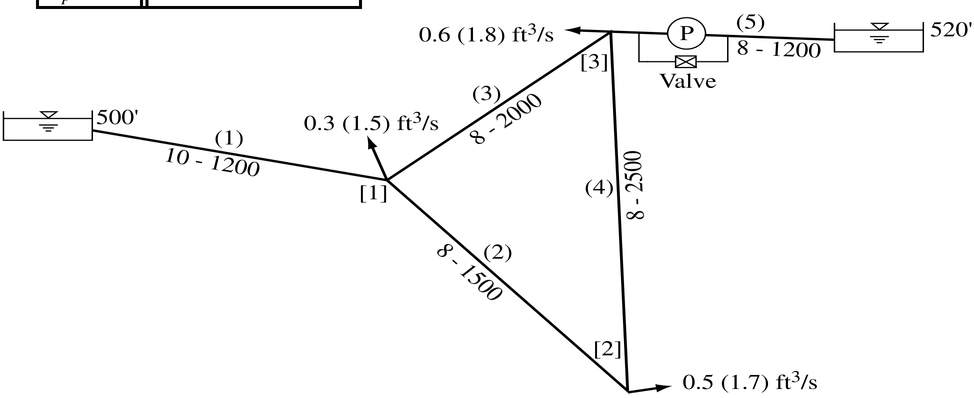
(c) Write the system of equations that would govern the solution of both steady state problems in parts (a) and (b). These should be general equations that would in principle allow pipe diameters or pump heads to be determined.

(d) Applying rigid-column theory, write the equations to be solved to obtain the unsteady discharges and nodal heads for this network if the demands change with time. What equation(s) will change, depending on whether the pump is operating?

(e) Obtain an unsteady solution with UNSTPIP when the pump is operating, the demands at all three nodes change linearly from the low values to the high values over 30 sec and thereafter remain constant over the next 70 sec.

Pump Characteristic Data

Q, ft^3/s	1.0	1.5	2.0
h_p, ft	50	48	45



12.16 An 8-in diameter ($e = 0.002$ in) horizontal pipeline obtains its water supply from a reservoir with a constant head of 50 ft above the pipeline. It has a closed cylindrical tank, with a total volume of 100 ft^3 and a cross-sectional area of 20 ft^2, connected to it at a pipe bend which is 6000 ft downstream from the reservoir. The pipe continues an additional 500 ft to a valve. The pipe that connects the pipeline to the closed tank has a 4-in diameter and is 100 ft long. The downstream valve has been open for a long time, and at $t = 0$ it is closed so the discharge through the valve is reduced linearly from the steady-state discharge to zero in 5 sec. The local loss coefficient for the valve is a function of the discharge through the valve in the form $K = 8000/e^{8Q}$. During steady flow the tank is half full of water and half full of air; when the water surface elevation in the tank is at this middle position, it has the same elevation as the centerline of the pipeline.

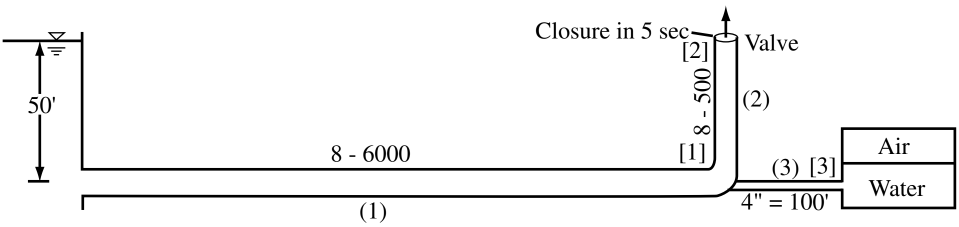
Complete the following tasks:

(a) Compute the steady-state head in the pipeline at the bend, which is node 1, when the valve is open. The steady discharge is $Q_o = 1.523$ ft^3/s.

(b) Determine the pressure and density of the air in the tank, assuming any changes are adiabatic from an initial temperature of 58 ^{o}F and pressure of 14.7 lb/in^2 absolute.

(c) Write a system of equations, using rigid-column theory, to describe the unsteady flow in the pipeline and surge tank that is caused by closing the valve in 5 sec (i.e. reducing the discharge at the end of the pipeline linearly from Q_o to 0 in 5 sec).

(d) Determine the time-dependent flow in the pipe lines and the surge tank.



12.17 Repeat problem 12.16, assuming that the total tank volume and cross-sectional area are reduced by one-half to 50 ft^3 and 10 ft^2, respectively.

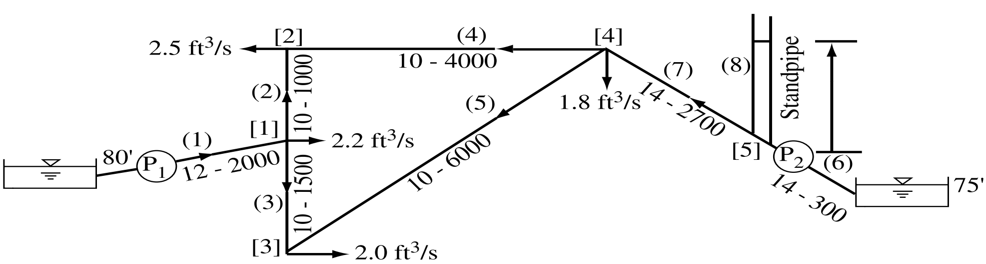
12.18 This network is supplied by two pumps. A 24-in-diameter standpipe exists downstream from the second pump, with a 6-in-diameter orifice at the entrance to the standpipe. Assume the nodal elevations are all zero feet and all pipes have roughnesses e = 0.005 in. The demand at node 4 is reduced by 0.1 ft^3/s per second until it becomes zero. Simulate with rigid-column theory the network performance over 10 sec, using 0.5 sec increments.

Pump 1

Q ft^3/s	h_p ft
1.5	60
2.5	52
3.5	41

Pump 2

Q ft^3/s	h_p ft
2.0	80
3.0	72
4.0	60



12.19 Determine the unsteady discharges and heads in the network in problem 12.18 when the demand at node 2 is reduced linearly in time to zero in 12.5 sec.

12.20 In problem 12.18 the demand at node 3 changes linearly in time from 2.0 ft^3/s to 2.5 ft^3/s and then to zero.

12.21 Program SURGNET, originally written to analyze the network in Example Problem 12.4, calls on DVERK to solve nine simultaneous first-order ODEs for the unsteady pipe discharges. Modify this program so that each call to DVERK requests the solution of only one first-order ODE. (In using DVERK in this way, note that the argument IND returns a value of 3 after a solution has been completed over the specified interval in anticipation that it will be called to continue the solution over additional increments of the independent variable.)

12.22 Modify program SURGNET, used in Example Problem 12.4, so ODESOL is used in place of DVERK to solve the system of ODEs.

12.23 Modify the program that was developed in problem 12.22 so that each call to ODESOL requests the solution of only one first-order ordinary differential equation.

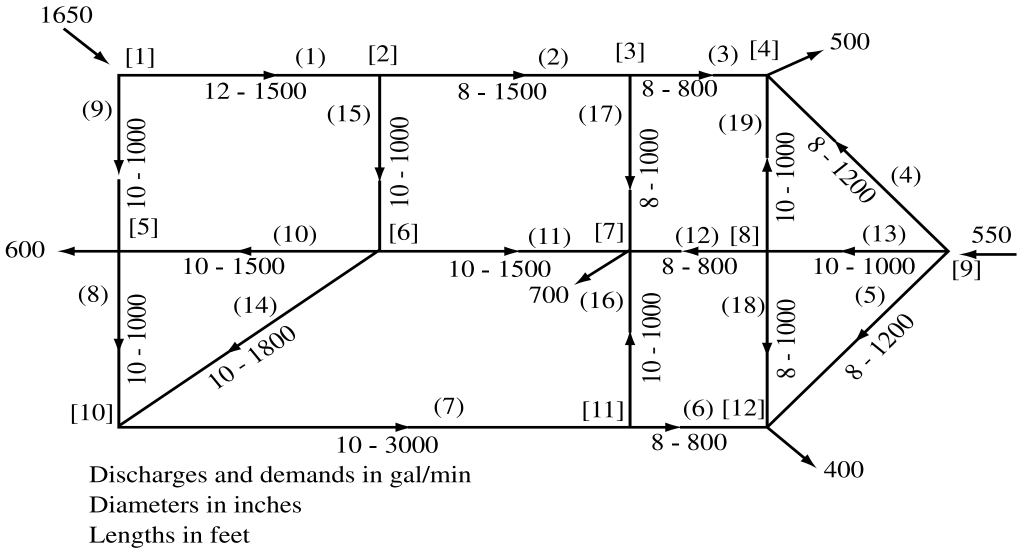
12.24 Modify the program that was developed in problem 12.23 so that the solution to the single first-order ODE is obtained by RUKUST rather than ODESOL or DVERK.

12.25 For the pipe network shown below the pressure at node 1 is 85 lb/in^2, and all pipes have a Hazen-Williams coefficient of 120. The network lies in a horizontal plane at an elevation of 1100 ft. Assume the wave speed in all pipes is 3000 ft/s.

(a) Obtain the steady state solution.

(b) Find the maximum and minimum pressures, their location and their time of occurrence if the demand at node 12 is instantly increased to 650 gal/min, and elastic effects are included in the analysis.

(c) Ignore elastic effects and increase the demand at node 12 linearly to 650 gal/min over 4 sec.



12.26 For the network of problem 12.25, predict the consequences of a sudden stoppage of the 1650 gal/min input to the network at node 1.

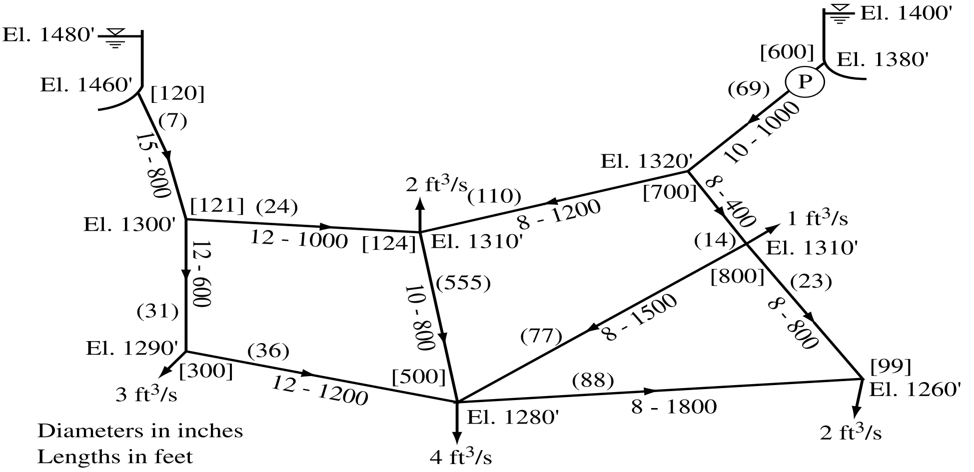
12.27 For the network of problem 12.25, find the maximum and minimum pressures, their location and time of occurrence if the valve at the downstream end of pipe 7 were suddenly closed.

12.28 For this network the pipe roughnesses are all $e = 0.02$ in, and the wave speed for each pipe is 3300 ft/s. The pump curve for the one pump in pipe 69 is defined by $h_p = -0.5Q^2 - 0.3Q + 90$, with Q in ft^3/s and h_p in ft. The pump runs at 1750 rev/min, and for this unit $Wr^2 = 40$ lb-ft^2.

(a) Obtain the steady state solution.

(b) Assuming the brake horsepower for the pump is constant at 40 hp, determine the consequences of pump power failure.

(c) Ignore elastic effects and determine the consequences if the head supplied by the pump dropped to zero in 4 sec.



12.29 For the network in problem 12.28, find the maximum and minimum pressures, their location and time of occurrence if the valve at the downstream end of pipe 7 is suddenly closed.

12.30 Solve problem 12.28 if pipe 23 is removed and the demand at node 99 is reduced to 1.0 ft^3/s.

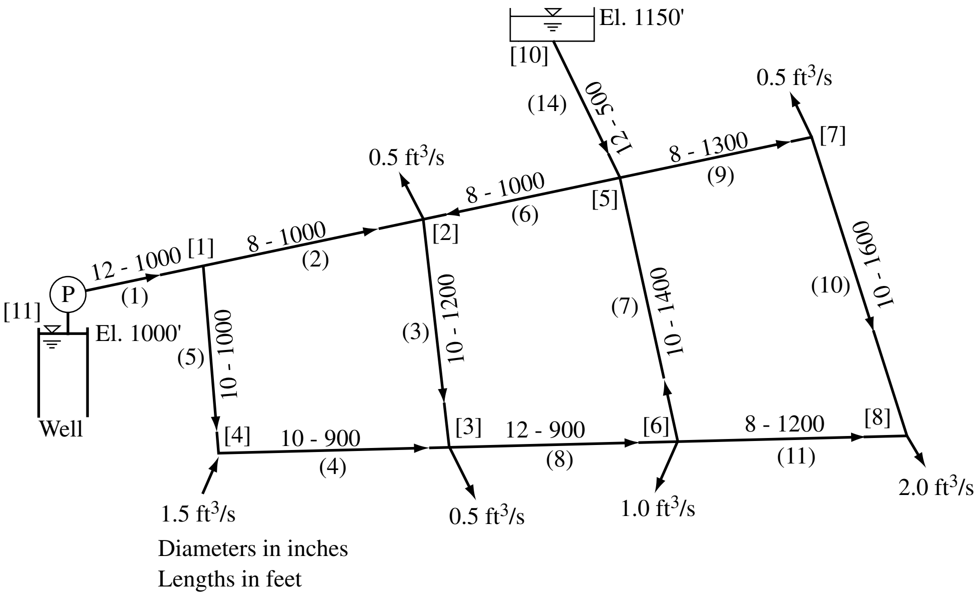
12.31 In problem 12.30 determine the consequences of sudden valve closure at the downstream end of pipe 88.

12.32 For the network shown atop the next page, all pipes have roughness $e = 0.008$ in and wave speeds of 2700 ft/s. The nodes are all at elevation 1050 ft. The pump curve for pipe 1 is $h_p = -1.5Q^2 - 1.5Q + 170$, with Q in ft^3/s and h_p in feet. The pump runs at 1180 rev/min, and for this unit $Wr^2 = 50$ lb-ft^2.

(a) Obtain the steady state solution.

(b) Assuming the brake horsepower is constant at 45 hp, determine the consequences of power failure.

(c) Let the head of the pump decrease to zero over 5 sec, ignore elastic effects, and determine how the pressures and discharges decrease throughout the network.



12.33 For the network in problem 12.32, find the maximum and minimum pressures, their location and time of occurrence if the valve at the downstream end of pipe 14 is suddenly closed.

CHAPTER 13

TRANSIENT CONTROL DEVICES AND PROCEDURES

Transients in a pipeline system can cause objectionably high or low pressures. Excessively high pressures can damage pumps, valves, and other pipeline appurtenances, as well as rupturing the pipe itself. However, "failure" may refer only to the inability to meet a given standard of performance; thus it is possible for a failure to occur in the absence of any physical damage. For example, it may be required under all conditions that the pressure in a pipeline remain above atmospheric pressure to prevent air from entering the lines through vacuum valves. If an analysis indicated that the pressure would drop below atmospheric pressure for even a single operating condition, the pipeline has "failed."

Excessively low pressures can lead to the release of large amounts of dissolved air, and extensive vaporization of the liquid can occur if the pressure drops to the liquid vapor pressure. The resulting low pressures, possibly enhanced by external pressures, could cause the pipe to collapse. Also, a vapor cavity closure event occurring at some point in the pipeline can produce high shock pressures which could lead to failure of the pipe. These cavity closure shocks are difficult to predict owing to the difficulty of simulating the actual physical phenomena occurring in the pipe. The approach outlined in Section 10.7 appears to be the most commonly used method of simulating this complex phenomenon. Brittle pipe materials such as concrete are particularly susceptible to this type of problem. For example, some types of reinforced concrete pipe contain a thin steel cylinder which is lined with cement mortar and then wrapped under tension with reinforcing wire (see Section 8.3). If pressure shocks cause the fracture and spalling of the internal cement lining, the thin steel cylinder has little support to prevent wall buckling and collapse. Even under less dramatic circumstances the loss of the mortar lining would potentially expose the steel to corrosion which could ultimately undermine the integrity of the pipe.

Another type of transient condition which can cause problems in a pipeline is vibration. A periodic pressure variation could excite some component of the pipeline which possesses a natural frequency near the pressure fluctuation frequency. Under this condition, large stresses, strains, and displacements could occur which at best would be undesirable and at worst could cause system failure. Because a good understanding of these phenomena requires a knowledge of the natural frequencies of the system components and how they are related to the periodic pressure fluctuations, the method of characteristics may not be as appropriate as other existing techniques for this analysis. Therefore, the analysis of periodic transient flow will not be addressed in this work. The reader may consult Wylie and Streeter (1993) and Chaudhry (1987) for details.

13.1 TRANSIENT PROBLEMS IN PIPE SYSTEMS

In this section we explore the most common causes of transient problems in pipe systems.

13.1.1. VALVE MOVEMENT

Probably the most common and well-known cause of transient flow problems is the movement of a valve. Any valve movement causes pressure waves to propagate through the system. The magnitude of the pressure waves depends on the type of valve, the way in which the valve is moved, the hydraulic properties of the system, and the elastic properties and restraint of the pipe system.

The proper evaluation of the impact of valve movement on the pressures in a system depends strongly on the loss characteristics of the valve. While there are charts and graphs available to *estimate* the effects of valve closure, it is far more reassuring to be able to *calculate* the effects in a specific situation. We have shown how this can be accomplished in Section 10.4.

13.1.2. CHECK VALVES

Check valves can cause large transient pressure differences if the flow backwards through them can occur before the valve closure is complete. Such a case is documented by Purcell (1997), in which check-valve slam was caused by an air chamber at the pump discharge. The high discharge pressure, maintained by the air chamber after pump power failure, caused the pump discharge to drop to zero rapidly, in turn causing the check valve, presum-ably undamped, to close abruptly. In this situation the slamming check valve creates the same problem that is caused by sudden valve closure.

Most modern check valves do not slam. In some cases a spring or weight causes the check valve to close at the instant forward flow ceases, thereby preventing the reverse-flow problem. Another type closes slowly, regulated by a damping mechanism, to bring the reverse flow to rest gradually.

13.1.3. AIR IN LINES

Filling empty lines, particularly in pumped systems, can produce velocities that are well above the expected steady-state velocities. At the low pump head that generally exists early in the filling process, the pump is operating on its curve at a point where the discharge is quite large. If the line ends at any device which acts as a flow obstruction, as Fig. 13.1 shows, e.g., a partially-closed valve or an open air-vacuum valve, then a serious water hammer situation can occur.

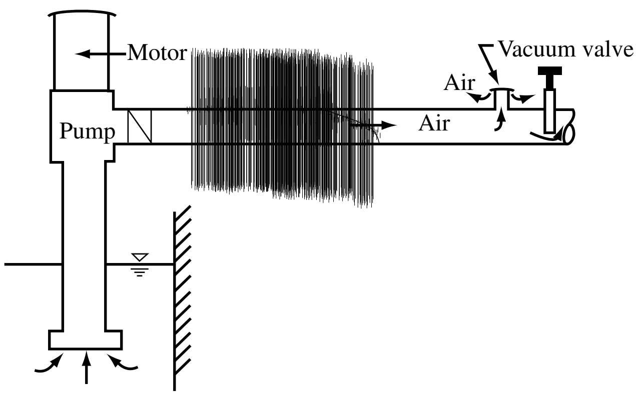


Figure 13.1 Filling an initially empty pump discharge line.

The air being vented from the pipe ahead of the oncoming water will leave the pipe much more easily than will the water behind it. When the last of this air leaves the pipe and the water hits the obstruction, there is a significant drop in water velocity which can cause a large increase in pressure. Research at Colorado State University by Kolp (1968), Andrews (1970), Diaz (1972), and Berlant (1974) has demonstrated the potential severity of the consequences of air exhaustion from pipelines. A Johns-Manville Corporation (1977) technical report nicely summarizes their work.

Another situation wherein air exhaustion can cause significant pressures is depicted in Fig. 13.2. Here the pump discharge column is initially empty, having been vented to the atmosphere by an air-vacuum valve. The water in the pipeline is restrained by a check valve. When the pump starts up, the water rushes up the discharge column and forces the air out through the open air-vacuum valve. When the last air leaves and the valve slams shut, large water hammer pressures can develop.

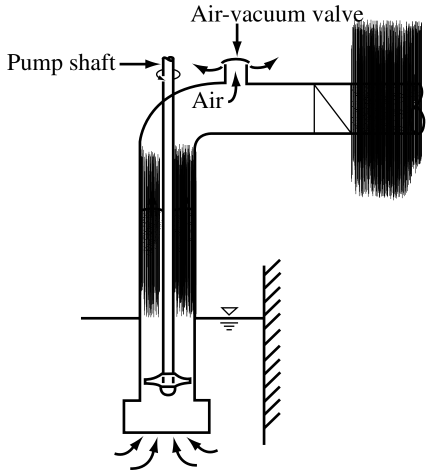


Figure 13.2 Filling a pump discharge column behind a closed check valve.

One other notable situation is a consequence of shutting down a pipeline in such a manner that air-vacuum valves open and large amounts of air enter the line. Upon restarting the flow in the line, care must be taken to insure that the air exhaustion problems discussed above do not occur here. This situation is insidious in that, after the pipeline has been successfully filled, it is easy to overlook the fact that significant amounts of air can be reintroduced by subsequent operation of the line.

13.1.4. PUMP STARTUP

As the pump starts up and comes "on line," a positive pressure surge is created in the downstream line. The magnitude of the pressure increment depends on the sudden increase in velocity which occurs when the check valve is forced open and the liquid in the pipeline begins to move. When there is no air in the line, the pressure increase is generally not large and does not exceed the pump shutoff head. If the pump has an objectionably high shutoff head, then there is a problem. Determining these pressures for various startup procedures using PROG8 was discussed briefly in Section 11.2.

If there is air in the discharge column or in the line, then substantial transient pressures can be developed. We have already discussed the problem of air in the discharge column. Martin (1976) analyzed this problem and concluded that head increases greater than ten times the original head can be generated under certain circumstances.

13.1.5. PUMP POWER FAILURE

Systems in which the static lift is large and the pipeline profile rises rapidly immediately downstream of the pumps can be subjected to the most severe transients upon power failure. If power is cut off from the pumps suddenly, either accidentally or purposefully, the pressure just downstream of the pumps drops rapidly, and this pressure drop propagates downstream at the wave speed (see Fig. 13.3). This drop in pressure can cause extensive column separation and lead to subsequent cavity closure shocks of large magnitude. In addition, a flow reversal in the system may also occur and lead to significant overpressures in the system, generally in the vicinity of the pumps, if the transient is not properly controlled.

If the pumps are booster pumps without a bypass line, power failure will initially cause the pressure to increase on the suction side of the pumps and drop on the discharge side.

Subsequent reflections from the upstream and downstream reservoirs may then cause unanticipated high or low pressures on either side of the pumps.

These situations are the most common causes of transient problems in pipe systems. Other situations are often combinations of these basic ones. We will now examine each of these situations individually and investigate the procedures and devices which can be employed to prevent objectionably high or low pressures.

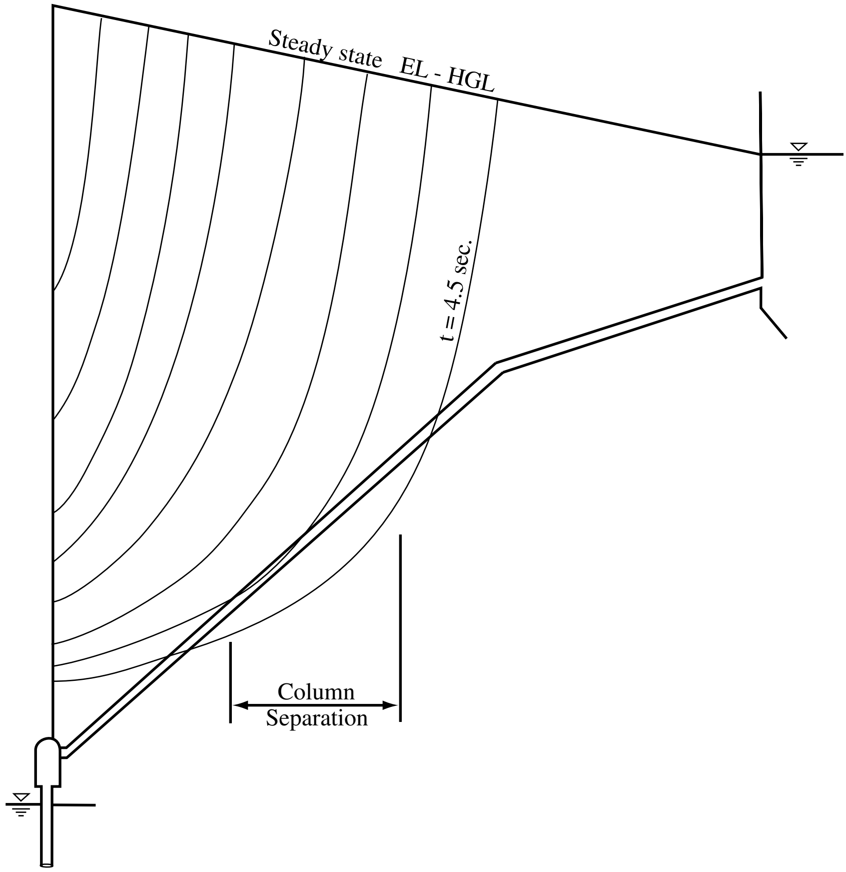


Figure 13.3 Propagation of a negative wave resulting from pump power failure.

13.2 TRANSIENT CONTROL

Transient pressure waves occur in pipelines because of changes in the fluid velocity that are commonly caused, for example, by valve movement, pump power failure, and/or column separation. Because the change in pressure is directly proportional to the change in velocity, the avoidance of sudden velocity changes will generally prevent serious transient pressures from developing. Most control devices and procedures are designed to function in a particular application to achieve this goal. We will now see how this approach can mitigate or sidestep these problems.

13.2.1. CONTROLLED VALVE MOVEMENT

In Section 10.4 we demonstrated how a valve closure schedule could affect the maximum transient pressure. For a gate valve we saw that the last 2-5% of the valve closure motion was critical in determining the maximum pressure. Different results will be found for other kinds of valves. The best way to determine the effect of a valve closure schedule on transient pressures is to obtain loss coefficients for the valve at various openings and conduct computer simulations of the system behavior in response to various proposed closure schedules. Once the proper closure schedule has been determined, a control system must be devised to implement it. The cost and availability of valve closure mechanisms in relation to funding limits will narrow the exploration of various closure schedules. For example, if the only option in closing a valve is to use a constant-speed motor, then two-rate closure schedules are not relevant to the study.

13.2.2. CHECK VALVES

The best check valves to use do not slam shut but instead close at the moment when forward flow ceases. Even in this case there may be some elastic energy in the system which will cause a pressure surge at the check valve. If a damped check valve is used, it must be treated in the same manner as a closing valve during the back flow period. It is important to assure that the valve either closes quickly before a reverse flow can become large or closes slowly over a time interval that is considerably greater than the critical time of closure $2L/a$. Otherwise an objectionably high pressure could occur at the time of check valve closure. Unfortunately, this problem is difficult to analyze; to do so requires a knowledge of the back-flow loss characteristics of the valve, which is rarely available.

13.2.3. SURGE RELIEF VALVES

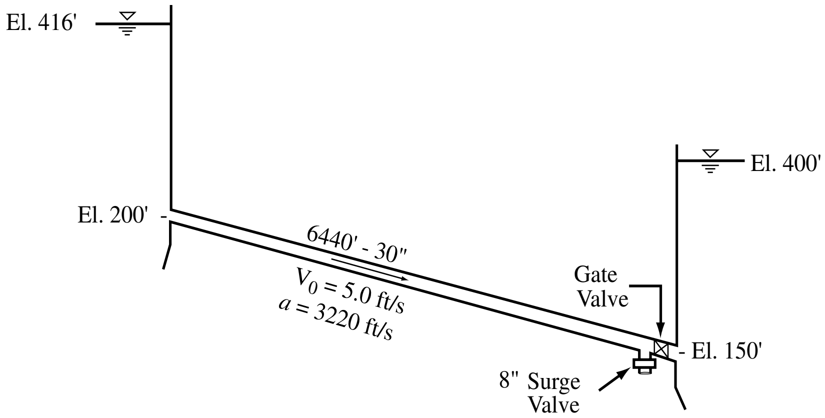
On occasion it is necessary to close valves rapidly or create other obstructions to the flow which cause abrupt decreases in velocity and result in high transient pressures. In these cases the most economical solution is often to use a surge relief valve. As Weaver (1972) describes, these valves open when a prescribed pressure is exceeded; they range from relatively inexpensive spring-loaded devices to rather expensive and complicated systems.

The surge relief valve is generally located adjacent to the device that is expected to cause the high pressure. The purpose of the valve is to provide an escape for the flowing liquid so that a sudden change in velocity and the consequent high pressures do not occur. A high-quality surge relief valve has little inertia in its actuating mechanism, so it can open almost instantaneously. It can be adjusted to operate to minimize the loss of liquid from the system and yet avoid unnecessarily high pressures during the closure process. These requirements can lead to a rather expensive valve which must be adjusted in the field for proper performance.

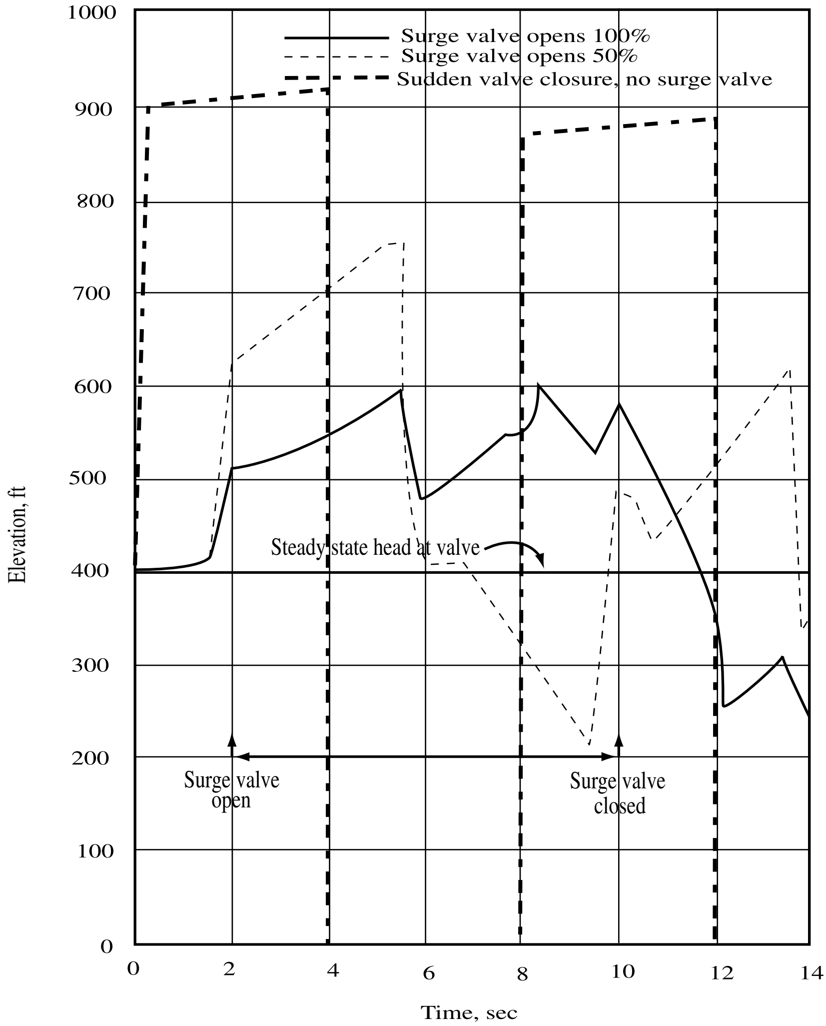
Large pipelines can be fitted with small surge relief valves because these valves can tolerate extremely high velocities for a short time period. To explore further the effectiveness of surge relief valves, we will look at an example.

Example Problem 13.1

This 30-in steel pipeline carries 11,020 gal/min between the two reservoirs. The gate valve closes in 2 sec, which is half of the critical closure time. The 8-in surge relief valve is set to open when the pressure exceeds 130 lb/in^2, and it will then close 8 sec later.



The results from three analyses are displayed in the plot and described in the paragraphs which follow.



The first analysis of the system assumed a sudden valve closure and an inoperative surge relief valve. The other two analyses treated cases with the relief valve (a) opening fully or (b) opening only 50 %.

With sudden valve closure and an inoperative surge relief valve, we obtain a typical nearly-square wave form similar to the one found in Chapter 7. When the surge relief valve is operative, the gate valve is closed in two seconds. If the surge relief valve only opens 50% of its full stroke, some pressure attenuation is achieved. However, to achieve a sig-nificant pressure reduction it is necessary to open the valve completely. The surge relief valve has reduced the potential surge pressure from over 500 ft to less than 200 ft.

The surge pressure can be regulated by the choice of surge relief valve size, opening pressure, percent of initial opening, and/or by the choice of closure schedule. Some surge relief valves are designed to close at the end of the surge in direct response to the pressure reduction in the line (spring-loaded). Others are designed to close after a specified (approximate) time interval has passed. Simulating well the behavior of a particular valve requires an intimate knowledge of the operational characteristics of that valve.

* * *

13.2.4. AIR VENTING PROCEDURES

Filling empty lines

The key to filling the empty lines of a pipeline system safely is *caution*. A means must be provided to introduce liquid slowly into the system at velocities of 1.0 ft/s or less (Johns-Manville Corp., 1977). Air release and air-vacuum valves must be located so that all air can be removed from the system slowly. Normally valves must be provided at the ends of lines so each line can be pressurized and all air can be forced out. This feature is also needed so that pressure tests can be conducted for leaks. Whatever the situation, operational procedures for the system must provide a way to control the rate of change of velocity so that severe transients do not occur.

The problem of air in the pump discharge column can be solved by replacing the vacuum valve on the pump discharge line with a valve which opens on sensing a vacuum but then closes slowly after the air is exhausted. Such a valve, much like a surge relief valve, can also be set to open at a prescribed high pressure, thus preventing the pump from ever operating at shutoff head.

In some pumped pipelines it may be necessary to provide a discharge bypass back to the sump to avoid the need to operate the pump under no-flow conditions. This bypass can prevent high pressures from developing, and it can also reduce the electrical load on the pump motor and the heat buildup in the pump itself. This feature is almost always required for axial-flow pumps.

Removing Air From Lines

Proper location and sizing of air-release and air-vacuum valves is an important consideration in pipeline design. Lescovich (1972) discusses this topic thoroughly. Seipt (1974) describes some operating techniques and addresses problems related to installation that can minimize air-related problems.

If the line is mostly filled with only relatively small pockets of air created by a shutdown, caution must still be used when the pumping system is restarted. The best approach is first to fill the empty lines by using the technique above and not resume normal operation until every air-vacuum and air-release valve has closed.

13.2.5. SURGE TANKS

Surge tanks can be used to mitigate both high and low pressures. They may act as temporary storage devices for excess liquid which has been diverted from the main flow. Such a diversion permits a much more gradual temporal change in velocity in the pipeline and a reduction in the magnitude of transient pressure waves. Surge tanks can also supply liquid to the pipeline to prevent excessive deceleration and objectionably low pressures. They may also act as damping devices on a pipeline where velocities surge back and forth frequently. There are numerous different types of surge tanks, each tailored to a particular purpose. The types that are most commonly used to protect pipelines are open-end, one-way, vented surge tanks, and air chambers. We will address each in turn.

Open-end surge tanks

The open-end surge tank is the simplest of the various types of tanks. Unfortunately, as a consequence of this simplicity, it is not commonly used in pipeline systems. The tank is connected to the pipeline so that the steady-state EL-HGL passes through the sur-face of the liquid in the tank. Any fluctuation in pressure at the surge tank connection causes flow to or from the tank, thereby moderating the pressure surges in the system. Unless the tank is quite tall and possibly rather large, it cannot accommodate large or extended pressure fluctuations. It is this disadvantage that limits its usefulness. It finds its greatest application in hydroelectric power projects where the damping features are valuable and the pressures are such that a reasonably sized tank, chamber, or tower can be employed. The cost of this type of project is generally so large that even a large surge tank or air chamber can be justified.

One-way surge tanks

In pumped flows in pipelines the one-way surge tank is commonly used because the EL-HGL is usually too far above the pipeline to employ an open-end surge tank. The one-way surge tank is used to prevent objectionably low pressures downstream from it. This tank can not prevent high pressures because the only flow is from the tank. A check valve in the connection prevents any return flow to the tank.

During normal steady-state operation the one-way surge tank is isolated from the system by the check valve. Figure 13.4 shows a typical one-way surge tank configuration. When transients occur which cause the pressure head at the tank connection to drop below the liquid level in the tank, the check valve will open, and flow from the tank into the line will occur. As a result, the liquid column is not required to decelerate so rapidly, and the pipeline EL-HGL is fixed nearly at the surge tank liquid surface. Figure 13.5 shows qualitatively how a series of one-way surge tanks placed along an uphill pipeline can prevent the column separation that is a common result of a pump power failure.

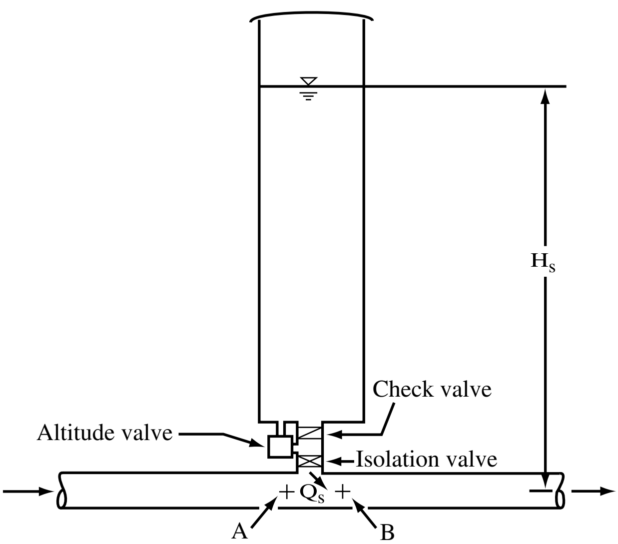


Figure 13.4 Diagram of a one-way surge tank.

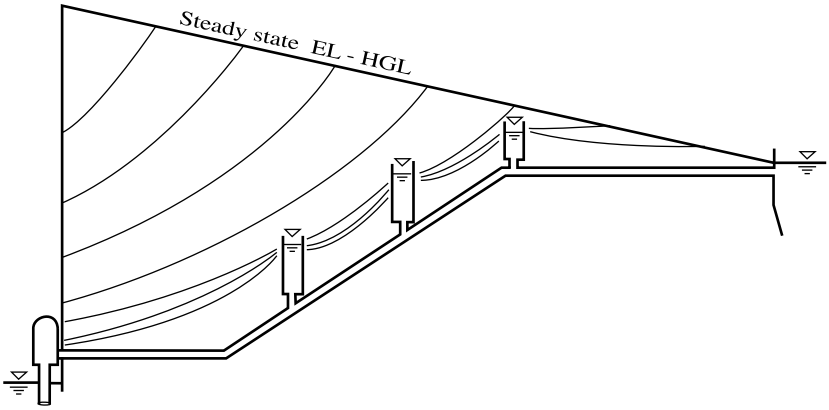


Figure 13.5 One-way surge tanks in a pumped pipeline.

To include one-way surge tanks in a transient analysis, it is necessary to model them with a particular set of interior boundary conditions. Input data to a computer program must specify the locations of the tanks, their geometry, and their hydraulic characteristics. We will now develop the equations to simulate the operation of these tanks.

We first assume that the surge tank is always sited at the junction of two series pipes. This is not a restrictive assumption, because we can always divide any pipe into a convenient number of series pipes, "breaking" the pipe at any convenient location. Referring to Fig. 13.4, we can write the following equations for the internal boundary conditions:

Upstream C+ $$V_{P_A} = C_3 - C_4 H_P \tag{13.1}$$

Downstream C– $$V_{P_B} = C_1 + C_2 H_P \tag{13.2}$$

Conservation of mass $$V_{P_A} A_A + Q_s = V_{P_B} A_B \tag{13.3}$$

Work-energy $$Q_s = C_{out} A_{out} \sqrt{2g\left(H_s + z_{AB} - H_P\right)} \tag{13.4}$$

Here C_{out} is the loss coefficient for the connecting pipe, A_{out} is the cross-sectional area of that connection, H_s is the height of the tank liquid surface above the centerline of the pipeline, and z_{AB} is the elevation of the center of the pipeline. The H_P's are not subscripted because the values of the head at locations A and B are identical. To determine when to activate this internal boundary condition, we continually monitor the pressure head at the surge tank connection. When the pressure head drops below the liquid level in the surge tank, flow from the tank begins, and the four equations must then be activated to simulate this condition.

In Eq. 13.4 the short-tube orifice equation describes the flow from the tank. The values of C_{out} can be calculated from the more readily available values for K_L for the components of the tank connection by using

$$C_{out} = \frac{1}{\sqrt{1.0 + \sum K_L}} \tag{13.5}$$

in which ΣK_L is the sum of the loss coefficients for the entrance, bends, check valve, and isolation valve, and 1.0 is the coefficient associated with the loss of one velocity head as the fluid from the tank enters the flow in the pipe. The pipe friction coefficient fL/D for the connector must be included if it is long. For a very well-designed connection C_{out} could be as large as 0.80; for a poorly designed connection C_{out} may be as low as 0.40.

In this surge tank model we have five unknowns but only four equations, so we need another equation. We resolve this problem by monitoring the height H_s of the liquid in the tank. Calling the initial height H_{s_0}, the height H_s at any later time can be found by direct integration via the equation

$$H_s = H_{s_0} - \frac{1}{A_s}\int_0^t Q_s dt \tag{13.6}$$

in which A_s is the cross-sectional area of the surge tank.

However, instead of performing the integration, we will keep a running record of the liquid height by finding the change at each time step and recomputing the new height. Thus H_s becomes known in the equation set. The relation which does this is

$$H_s(t + \Delta t) = H_s(t) - \frac{\Delta t}{A_s} Q_s(t) \tag{13.7}$$

Here we have treated the flow from the tank as a quasi-steady flow by neglecting inertial effects and assuming that the steady-flow equation of motion applies.

Solving Eqs. 13.1 through 13.4 simultaneously for the discharge from the tank, we obtain

$$Q_s = 0.5\, C_5\left(-1 + \sqrt{1 + \frac{4C_6}{C_5^2}}\right) \tag{13.8}$$

in which

$$C_5 = \frac{2gC_{out}^2 A_{out}^2}{C_2 A_B + C_4 A_A} \tag{13.9}$$

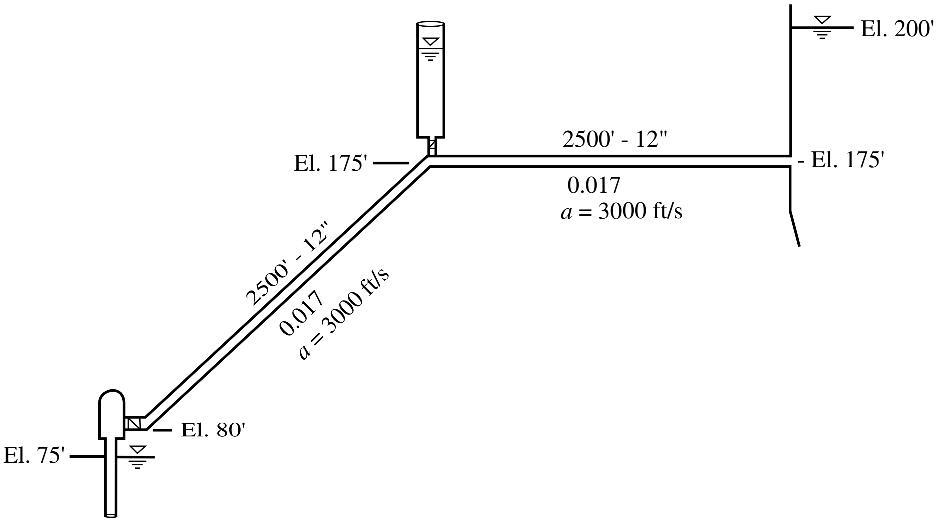
$$C_6 = \frac{2gC_{out}^2 A_{out}^2}{C_2 A_B + C_4 A_A}\left[C_1 A_B - C_3 A_A + \left(C_2 A_B + C_4 A_A\right)\left(H_s + z_{AB}\right)\right] \tag{13.10}$$

Once the surge tank begins to empty, the value of C_6 is continually tested to determine whether Q_s is going to become negative. If $C_6 < 0$, then Q_s is set to zero, thereby closing the surge tank check valve. An example is presented to show the input data that are required for a one-way surge tank and to illustrate their effect on transient pressures.

Example Problem 13.2

Two five-stage Johnston 14BC pumps with 10-in impellers (see Appendix B) are used to pump water from a river at elevation 75 ft to a reservoir with a surface elevation of 200 ft. The ductile iron pipeline is 12 in inside diameter with a friction factor of 0.017 and a wave speed of 3000 ft/s. The pipeline profile is shown on the next page.

The one-way surge tank is located at the end of the uphill run of the pipe from the pump station. The tank is connected to the pipeline with a short 12-in-diameter pipe incorporating a slant-disk check valve and an isolation gate valve. The connection loss coefficient is estimated to be $C_{out} = 0.6$.



The input data file for the final analysis follows:

```
DEMONSTRATION OF PROGRAM NO. 8 - INPUT DATA FILE "EP132.DAT"
USE OF A ONE-WAY SURGE TANK TO ELIMINATE COLUMN SEPARATION
&SPECS NPIPES=2,NPARTS=5,IOUT=1000,NSURGE=1,HRES=200.,
       HSUMP=75.,HATM=33.,ZEND=175.,TMAX=50.,QACC=0.5,AIR=F,
       PFILE=F,HVPRNT=T,PPLOT=F,GRAPH=T/
1  12.  2500.  0.017  3000.   80.
2  12.  2500.  0.017  3000.  175.
&PUMPS NPUMPS=2,NSTAGE=5,RPM=1175.,WRSQ=125.,
       QN=0.,200.,400.,600.,900.,1300.,
       HNSQ=43.,40.,38.,36.,26.,0.,
       TNSQ=4.,4.8,6.,7.2,8.,4./
1  25.  3.  12.  0.6
&GRAF NSAVE=4,IOUTSA=5,PIPE=1,1,2,2,NODE=1,4,1,3/
```

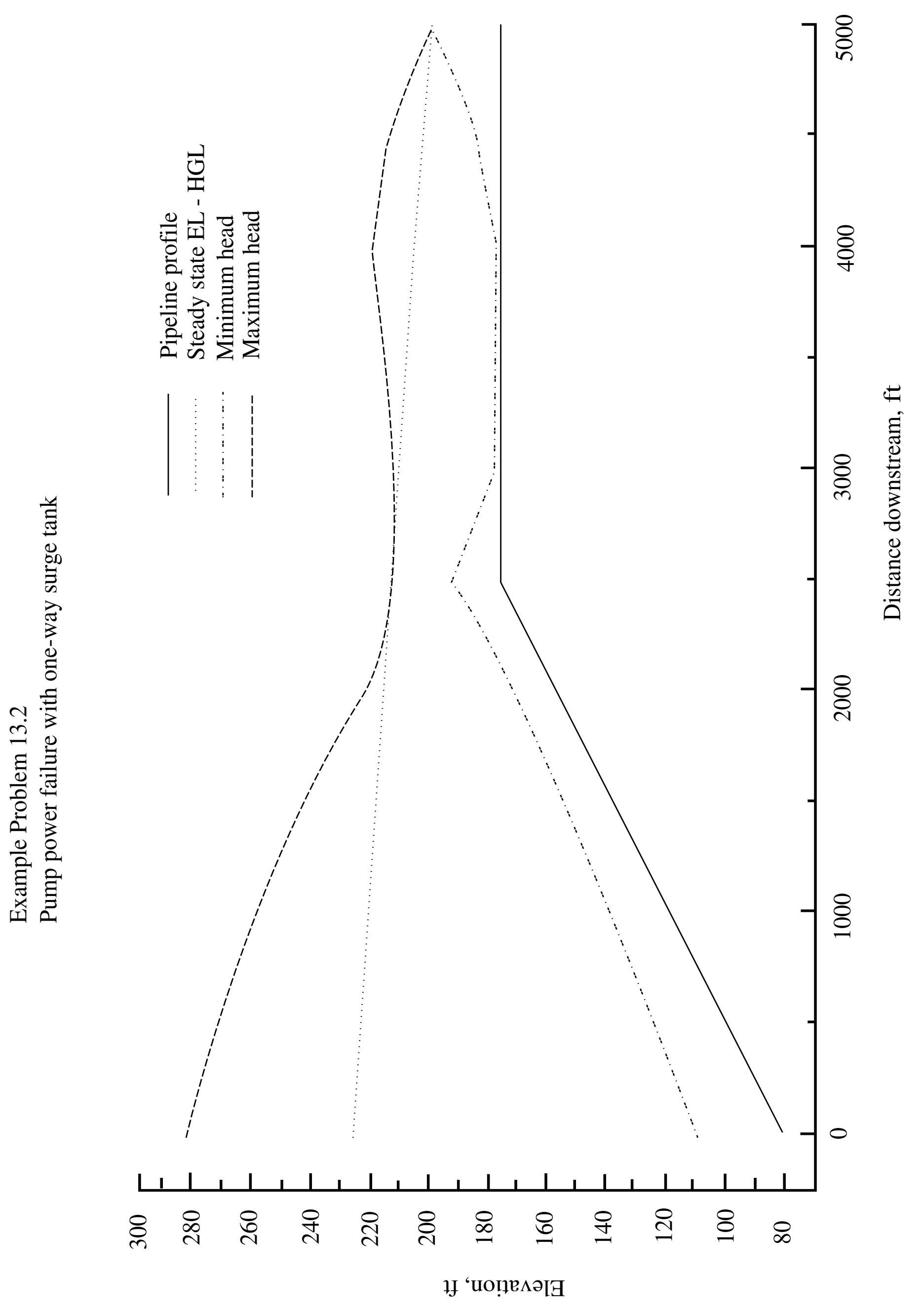
A one-way surge tank 25 ft high and 3 ft in diameter will meet the requirements. The minimum pressure of 1 lb/in^2 occurs in the horizontal section of pipe downstream of the surge tank. A plot of the maximum and minimum pressures along the pipeline is presented on the following page.

* * *

13.2.6. AIR CHAMBERS

An open-end surge tank placed on the discharge side of a pump station would be an excellent device for the control of both positive and negative surges. However, because the discharge pressure of the pumps is often quite high, the surge tank would have to be very tall to extend above the EL-HGL. This height requirement generally causes the open-end surge tank to be uneconomical, not to mention unsightly. However, there is a device which can play the role of an open-end surge tank without the height problem. The device is an air chamber (sometimes called a hydro-pneumatic tank, an air bottle or a shock trap). It is a relatively small pressurized vessel, containing both air and liquid, which is connected to the discharge line from the pump station.

The primary purpose of the air chamber is to prevent negative pressures and column separation in the pipeline downstream of the pump station during power failure rundown. However, the device can be an excellent positive surge suppresser as well. As Fig. 13.6



shows, the chamber is sealed and compressed air overlays the liquid in the chamber. After power failure occurs, liquid is drawn into the pipeline from the chamber, permitting the flow in the pipeline to decelerate more slowly and keeping the pressure relatively high. As the amount of liquid in the chamber decreases, the air volume expands, decreasing the pressure at the pump discharge. The rate at which the air pressure drops is dependent on the

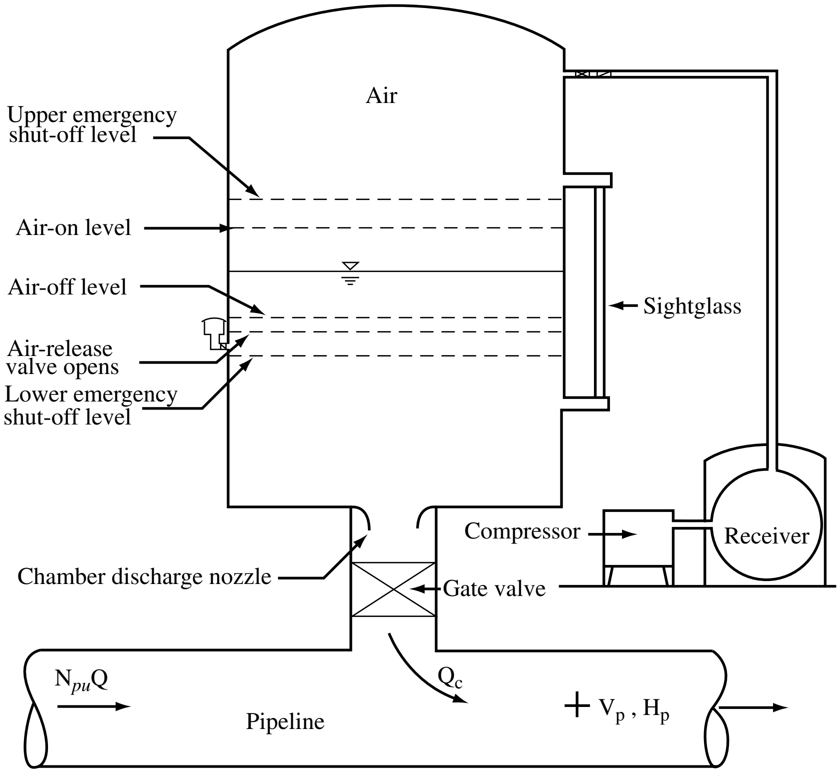


Figure 13.6 Diagram of an air chamber and its appurtenances.

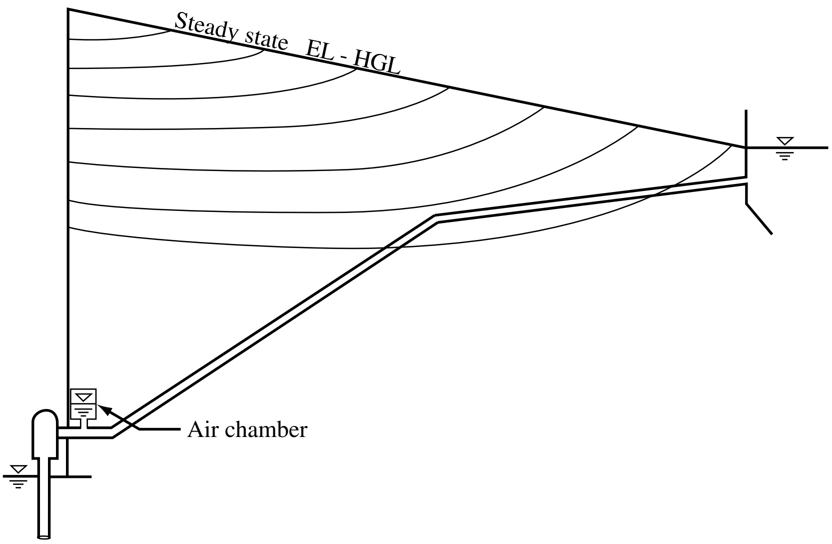


Figure 13.7 Propagation of a negative wave after pump power failure with an air chamber at the pump.

initial air volume, the rate at which liquid is drawn from the chamber, and the thermodynamic process which the air undergoes. This process simulates the dropping liquid level of an open-end surge tank and in many cases is able to bring the pipeline flow gently to rest without causing objectionably low pressures. Figure 13.7 illustrates how the air chamber affects the pressure profile during a transient incident. Compare this behavior with the scenario shown in Fig. 13.3.

The air chamber must be sufficiently large to supply the needs of the pipeline without emptying and permitting air to enter the pipeline. Also, the initial air volume must be large enough to prevent the rate of pressure drop from being excessively high. An initial air volume that is too small will cause the pump discharge pressure to behave as if the air chamber is absent, thereby giving little or no assistance in preventing low pressures. When the flow finally reverses and begins to move back toward the pumps, the check valve closes (actually it usually is already closed), and flow occurs into the chamber. To provide some damping for the system, the losses for flow *into* the chamber are deliberately made higher than the losses for flow *from* the chamber. This can be done by using a nozzle similar to the one shown in Fig. 13.6 or by having two connections to the chamber, one with a low loss for outflow and one with a higher loss for inflow. Generally, good damping can be accomplished without causing high pressures during the backflow phase.

Occasionally the air chamber alone is not adequate to prevent column separation. Low pressures can occur at local summits along the pipeline where the effect of the air chamber at the pump is inadequate. In these cases a one-way surge tank at each summit can be used to "drape" the EL-HGL above the pipeline on both sides of the summit. Figure 13.8 illustrates this technique.

The set of equations describing the behavior of an air chamber is rather complex. We will assume that the air chamber is at the upstream end of the pipeline so the boundary condition at this point will consist of the relations for both the pumps and the air chamber. We will assume that all of the pumps fail simultaneously.

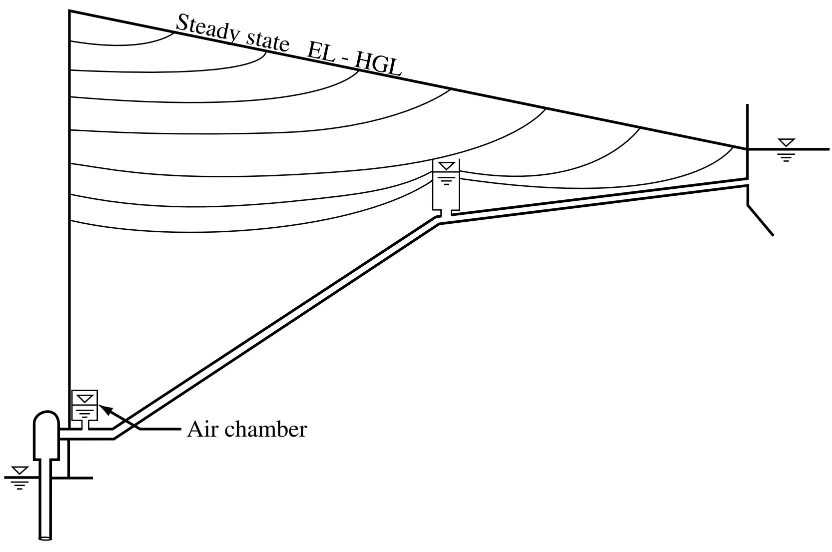


Figure 13.8 Propagation of a negative wave after power failure with an air chamber and a one-way surge tank.

The appropriate equations are the following:

C^{-} $V_P = C_1 + C_2 H_P$ (13.11)

Conservation of mass $N_{pu} Q + Q_c = V_P A$ (13.12)

Pump work-energy $$H_{sump} + h_p = H_P \tag{13.13}$$

Chamber work-energy $$Q_c = C_{out} A_{out} \sqrt{2g\left(H_c - H_P\right)} \tag{13.14}$$

Pump head increase $$h_p = N^2 N_{st}\left(C_7 \frac{Q}{N} + C_8\right) \tag{13.15}$$

In these equations H_c is the head in the chamber, C_{out} is the outflow coefficient, A_{out} is the outflow cross-sectional area, and Q_c is the discharge from the chamber.

We have six unknowns in these five equations, so another relation is required. This equation will describe the thermodynamic process that the air in the chamber undergoes. The most commonly used process is the polytropic process

$$\frac{p}{\gamma^{\eta}} = \frac{p_o}{\gamma_o^{\eta}} \tag{13.16}$$

in which p_o and γ_o are the absolute pressure and specific weight of the air in the chamber under steady-flow conditions, p and γ are those values at a later time, and η is the polytropic exponent, generally chosen to be 1.2. There is some disagreement over the appropriateness of this value. Graze (1972) and Graze et al. (1976) have shown that this process does not describe precisely the thermodynamic behavior of the air. One complicating feature during the air expansion process is that the freezing temperature of the liquid is often reached. The latent heat released by the freezing of condensed liquid vapor complicates the thermodynamic process beyond the simplicity of the polytropic model. However, in light of the many other uncertainties in the analysis, we will continue to use the polytropic equation until a better model that is reasonably easy to use appears.

With H_{atm} as the atmospheric pressure head, the polytropic equation can be written

$$\frac{\left(H_c + H_{atm} - z_P\right)\gamma_w}{\gamma^{\eta}} = \frac{\left(H_{c_o} + H_{atm} - z_P\right)\gamma_w}{\gamma_o^{\eta}} \tag{13.17}$$

If the initial air volume is $\forall_{c_o}$ and the volume at a later time is $\forall_c$, the equation can be written as

$$H_c = z_P - H_{atm} + \left(H_{c_o} - z_P + H_{atm}\right)\left(\frac{\forall_{c_o}}{\forall_c}\right)^{\eta} \tag{13.18}$$

in which the air volume at any time can be calculated in a way that is similar to the fluid volume computations for the one-way surge tank:

$$\forall_c(t + \Delta t) = \forall_c(t) + Q_c(t)\,\Delta t \tag{13.19}$$

Now we solve the six equations simultaneously.

For the most general case we assume flow continues both through the pumps and from the chamber. Knowing $\forall_c$ from Eq. 13.19, we can calculate H_c from Eq. 13.18, reducing the number of unknowns to five. Solving the five equations for Q_c produces

$$Q_c = 0.5C_5\left(1-\sqrt{1-\frac{4C_6}{C_5^2}}\right) \tag{13.20}$$

in which

$$C_5 = \frac{2gC_{out}^2A_{out}^2NN_{st}C_7}{N_{pu} - NN_{st}AC_7C_2} \tag{13.21}$$

and

$$C_6 = 2gC_{out}^2A_{out}^2\left[-H_c + \frac{H_{sump} + \left(NN_{st}/N_{pu}\right)\left(C_1C_7A + NN_{pu}C_8\right)}{1-\left(NN_{st}AC_2C_7\right)/N_{pu}}\right] \tag{13.22}$$

After calculating Q_c, Q is found from

$$Q = \frac{1}{N_{pu}}\left[AC_1 + AC_2\left(H_c - \frac{Q_c^2}{2gC_{out}^2A_{out}^2}\right) - Q_c\right] \tag{13.23}$$

If $Q \geq 0$, then the solution is acceptable, and the remaining unknowns may be calculated from Eqs. 13.11 through 13.15.

If $Q < 0$, then the pump check valves must be closed, and Q must be set to zero. Now Q_c must be calculated from

$$Q_c = 0.5C_5\left(-1+\sqrt{1+\frac{4C_6}{C_5^2}}\right) \tag{13.24}$$

where

$$C_5 = \frac{2gC_{out}^2A_{out}^2}{C_2A} \tag{13.25}$$

and

$$C_6 = 2gC_{out}^2A_{out}^2\left(H_c + \frac{C_1}{C_2}\right) \tag{13.26}$$

Once the pumps are off line and the check valves are closed, they are never reopened. Flow from the air chamber will continue until the flow reverses its direction.

To represent flow into the air chamber correctly, a different set of equations must be solved. With flow into the chamber, the pumps are not a factor, so the following equations are used:

C^- $V_P = C_1 + C_2H_P$ (13.27)

Conservation of mass $\quad Q_c = V_PA \quad$ (13.28)

Chamber work-energy $\quad Q_c = -\ C_{in}A_{in}\sqrt{2g\left(H_P - H_c\right)} \quad$ (13.29)

Here C_{in} is the discharge coefficient for flow into the air chamber, and A_{in} is the inflow cross-sectional area.

Solving these equations simultaneously for Q_c leads to

$$Q_c = 0.5C_5\left(1 - \sqrt{1 - \frac{4C_6}{C_5^2}}\right) \tag{13.30}$$

in which

$$C_5 = \frac{2gC_{in}^2 A_{in}^2}{AC_2} \tag{13.31}$$

and

$$C_6 = 2gC_{in}^2 A_{in}^2\left(H_c + \frac{C_1}{C_2}\right) \tag{13.32}$$

The values of C_{out} and C_{in} can be estimated from Eq. 13.5.

Sizing the air chamber

At the upstream end of a pipeline where an air chamber is located, the variation of pressure with time depends primarily on the initial air volume in the chamber when the power failure occurs. The pressure drops more rapidly with a smaller initial air volume.

The first step in the sizing procedure is to try successive values of the initial air volume until the minimum pressures along the pipeline are acceptable. This air volume establishes the upper emergency level (see Fig. 13.6). If power failure occurs when the air volume is smaller than this, undesirably low pressures will occur.

Because pressures fluctuate during the normal operation of the system, there must also be some space in the air chamber to accommodate this variation. Evans and Crawford (1953) recommend 25% of the initial air volume at the upper emergency level for this purpose; however, the chosen value can be based on the actual operation of the pumping system.

Because it is possible for the power to fail when the initial air volume corresponds to the upper emergency level plus 25%, we must make sure the chamber is sufficiently large that it will not empty during the downsurge. Hence we make one last analysis using this larger air volume as the initial air volume. This initial air volume is associated with the lower emergency level (see Fig. 13.6). The maximum air volume which exists during this analysis establishes the minimum total volume of the air chamber. This value should be increased by another 10% or more as a factor of safety against emptying the air chamber.

While the configuration in Fig. 13.6 may be a typical schematic design for a small chamber, it may not be appropriate when large chambers are required. If a large chamber is needed, it can be replaced with several smaller chambers similar to the one in Fig. 13.6. Or it can be fabricated as a single horizontal tank that looks much like a large propane tank. Both approaches are commonly used.

Air chamber appurtenances

Air chambers do require some special appurtenances for proper operation. Because the liquid level in the chamber must be kept between the bounds of the upper and lower emergency levels (except for short term fluctuations), some provision must be made to accomplish this. If the liquid level gets too high and remains there too long, compressed air from the receiver (see Fig. 13.6) is injected into the chamber to force the liquid down. Conversely, if the water level drops too low, air is removed from the chamber via an air release valve to raise the liquid level.

Should the liquid level move above the upper emergency level and resist all efforts to bring it back down, the system should be shut down carefully to determine the cause. If the system cannot be shut down, an alarm should be sounded to alert personnel of the problem so corrective action can be initiated.

Flow from the chamber should be achieved with minimum head loss so that the pressure in the pipeline downstream remains as high as possible. If the smooth nozzle shown in

Fig. 13.6 is not practical, at least the outflow connection should be sufficiently large that the fluid velocity in it is moderate. Because the air chamber may take over for the pumps rather quickly, a good estimate of the initial flow from the chamber is the steady-state discharge from all of the pumps.

Flow into the chamber typically is designed to undergo a greater head loss than is experienced by an outflow. This situation will damp the oscillatory flow over time. The nozzle in Fig. 13.6 will accomplish this. However, there are other ways to reach the same end. Separate inflow and outflow connections can be used, with the inflow connection being smaller to create higher velocities and greater losses.

A sight glass is needed to permit observation of the liquid surface in the chamber. Water level sensors are needed to determine when to turn the compressed air flow on or off and when to signal a violation of the upper or lower emergency levels. They can be located externally to the chamber via connecting piping so that maintenance or replacement of these sensors can be done easily. A man-door is generally provided on larger chambers. A variety of drains, pressure regulators, pressure gages, air release valves etc. complete the list of devices. In cold climates the chambers are usually enclosed in a heated structure to prevent their freezing.

Example Problem 13.3

Example Problem 11.2 is reconsidered here with the objective of providing corrective devices to prevent column separation. In fact, we will attempt to eliminate negative pressures from the entire pipeline. We will present the results of five different analyses using PROG7. The five analyses consider the following scenarios:

(a) No corrective devices.

(b) Air chamber with an initial air volume of $320\ ft^3$.

(c) Air chamber with an initial air volume of $400\ ft^3$.

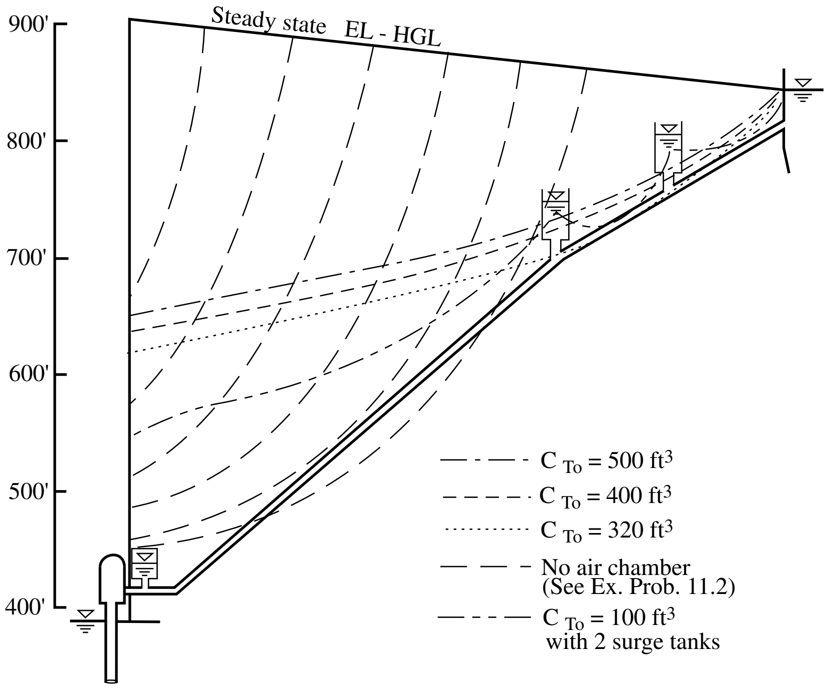
(d) Air chamber with an initial air volume of $500\ ft^3$.

(e) Air chamber with an initial air volume of $100\ ft^3$ and two one-way surge tanks.

The input data file for run (e) is presented here:

```
DEMONSTRATION OF PROGRAM NO. 7 - INPUT DATA FILE "EP133.DAT"
USE OF AN AIR CHAMBER & ONE-WAY SURGE TANKS TO ELIMINATE
COLUMN SEPARATION
&SPECS NPIPES=4,NPARTS=3,IOUT=1000,NSURGE=2,HRES=840.,
       HSUMP=395.,HATM=32.,ZEND=810.,TMAX=50.,QACC=0.50,
       AIR=T,NOPUMP=F,PFILE=F,HVPRNT=T,PPLOT=T,
       GRAPH=T,RERUN=F/
1  30.   2000.  0.013  3590.  415.
2  30.  15840.  0.013  3590.  415.
3  30.   5280.  0.019  3490.  700.
4  30.   5280.  0.019  3490.  755.
&PUMPS NPUMPS=4,NSTAGE=5,RPM=1775.,WRSQ=475.,
       QN=0.,1000.,2000.,3000.,4000.,4500.,
       HNSQ=129.,127.5,121.,103.5,67.5,0.,
       TNSQ=50.,58.,78.,92.,97.,80./
&CHAMB CTZERO=100.,COUT=0.80,CIN=0.50,DNOZ=12.00,EXPON=1.20/
2  40.  6.  12.  0.80
3  50.  6.  12.  0.80
&GRAF NSAVE=3,IOUTSA=2,PIPE=1,3,4,0,NODE=1,1,1,0/
```

The curves for analysis (a) were copied from Example Problem 11.1 and depict the progression of the negative wave downstream. Included in this plot are the results from runs (b) through (e) which represent the lower bound on pressure heads along the pipeline for that configuration.



The initial air volumes each differ in size by 25%. This approach is useful in determining the final size of the air chamber. For example, in the cases where the initial air volume was 500 ft^3, the pressures were found to be well above zero. The next air volume was 500/1.25 = 400 ft^3. This analysis still gave positive pressures so we next tried 400/1.25 = 320 ft^3. This time the minimum pressure was about zero at one point in the line, so we have now established the initial air volume at the upper emergency level. Following the 25% rule for pressure fluctuation space, we next conduct an analysis for 1.25x320 = 400 ft^3. However, this analysis has already been completed, and the information for sizing the tank is already available.

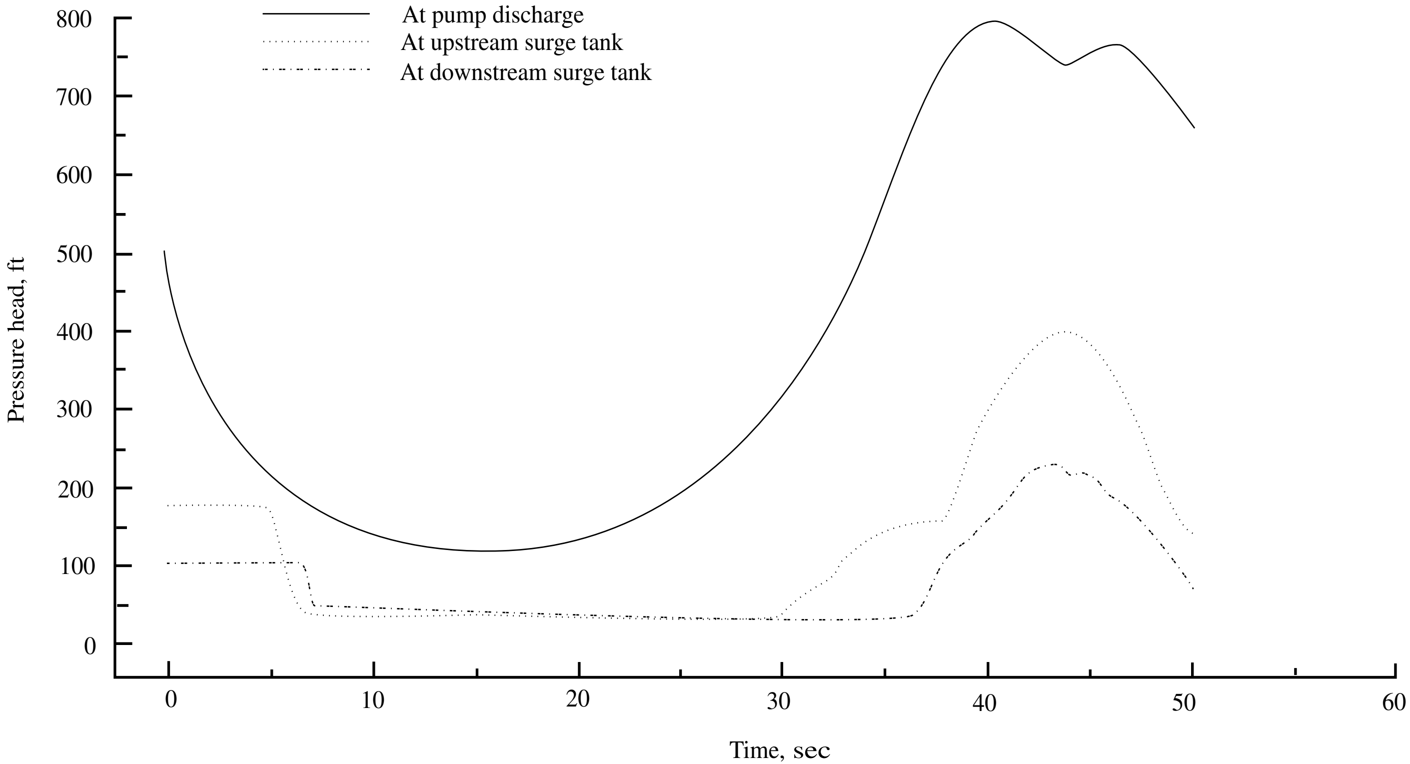
In the analysis of alternative (c) the maximum air volume reached approximately 730 ft^3. We can now proceed to the design decisions about the number and size of the air chamber units.

By using two one-way surge tanks, it is possible to reduce the air chamber volume significantly. The practicality of this alternative depends on the economics of the design and on aesthetic considerations at the site. Two one-way surge tanks, 40 ft and 50 ft high, respectively, may be more expensive and unsightly than a larger air chamber. This alternative is presented mainly to show that combinations of surge control devices may be more effective than any one device. In fact, if there were a local summit in the pipeline, it is likely that a one-way surge tank would be needed to prevent column separation downstream of the summit, and an air chamber would be needed at the pump to prevent the same problem upstream of the summit.

Design of the physical configuration of the pump station and pipeline, into which the various control devices are incorporated, is outside the subject matter of this book. For more information consult Sanks et al. (1981) for a comprehensive treatment of pumping station design.

The following two pages present a plot of head vs. time for three locations along the pipeline and a plot of the maximum and minimum heads along the pipeline.

Example Problem 13.3
Pump power failure with air chamber and one-way surge tanks

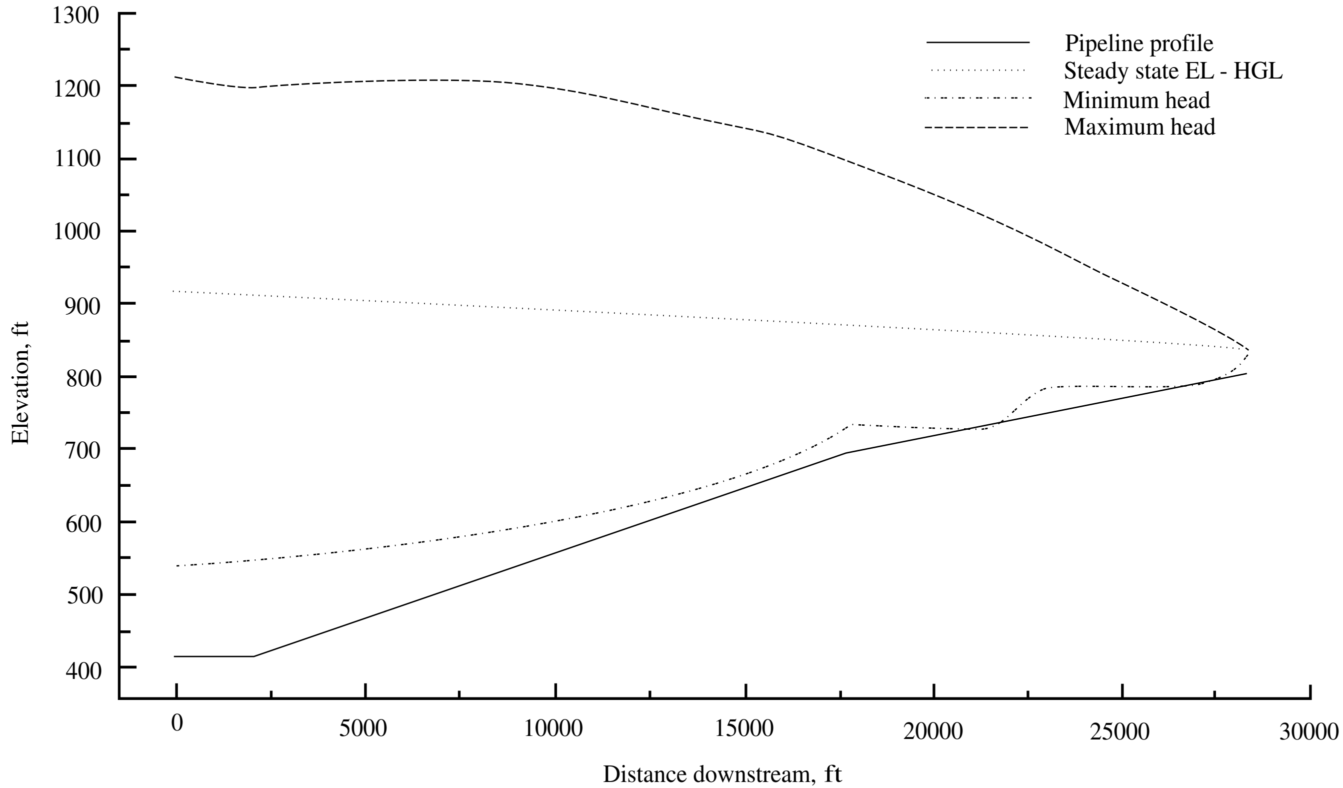


Example Problem 13.3
Pump power failure with air chamber and one-way surge tanks

*

*

*



Vented surge tanks

A variant of the one-way surge tank is the vented surge tank. This device is conceptually a one-way surge tank for downsurge and an air chamber for upsurge. The tank is sealed and equipped with a vacuum valve (actually a spring-loaded check valve exposed to the atmosphere) to permit air to enter the tank during downsurge but not exhaust air when the flow reverses (see Fig. 13.9). It also has an air release valve to bleed the ingested air out slowly. The tank is connected to the pipeline through an open line so that the line pressure is continually communicated to the tank, i.e. the tank is continually "on line." When the EL-HGL drops to the level of the liquid in the tank during a downsurge, the vacuum valve opens, permitting air to enter the tank to replace the liquid flowing into the line; this behavior mimics the behavior of a one-way surge tank. With an upsurge the liquid flows into the tank, but the air is prevented from exhausting rapidly by the vacuum (check) valve, so it is compressed much like the air in an air chamber on upsurge. The compressed air acts as a shock absorber in the system. The size of the air release valve is sufficiently small that it does not materially affect the compression process. After a short time the air release valve has bled all of the air from the tank, and it is once again full of liquid and ready to function.

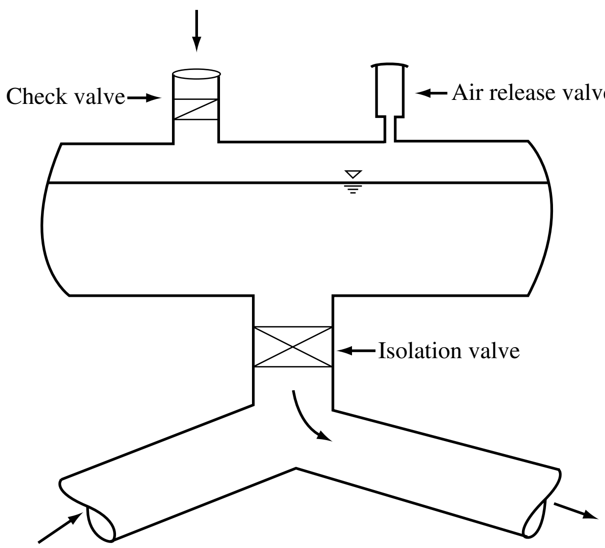


Figure 13.9 Schematic diagram of a vented surge tank.

The additional versatility of the vented surge tank in comparison with the one-way surge tank can be useful if reverse flow problems are anticipated. However, the tank must be designed to withstand the line pressure. Unless it is as tall as the one-way surge tank, it loses some of its effectiveness in preventing column separation. It is most effective at a well-defined summit of a pipeline.

Computationally, the vented surge tank is modeled as a one-way surge tank on downsurge (keeping track of the accumulating air volume). When flow begins to enter the tank, it is then treated like an air chamber. Computer code from the previous two sections can be utilized to accomplish such an analysis.

13.2.7. OTHER TECHNIQUES FOR SURGE CONTROL

Air-vacuum valves

Air-vacuum valves are a potential source of surge control. By opening when the pressure in the line drops below atmospheric pressure, they expose the pipeline to the atmosphere, which permits the liquid to decelerate more slowly. This does not prevent the occurrence of column separation at points in the pipeline that are remote from the location of the air-vacuum valves, but it may reduce the intensity of a cavity closure shock by keeping the velocities lower. However, a problem may develop when (1) a pump restart or (2) a reversed flow driven simply by gravity causes the air to exhaust from the pipe and slam the air-vacuum valve shut. To determine whether air-vacuum valves are an asset or a liability in a pumped pipeline system, one can use PROG8 to analyze the system. For a pumped pipeline this analysis will permit one to develop an operational strategy which will minimize transient pressures.

Surge anticipation valves

In some cases the low pressures associated with column separation are not a problem. Rather the problem to avoid is the creation of potentially high shock pressures from the closure of a cavity. The surge anticipation valve provides this service. Although it does not prevent column separation, it minimizes the impact of cavity closure. It is most effective in pumping systems which run uphill with no major intermediate summits in the pipeline profile. Because the pressure could drop below atmospheric for some time over a large fraction of the pipeline, air may be ingested into the pipeline through air-vacuum valves. This may or may not be a positive occurrence for surge control, as it depends on the amount of air ingested and a host of other considerations. We are also then faced with the removal of the air in the system as a part of the pump restart procedure. Finally, the pipeline should be designed to withstand vacuum pressures because they are likely to occur somewhere along the pipeline.

The surge anticipation valve is a specially-operated surge relief valve placed at the upstream end of a pipeline. It is adjusted so that the valve opens after pump power failure when the pressure at the valve drops below a set value. As a consequence, the pressure at the valve quickly drops nearly to atmospheric pressure, causing the pressure in the line downstream to drop sharply. The liquid in the line decelerates rapidly with extensive column separation likely to occur. When the flow reverses, the surge valve is already open so the reverse flow is routed out of the system without a sudden decrease in velocity which would cause cavities to close and lead to high shock pressures. That is, the nearby cavities may be washed out of the system through the open surge anticipation valve. The valve then closes slowly to bring the reverse flow gently to rest.

Pump inertia control

Because column separation in pumped pipelines is normally the direct result of the low rotational inertia of the pumping system, increasing the inertia is another means of mitigating column separation. The impact of increasing the moment of inertia of the pump and motor unit was demonstrated by Streeter and Wylie (1967); for a long pipeline where most of the pumping head is used to overcome pipe friction, they show that a quadrupling of the rotational inertia would prevent column separation. This can be accomplished by incorporating a flywheel into the linkage between the pump and motor. It is easiest to manage this for pumps driven by diesel or gasoline engines. This alternative is attractive because it is simple, low maintenance, and relatively inexpensive. However, owing to the practical limits on how much inertia can be added, the method has not found wide application.

13.3 PROBLEMS

13.1 For Problem 11.1 use PROG7 to determine the appropriate size for an air chamber which will prevent any negative pressures from occurring in the pipeline.

13.2 For Problem 11.4 what is the size of the air chamber which will prevent any negative pressures from occurring in the pipeline? Use PROG7.

Complete another design which employs both an air chamber and one-way surge tanks to accomplish the same purpose.

13.3 For Problem 11.5 use PROG7 to determine the size of an air chamber which will prevent any negative pressures from occurring in the pipeline.

Investigate the feasibility of a design using only one-way surge tanks.

Complete a third design using an air chamber and one-way surge tanks.

13.4 For Problem 11.3 use PROG7 to determine the size of an air chamber which will prevent negative pressures from occurring over any significant portion of the pipeline.

13.5 A preliminary design is being prepared for a pumped pipeline which lifts water from a reservoir at elevation 5760 ft to a canal at elevation 6220 ft. The steel pipe is 72 in in diameter with a friction factor of 0.020 and a wave speed of 3100 ft/s. The pipeline profile is described in the following table:

Station, ft	Elevation, ft
0	5765
350	5875
1100	5905
2100	6165
3150	6095
6400	6115
7700	6185
13,400	6055
14,000	6200

The pumping station employs ten six-stage pumps operating at 1170 rev/min. Each pump and motor unit has Wr^2 = 1500 lb-ft^2. The pump characteristics are given in the following table:

Discharge, gal/min	Head/stage, ft	BHP/stage, hp
0	145	180
2500	124	180
5000	104	180
6500	87	180
8500	62	180
10,500	10	180

Use PROG7 to devise an air chamber and one-way surge tank configuration which will prevent negative pressures from occurring in the pipeline. Consider reducing the number of series pipes by approximating the pipeline profile. This approach will allow more freedom in choosing Δt and save both computational time and machine storage.

13.6 A ten-stage pump lifts water through an 8-inch steel pipeline (7.85 in inside diameter) from a reservoir at elevation 4700 ft to a reservoir at elevation 5120 ft. The steel pipe is 14 ga (e = 0.075 in) with a friction factor of 0.023. The pump characteristics are for a speed of 1770 rev/min. For the pump and motor unit Wr^2 = 70 lb-ft^2.

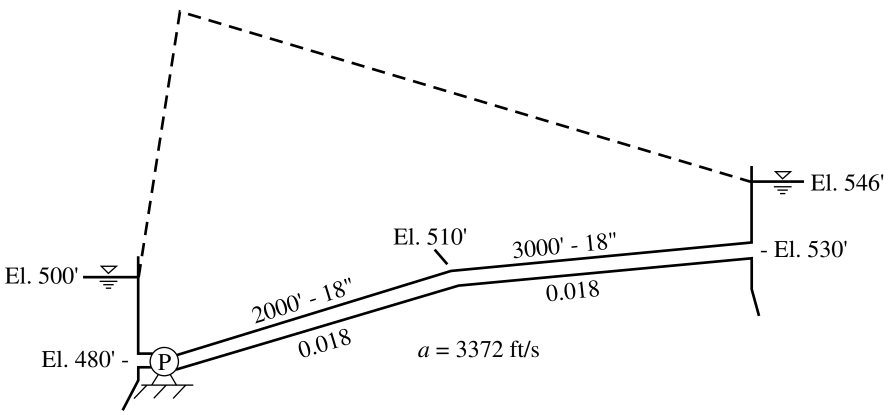
Discharge, gal/min	Head/stage, ft	BHP/stage, hp
0	80	8
300	71	9
600	65	10
900	60	13
1200	48.5	15
1500	32	14

The pipeline profile is as follows:

Station, ft	Elevation, ft
0	4700
2210	4950
4780	5040
5120	5040
5600	5100
6400	5115

Use PROG3 to analyze the system for pump power failure. If column separation occurs, use PROG7 to find the air chamber size which will prevent negative pressures from occurring over a significant portion of the pipeline.

13.7 In the pipeline system shown below, two two-stage Ingersoll-Dresser 20KKH pumps are placed in parallel to pump water into the upper reservoir (For pump characteristic diagrams see Appendix B). Driven by diesel engines, the pumps use 15-in impellers and have $Wr^2 = 330$ lb-ft^2 for the pump and motor combination.



The design engineer suspects that a power failure will cause column separation in the pipeline. Use PROG3 to determine whether this suspicion is correct.

If so, the decision has been made to add flywheels to the pump shafts to slow the pump deceleration and prevent column separation. Apply PROG3 again to determine the value of the moment of inertia for each of the proposed flywheels that will be required to prevent column separation.

How large must each value of the moment of inertia be to prevent any negative pressures from occurring?

CHAPTER 14

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APPENDIX A

NUMERICAL METHODS

A.1 INTRODUCTION

The goal of Appendix A is to provide enough information so the reader can effectively use some subroutines (functions) that implement commonly used numerical methods. For detail about the methods readers may refer to any of a number of books on numerical analysis; for example, one "oldy but goody" is *Applied Numerical Methods*, by Carnahan et al. (1969). *Numerical Recipes: The Art of Scientific Computing*, by Press et al. (1992) with versions that emphasize either Fortran, Pascal, C or Basic, provides detail on effectively implementing these methods in computer codes. The order in which numerical methods will be described in this appendix is (1) linear algebra, (2) numerical integration, and (3) the solution of ordinary differential equations (ODEs).

If the derivative of the dependent variable y with respect to the independent variable x is only a function of the independent variable, then the solution $y = f(x)$ can be obtained by direct integration. If dy/dx depends upon both y and x, then the methods for solving ODEs must be used. Sometimes it is possible to rearrange the form of the original equation so only x appears on one side of the equal sign, and y on the other, i.e. separate variables, and then integration will provide the solution. The same principles apply for second derivatives etc. Since the methods for solving ODEs normally let $dy/dx = f(x, y)$, and this implies dy/dx may only be a function of x or y, the methods for solving ODEs can be used to solve a problem for which numerical integration could be used. However, the reverse is not true.

A.2 LINEAR ALGEBRA

A.2.1. GAUSSIAN ELIMINATION

The simplest method for solving a linear system of equations is Gaussian elimination; in this method we multiply an equation, or row in the coefficient matrix, by a value so that the first term in a resulting equation becomes zero, or is eliminated, when we subtract that equation from a given equation . This process is continued until all elements before the diagonal are zero. Then the solution vector is obtained by back substitution. (If you are unfamiliar with Gaussian elimination, you should read about it in a book on linear algebra, because the following discussion will assume you have some understanding of this method.) While it is simple and straightforward in its implementation in computer codes, Gaussian elimination can produce inaccurate solutions due to truncation error. For example, in the elimination process the product of two values, when it is subtracted from another value, can produce a difference that is several digits less accurate than can be carried in the word length being used by the computer. Therefore, especially when using single precision in a computer program, it is well to improve the accuracy by applying iterative corrections to the solution vector. The subroutine GAUSEL on the accompanying CD is a relatively simple program that uses one iterative correction to the Gaussian elimination method in solving a linear system of equations $[A]\{x\} = \{b\}$. You should obtain a listing of this code and study it as you continue reading this section. Comments in the code indicate what is done by the statements in the section which follows.

Gaussian elimination first solves the linear system $[A]\{x\} = \{b\}$ for the approximate solution $\{x_a\}$. This solution can be denoted as $[A]^{-1}\{b\}$. Next the residual vector $\{r\}$ is computed, which is defined by $[A]\{x_a\} = \{b\} + \{r\}$. By subtracting the original matrix equation from this matrix equation, we obtain

$$[A](\{x_a\} - \{x\}) \equiv [A]\{e\} = \{r\} \tag{A.1}$$

or

$$(\{x_a\} - \{x\}) \equiv \{e\} \approx [A]^{-1}\{r\} \tag{A.2}$$

The first of these equations indicates, if $\{r\}$ is used in place of $\{b\}$, that the same solution process that has obtained $\{x_a\}$ can be used to find the error vector as $\{e\} = [A]^{-1}\{r\}$. In this case $\{e\}$ can be considered an approximate value for the error vector $\{e\} = \{x_a\} - \{x\}$. By rearrangement of the terms, with the subscript i to indicate the iteration number, we might write $\{x\}_i = \{x_a\}_i - \{e\}_i$. This iterative equation indicates for any solution component $\{x\}_i$ that an improved approximation is the original calculated value minus the calculated error value. Each subsequent error should be smaller than the current estimated error, and therefore $|\{e\}_i|$ will be a conservative estimate of the error in the new approximation. The relative error is defined as $|\{e\}_i|/|\{x_a\}_i - \{e\}_i|$. Generally one iterative improvement is all that is necessary, and that is what is done in GAUSEL.FOR.

The call to this subroutine should contain a statement of the form

```
CALL GAUSEL(N, M, A, B, DET, ERRNOR)
```
(A.3)

The arguments are the following:

N = number of equations to be solved; A must then contain N x N values, and B must contain N values.

M = the dimensions of arrays A and B in the main calling program, A(M, M) and B(M).

A = the coefficient matrix [A]. This matrix will always be a two-dimensional array that is dimensioned A(M, M).

B = the known vector {B}, which will be a one-dimensional array in the calling program that is dimensioned B(M).

DET = the determinant for the matrix [A]. Its value is returned from GAUSEL.

ERRNOR = the estimate of the relative error; it must be dimensioned in the calling program as a one-dimensional array ERRNOR(M). The values in this array are returned by GAUSEL and provide a way to decide whether the solution has sufficient accuracy.

The subroutine always returns the determinant in DET and the relative error for each component of the unknown vector in ERRNOR. If these are not wanted, then they can be eliminated from the statements in the subroutine. When GAUSEL is written to use double precision, then A, B, DET, and ERRNOR must also be double precision in the calling program.

A.2.2. USE OF THE LINEAR ALGEBRA SOLVER SOLVEQ

This section describes subroutine SOLVEQ, a more sophisticated subroutine than GAUSEL. It will (1) solve a linear system of equations, given the coefficient matrix and the known vector, (2) provide the inverse matrix of a square matrix, (3) evaluate the determinant, (4) evaluate the determinant and produce the inverse matrix, and/or (5) evaluate the determinant, produce the inverse matrix and solve the system of equations. SOLVEQ is used by a number of programs described in this text. You should extract it (the object element if you are using MS-Fortran, or the source code if you are using another compiler so you can create an object element, or the C source if you are a C user) from the CD, so it will be available to link with programs that use it.

Subroutine SOLVEQ must be called by a program that defines the problem it is to solve; the program does this by supplying the coefficient matrix in a two-dimensional array and, if it is required, the known vector in a one-dimensional array. In Fortran the matrix and vector indexes begin with default subscript 1 and end with subscript N. Thus these arrays are dimensioned as REAL A(100, 100), B(100). A call to subroutine SOLVEQ should consist of the following (the names of the arguments can be different, but the types must be as described below, and the dimensions of arrays must be as indicated.):

```
CALL SOLVEQ(N, NPROB, NDIM, A, B, ITYPE, DET, INDX)
```
(A.4)

The arguments in the call are as follows:

N = the integer number of equations to be solved, or the size of the matrix if only the inverse is requested. The program that calls SOLVEQ must supply values for a square coefficient matrix with N rows and N columns.

NPROB = the integer number of problems to be solved by providing solution vectors, i.e., we seek NPROB separate solutions from NPROB known vectors. (A modified version of SOLVEQ may omit this argument.)

NDIM = the integer number of dimensions of matrix A(NDIM, NDIM) and vector B(NDIM). NDIM can be larger than, or equal to, N. Its value allows SOLVEQ to locate the proper positions of the elements within the two-dimensional coefficient array A.

A = the real two-dimensional array in the calling program which contains the coefficient matrix; it must be square with N rows and N columns. The correct coefficient values for the problem to be solved must all be contained within this array upon entry into subroutine SOLVEQ. Upon returning from SOLVEQ, this two-dimensional array will contain the inverse matrix, if it is requested. The values of the coefficient matrix are altered during the execution of SOLVEQ.

B = a real array containing the known vector {b} in the linear system of equations [A]{x} = {b}, and the correct values for this known vector must be in the elements of B when SOLVEQ is called. Upon returning from the call to SOLVEQ, this array will contain the solution vector {x} for the linear algebra problem. Generally B will be dimensioned as a one-dimensional array. However, if NPROB is greater than one so that more than one linear algebra problem is to be solved with the same coefficient matrix [A], then B can be a two-dimensional array with the second dimension being NPROB.

ITYPE = the integer that tells SOLVEQ which tasks are to be done, described by the value selected from the following menu of choices:

- = 1, solves the linear system of equations;
- = 2, produces the inverse matrix (in A);
- = 3, evaluates the determinant and places the result in DET;
- = 4, solves the equation set and produces the inverse matrix;
- = 5, evaluates the determinant and produces the inverse matrix;
- = 6, finds the determinant and the inverse matrix and solves the equations.

DET = a real variable that returns the value of determinant if it is requested.

INDX = an integer *2 one-dimensional array with the size NDIM which is used for work space. Upon entry to SOLVEQ, it can be empty or it can be another integer array used subsequently in the calling program. The values in this array will be destroyed upon returning from SOLVEQ, so if it is an INTEGER*2 array used for some other purpose, this purpose must be located after all calls to SOLVEQ have been completed.

Example programs in the body of the text can be used as examples of how to implement a call to SOLVEQ properly. If you wish to work in C (C++), then you should print the file SOLVEQC.DOC from the CD to obtain additional help in using the function solveq.

A.3 NUMERICAL INTEGRATION

A.3.1. TRAPEZOIDAL RULE

The trapezoidal rule (TR) states that an approximation to the integral of a function $f(x)$ from x_b (beginning value of the independent variable) to x_e (ending value of the independent variable) equals the average of the function, evaluated at these two end points, times the interval $\Delta x = x_e - x_b$, or

$$\Delta F = \int f(x)dx = (x_e - x_b)\{f(x_b) + f(x_e)\}/2 \tag{A.5}$$

The accuracy of the numerical integration depends upon the size of the interval Δx. In other words, to satisfy an error requirement the Δx in the process must be chosen to be small enough. How can it be determined what is small enough? Normally Δx is small enough when the result that is obtained by using an increment $\Delta x/2$ produces the same final answer as when Δx is used. In other words, the numerical integration can be repeated after reducing Δx (usually by a factor of two); then these results are compared with those previously obtained, until the difference is less than some chosen error criterion. If this process is implemented without a consideration of how to minimize the amount of computation, much more arithmetic will be done than is required. We now describe an algorithm that allows a reduction in interval size without losing the benefit of the previous arithmetic. To facilitate this discussion a first order approximation, the trapezoidal rule (TR) will be used.

Applying the TR repeatedly over consecutive intervals Δx, in which the integration interval has been divided into N intervals $\Delta x = (x_e - x_b)/N$, produces the following result:

$$F(x_e) - F(x_b) = \Delta x\left\{f_0/2 + f_1 + f_2 + \ldots + f_{N-1} + f_N/2\right\} \tag{A.6}$$

Thus the function values at all intermediate points are added together, except for the first and the last which are halved before being added. Here $f_0 = f_b$ (the function at the beginning of the interval) and $f_N = f_e$ (the function at the end of the interval). Now how can this extended TR be implemented repeatedly with new Δx's that are each equal to one-half the previous value without losing the previous evaluations of the function, i.e. the previous arithmetic? To visualize how such an algorithm can be developed, consider first the coarsest implementation of the TR as the average of the function at the two end points x_b and x_e, multiplied by $(x_e - x_b)$, as shown in Fig. A.1 with $N = 1$. When the range of integration is divided into two intervals with $\Delta x = (x_e - x_b)/2$, the function must be evaluated at one additional point, the midpoint shown for $N = 2$ in Fig. A.1. The application of the extended TR will multiply the previous end values by 1/2 because Δx is now half as large as before, and this result is then added to the value of the function at the midpoint, multiplied by Δx. Upon dividing the range of integration into 4 intervals so $\Delta x = (x_e - x_b)/4$, the function must be evaluated at two additional points, i.e., at $x = x_b + (\Delta x)_{i-1}/2$ and at $x = x_b + (\Delta x)_{i-1}/2 + (\Delta x)_{i-1}$, in which $(\Delta x)_{i-1}$ is the previous increment. These two additional points are shown on the line associated with $N = 3$. For $N = 4$ the function must be evaluated at four additional points, and this process continues. As shown in Fig. A.1, the sum of all of the evaluations provides all of the values that are needed to implement the extended TR. This process could continue until the evaluations of the integral between consecutive increases in N produce the same value within the error limit that has been selected.

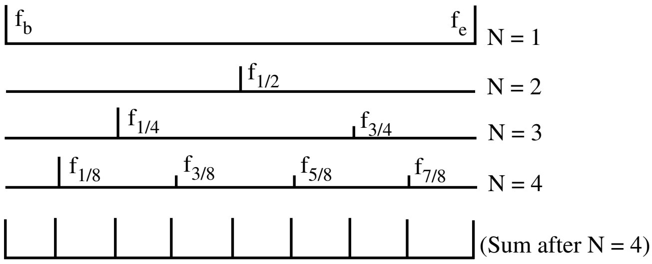


Figure A.1 Implementation of trapezoidal rule with ever decreasing increments Δx that are half of the previous increment, so that only functions at the new points are evaluated as Δx is decreased.

The Fortran listing (a C function is on the CD) in Fig. A.2 implements this algorithm; thus it is a subroutine that numerically evaluates an integral using the TR.

```
      SUBROUTINE TRAPR(EQUAT, XB, XE, VALUE, ERR, MAX)
      EXTERNAL EQUAT
      EV=-1.E30
      VALUE=0.5*(XE-XB)*(EQUAT(XB)+EQUAT(XE))
      M=1
      I=1
   10 I=I+1
      DELX=(XE-XB)/FLOAT(M)
      X=XB+0.5*DELX
      SUM=0.
      DO 20 J=1,M
      SUM=SUM+EQUAT(X)
   20 X=X+DELX
      VALUE=0.5*(VALUE+(XE-XB)*SUM/FLOAT(M))
      M=2*M
      IF(ABS(VALUE-EV).LT.ERR*ABS(EV)) RETURN
      EV=VALUE
      IF(I.LT.MAX) GO TO 10
      WRITE(*,*)' FAILED TO SATISFY ERROR REQ.', VALUE-EV
      RETURN
      END
```

Figure A.2 Program TRAPR.FOR. It integrates a function by repeatedly reducing Δx until the selected error criterion is satisfied.

To use this subroutine, two other programs are needed. The first program is a FUNCTION subroutine (with the name EQUAT as the first argument in the call to TRAPR) that evaluates the integrand (the function being integrated) at the argument X; so this program begins FUNCTION EQUAT(X). The second program is a main program that, among other tasks, properly calls TRAPR.

The arguments for TRAPR are as follows:

EQUAT = the name of the external FUNCTION SUBPROGRAM that evaluates the integrand at the given X value.

XB = the real value of the independent variable at the beginning of the interval.

XE = the real value of the independent variable at the end of the interval, i.e., the integral is from XB to XE.

VALUE = the real value of the integral that is returned to the main program.

ERR = the relative error criterion, a real number. The increment Δx will be repeatedly reduced by one half until the absolute difference between two successive values of the integral is less than the product of ERR and the absolute value of the integral from the previous evaluation. A value for ERR of 1.0×10^{-6} is near the limit that can be used with 32 bit arithmetic and not have truncation error cause the algorithm to fail to meet the criterion.

MAX = the maximum integer number of reductions in Δx that are allowed.

If the function can be defined as a single statement in the declaration portion of the main Fortran program, then a function statement can be used in place of the FUNCTION EQUAT. This approach is used in Example Problem A.1 below.

A.3.2. SIMPSON'S RULE

Simpson's rule is a double interval integration formula; that is, it evaluates the integral over $2\Delta x$ and produces a second-order approximation by passing a second degree polynomial through three consecutive, evenly spaced points. Simpson's Rule is

$$\Delta F_{i-1}^{i+1} = \Delta x\left\{f_b + 4f_m + f_e\right\}/3 \tag{A.7}$$

in which f_m is the integrand at the midpoint of the interval $2\Delta x$, that is, at $x = \Delta x$.

As with the trapezoidal rule, Simpson's rule can be implemented in an algorithmic form that is arithmetically efficient by comparing the result with a new interval size Δx with that which was previously obtained by using $2\Delta x$. The algorithm works in the following way. We start with the entire range of the independent variable as the increment $\Delta x_0 = x_e - x_b$. An approximate (TR) value for the integral is $\text{VALU1}_0 = \Delta x_0(f_b + f_e)/2$. If we then divide this interval by 2 so $\Delta x_1 = \Delta x_0/2$ and evaluate the function at the original midpoint to obtain f_m, we can apply this rule twice and add to obtain an approximate value for the integral from x_b to x_e as $\text{VALU1}_1 = \Delta x_1(f_b + 2f_m + f_e)/2$. Simpson's rule will be obtained if we multiply this new value by 4, subtract the first value and divide the ensuing result by 3, or

$$\begin{aligned} VALUE &= (4VALU1_1 - VALU1_0)/3 \\ &= \left\{4\Delta x_1\left(f_b + 2f_m + f_e\right)/2 - \Delta x_1\left(f_b + f_e\right)\right\}/3 \\ &= \Delta x_1\left(f_b + 4f_m + f_e\right)/3 \end{aligned} \tag{A.8}$$

This algorithm can be applied with a successive halving of Δx_i for i = 2, 3, ..., and the new approximate value of the integral for Simpson's rule, *VALUE*, that is associated with each new, halved interval is

$$VALUE = (4VALU1_i - VALU1_{i-1})/3 \tag{A.9}$$

in which the $VALU1$'s are obtained by the trapezoidal rule algorithm. Each $VALU1_i$ is evaluated with the new Δx, and each $VALU1_{i-1}$ is evaluated with the previous Δx.

A Fortran listing of SIMPR, which implements Simpson's rule to evaluate integrals numerically, appears in Fig. A.3. (A similar C function is on the CD.) Its arguments are identical to those for subroutine TRAPR. In fact, to use it in a program that previously called TRAPR, just change the name itself to SIMPR.

```
      SUBROUTINE SIMPR(EQUAT, XB, XE, VALUE, ERR, MAX)
      EXTERNAL EQUAT
      EV1=-1.E30
      EV=-1.E30
      VALU1=0.5*(XE-XB)*(EQUAT(XB)+EQUAT(XE))
      M=1
      I=0
   10 I=I+1
      DELX=(XE-XB)/FLOAT(M)
      X=XB+0.5*DELX
      SUM=0.
      DO 20 J=1,M
      SUM=SUM+EQUAT(X)
   20 X=X+DELX
      VALU1=0.5*(VALU1+(XE-XB)*SUM/FLOAT(M))
      M=2*M
      VALUE=(4.*VALU1-EV1)/3.
      IF(ABS(VALUE-EV).LT.ERR*ABS(EV)) RETURN
      EV=VALUE
      EV1=VALU1
      IF(I.LT.MAX) GO TO 10
      WRITE(*,*)' FAILED TO SATISFY ERROR REQ.', VALUE-EV
      RETURN
      END
```

Figure A.3 Program SIMPR.FOR. It integrates a function by repeatedly reducing Δx until the selected error criterion is satisfied.

Example Problem A.1

Integrate the function $f(x) = x^2(x^2 - 2)sin(x)$ between the limits of 0 and $\pi/2$ using first the trapezoidal rule and then Simpson's rule, and compare the results with the exact integral.

The exact indefinite integral is $F(x) = 4x(x^2 - 7)sin(x) - (x^4 - 14x^2 + 28)cos(x)$. The following programs have been written to complete the solution to this problem:

```
FORTRAN MAIN program and SUBROUTINE EQUAT to solve problem:
      PARAMETER (NMAX=21, A=0, B=1.5707963, ERR=1.E-5)
      EXTERNAL EQUAT
      CLOINT(X)=4.*X*(X**2-7.)*SIN(X)-(X**4-14.*X**2+28.)*COS(X)
      CALL  TRAPR(EQUAT,A,B,VALUE,ERR,NMAX)    !  The  name  is  changed  to
SIMPR  to use Simpson's rule.
      WRITE(*,*) VALUE,CLOINT(B)-CLOINT(A)
      END
      FUNCTION EQUAT(X)
      EQUAT=X**2*(X**2-2.)*SIN(X)
      RETURN
      END
```

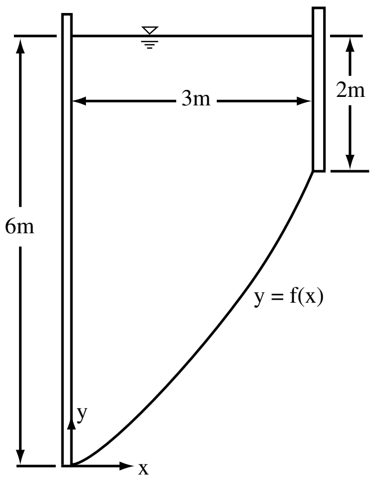
Upon executing the program, the following results can be compared:

Method	Result
Exact integral	- 4.791598E-1
Trapezoidal rule	- 4.791531E-1
Simpson's rule	- 4.791583E-1

Example Problem A.2

Find the equivalent concentrated vertical component of force, and its location, on the bottom surface that is 3 m long with water standing to a height of 6 m. The distance between vertical walls is 3 m. The surface is defined by the equation

$$y = f(x) = 2\left[x - cos\left\{\frac{\pi}{2}\left(1 - \frac{x}{3}\right)\right\}\right]$$



The vertical component of the hydrostatic force on the bottom of the tank is simply the weight of fluid above it, that is, the product of the specific weight of the water and the fluid volume, which in turn is the product of the area and the 3 m length. Since the bottom of the tank is given as a function of x, the area can be determined by numerically integrating the differential area $dA = (6 - y)\,dx$ or

$$A = \int_0^3 \left(6 - 2\left[x - cos\left\{\frac{\pi}{2}\left(1 - \frac{x}{3}\right)\right\}\right]\right) dx$$

Thereafter the centroid of this area will be determined from $Ax_c = \int_0^3 x(6 - y)dx$ The programs to obtain the area and the first moment of the area are listed next:

```
PROGRAM  SIMP1.FOR  TO INTEGRATE THE AREA
      PARAMETER (NMAX=21,A=0,B=3.,ERR=1.E-5)
      EXTERNAL EQUAT
      CALL SIMPR(EQUAT,A,B,VALUE,ERR,NMAX)
      WRITE(*,*) VALUE
      END
      FUNCTION EQUAT(X)
      EQUAT=6.-2.*(X-COS(1.5707963*(1.-X/3.)))
      RETURN
      END
PROGRAM  SIMP2.FOR  TO FIND THE FIRST MOMENT OF THE AREA
      PARAMETER (NMAX=21,A=0,B=3.,ERR=1.E-5)
      EXTERNAL EQUAT
```

```
CALL SIMPR(EQUAT,A,B,VALUE,ERR,NMAX)
      WRITE(*,*) VALUE
      END
      FUNCTION EQUAT(X)
      EQUAT=X*(6.-2.*(X-COS(1.5707963*(1.-X/3.))))
      RETURN
      END
```

The area is computed to be $12.820\ m^2$, and the solution for the first moment of the area is $16.295\ m^3$. Therefore the force on the bottom of the tank is $F = \gamma V = \gamma A b = 9.806(12.820)(3) = 377.1$ kN. It acts downward at the position $x_c = 16.295/12.820 = 1.271$ m from the origin.

A.4 SOLUTIONS TO ORDINARY DIFFERENTIAL EQUATIONS

A.4.1. INTRODUCTION

The need to solve ordinary differential equations (ODE's) occurs frequently in many fields. Often closed-form solutions to these equations do not exist, and they must be solved numerically. General purpose mathematics application software, such as MathCAD and TK-Solver, facilitates the solution of ODE's and allows the user to select the method to be used. Pocket calculators, such as the HP48G(X), also have the ability to solve ODE's (in addition to numerical integration and algebraic integrations). In the following paragraphs a brief description of the Runge-Kutta method (one of many methods, but a widely used method) for solving ODE's is presented, to be followed by descriptions of how more sophisticated ODE solvers can be used.

A.4.2. RUNGE-KUTTA METHOD

The Runge-Kutta method of numerical integration is well known as a very dependable method, although it is neither very fast or efficient. This section will describe how to implement the Runge-Kutta method. The solver DVERK in IMSL (International Mathematical Statistical Libraries), which has been widely used for years and is included in Microsoft Fortran Powerstation and its descendants, uses a Runge-Kutta method. The logic, methods etc. in DVERK are more comprehensive than that in this description, but since several programs in this text call on DVERK, the reader should extract it from the CD along with ODESOL and RUKUST, which is the program that will be described herein. A description of how to use DVERK is in the file DVERK.DOC on the CD.

The Runge-Kutta method evaluates the dependent variable y after the next increment with $y_{i+1} = y_i + \Delta y$. The Euler predictor obtains Δy by multiplying the increment Δx in the independent variable by the derivative $dy/dx = y'$, evaluated at x_i, so that $\Delta y = \Delta x y'(x_i, y_i)$. Consider a trial step to the midpoint of the increment; now use x and y here to compute Δy, or $\Delta y = \Delta x y'(x_i + \Delta x/2,\ y_i + \Delta y_m)$, in which Δy_m is the Δy obtained from the Euler predictor for the midpoint. This way of obtaining Δy is a second-order approximation since the first-order terms cancel. This method of evaluating Δy is called the second-order Runge-Kutta, or midpoint, method. The derivative y' can be evaluated by using different combinations of the independent and dependent variables, and from these combinations different values of Δy can be obtained by multiplying by the appropriate Δx's. We define the following increments:

$$
\begin{aligned}
\Delta y_1 &= \Delta x \cdot y'(x_i, y_i) \\
\Delta y_2 &= \Delta y_m = \Delta x \cdot y'(x_i + \Delta x/2, y_i + \Delta y_m) = \Delta x \cdot y'(x_i + \Delta x/2, y_i + \Delta y_1/2) \\
\Delta y_3 &= \Delta x \cdot y'(x_i + \Delta x/2, y_i + \Delta y_2/2) \\
\Delta y_4 &= \Delta x \cdot y'(x_i + \Delta x, y_i + \Delta y_3)
\end{aligned}
\tag{A.10}
$$

The first of these four equations is the Euler predictor, and the second of these equations is the midpoint method.

The fourth-order Runge-Kutta formula can be developed from these relations as

$$y_{i+1} = y_i + (\Delta y_1 + 2\Delta y_2 + 2\Delta y_3 + \Delta y_4)/6 \tag{A.11}$$

This formula requires the derivative to be evaluated four times in order to advance one increment Δx, and an analysis of the terms that have been truncated from the final result would show that terms involving Δx^5 are dropped; therefore the result provides a fourth-order approximation. Computer code to implement this fourth-order Runge-Kutta formula can consist of Fortran statements (two versions are presented) in a subroutine, as presented in Fig. A.4 (C and Pascal statements are on the CD). In these subroutines the current values for the independent variable x, the increment Δx and the dependent variable y are passed as arguments X, DX, and Y, respectively. (In the C and Pascal programs these variables must be global, and consequently they are defined in the function or procedure.) The Fortran routine(s) could be modified so that these variables appear in a common statement rather than in arguments.

```
SUBROUTINE RUKU4(X, DX, Y)
XH=X+0.5*DX
DY1=DX*SLOPE(X,Y)
DY2=DX*SLOPE(XH,Y+0.5*DY1)
DY3=DX*SLOPE(XH,Y+0.5*DY2)
Y=Y+(DY1+DX*SLOPE(X+DX,Y+DY3))/6.+(DY2+DY3)/3.
RETURN
END

SUBROUTINE RUKU4A(X, DX, Y)
DX5=0.5*DX
XH=X+DX5
DY1=SLOPE(X,Y)           ! 1st sub-step
DY2=SLOPE(XH,Y+DX5*DY1) ! 2nd sub-step
DY3=SLOPE(XH,Y+DX5*DY2) ! 3rd sub-step
Y=Y+DX*((DY1+SLOPE(X+DX,Y+DX*DY3))/6.+(DY2+DY3)/3.)
RETURN
END
```

Figure A.4 Two alternative Fortran subroutines for the Runge-Kutta fourth-order formula.

The use of either Runge-Kutta subroutine requires a main program that calls it appropriately, and a FUNCTION Subprogram SLOPE to evaluate the derivatives. The listings in Fig. A.4 are designed to solve a single ODE.

If a system of ODE's is to be solved, as accommodated by solvers like DVERK and ODESOL, whose use is described below, then arrays for Y and its derivatives are needed. Let SLOPE be a subroutine that returns N derivatives for N ODE's in its last array argument, evaluated at X and Y, its first two arguments. (Y must also be an array.) Then the solver could consist of the subroutine that is listed in Fig. A.5.

The deficiency in using RUKU4 (or RUNK4S) is that the accuracy of the solution will be dependent upon the step size Δx that is used. One way to proceed would be to solve the ODE twice, once with some Δx and then with $\Delta x/2$, and if the solution agrees within an allowable error, accept the solution; otherwise reduce Δx again by one-half, etc. Rather than putting this burden on the user, it is much better to adjust the step size to satisfy some error criterion. The step sizes may then be decreased or increased, as suggested by the accuracy of the solution being obtained. To automate this step, an estimate of the

```
SUBROUTINE RUKU4S(N, X, DX, Y)
      PARAMETER (NM=5)
      REAL Y(N),YT(NM),DY1(NM),DYT(NM),DYM(NM)
      DX5=0.5*DX
      XH=X+DX5
      CALL SLOPE(X,Y,DY1)        ! 1st sub-step
      DO 10 I=1,N
   10 YT(I)=Y(I)+DX5*DY1(I)
      CALL SLOPE(XH,YT,DYT)      ! 2nd sub-step
      DO 20 I=1,N
   20 YT(I)=Y(I)+DX5*DYT(I)
      CALL SLOPE(XH,YT,DYM)      ! 3rd sub-step
      DO 30 I=1,N
      YT(I)=Y(I)+DX*DYM(I)
   30 DYM(I)=DYM(I)+DYT(I)
      CALL SLOPE(X+DX,YT,DYT)    ! 4th sub-step
      DO 40 I=1,N
   40 Y(I)=Y(I)+DX*((DY1(I)+DYT(I))/6.+DYM(I)/3.)
      RETURN
      END
```

Figure A.5 Two alternative Fortran subroutines for the Runge-Kutta fourth-order formula.

error is needed. This estimate can be obtained with a "step doubling;" i.e. each step is repeated, once using the full Δx and then independently as two half steps $\Delta x/2$. Each of the three separate Runge-Kutta steps that are needed in using this procedure require four evaluations of y', but the single and double computations initially share common arguments of x and y, so the required total number of evaluations of y' is 11. Let the exact solution be denoted by y (without a subscript), the solution based on Δx by y_1, and the solution based on $\Delta x/2$ by y_2. Using the fourth-order Runge-Kutta method, the exact and two numerical solutions are related by

$$\begin{aligned} y(x+\Delta x) &= y_1 + C(\Delta x)^5 \\ y(x+\Delta x) &= y_2 + 2C(\Delta x/2)^5 \end{aligned} \tag{A.12}$$

in which C should remain constant over the step since the Taylor series representation of $C = (d^5y/dx^5)/5!$. Since y_1 contains $C\Delta x5$ and y_2 contains $C\Delta x^5/16$, the difference between the two solutions provides a convenient estimate of the error as

$$ERR = y_2 - y_1 \tag{A.13}$$

Hence the exact solution can be expressed as

$$y(x+\Delta x) = y_2 + ERR/15 + O(\Delta x^6) \tag{A.14}$$

To develop a criterion to decide whether Dx should be changed to satisfy an accuracy requirement, let ERR_1 be the error from using Δx_1. Then the step size Δx_o to produce an error of ERR_o can be estimated as

$$\Delta x_o = \Delta x_1 \,|\, ERR_o / ERR_1 \,|^{1/5} \tag{A.15}$$

Let ERR_o be the error associated with the desired accuracy. If $|ERR_1|$ is larger than ERR_o, then the above equation gives the $\Delta x = \Delta x_o$ to use to recompute the solution over the failed increment to satisfy the error condition ERR_O. In other words, if $|ERR_1|$ is larger than ERR_O, then the computations over Δx_1 did not satisfy the error requirement and must be repeated with a smaller increment given by Eq. A.15. If $|ERR_1|$ is less than ERR_O, then Eq. A.15 provides the Δx to use for the solution over the next step. Thus the most recent step in the solution exceeds the desired accuracy and will be accepted, but the next increment will be enlarged to avoid doing more arithmetic than is necessary to satisfy the error ERR_O, as given by Eq. A.15. For a system of ODE's the errors ERR_1 are an array of values, and the largest in magnitude should be used in Eq. A.15. The listing in Fig. A.6 includes logic to redetermine the step size to satisfy the error condition associated with the magnitude of ERROR.

```
      SUBROUTINE RUKUST(N, DXS, XBEG, XEND, ERROR, Y, YTT)
      PARAMETER (NM=5)
      REAL Y(N),YTT(N),YORI(NM)
      X1=XBEG
      DX=DXS
    1 DO 10 I=1,N
      YTT(I)=Y(I)
   10 YORI(I)=Y(I)
      X=X1
      IF(ABS(X+DX).GT.ABS(XEND)) DX=XEND-X
   20 DX5=0.5*DX
      CALL RUKU4S(N,X,DX5,Y)     ! Solve with half increment
      CALL RUKU4S(N,X+DX5,DX5,Y)
      X1=X+DX
      IF(ABS(X1).GT.ABS(XEND)-1.E-8) RETURN
      CALL RUKU4S(N,X,DX,YTT)     ! Solve with full increment
      ERRM=0.0
      X1=X+DX
      DO 30 I=1,N
      YTT(I)=Y(I)-YTT(I)
   30 ERRM=MAX(ERRM,ABS(YTT(I)/Y(I)))
      IF(ERRM.EQ.0.0) THEN
      DX=5.*DX
      DXS=DX
      GO TO 1
      ELSE
      ERRM=ERRM/ERROR
      DX=DX/ERRM**0.2
      DXS=DX
      IF(ERRM.GT. 1.0) THEN
      DO 40 I=1,N
      YTT(I)=YORI(I)
40    Y(I)=YORI(I)
      GO TO 20
      ENDIF
      ENDIF
      DO 50 I=1,N
50    Y(I)=Y(I)+YTT(I)/15.    ! Accounts for truncation error
      GO TO 1
      END
```

Figure A.6 A fourth-order Runge-Kutta code with automatic adjustment of step size.

The arguments for RUKUST now have the following meanings:

N = number of ODE's to be solved, and for which derivatives will be given.

DXS = an initial value for Δx. This value will be decreased or increased, depending upon what is needed to satisfy the error criterion. In previous Runge-Kutta subroutines DX was the interval over which the problem was solved. Now DXS normally will be smaller than this value. Upon returning from this subroutine, DXS is the Δx that was found to be satisfactory at the end of the solution, and it can be used as the initial increment in a subsequent call to RUKUST.

XBEG = the initial value of the independent variable.

XEND = the end value of the independent variable. The difference between XEND and XBEG was called DX in the previous subroutines.

ERROR = the error criterion to meet in obtaining the numerical solution.

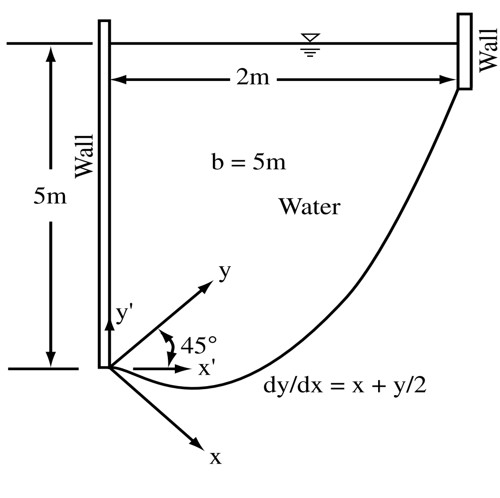
Y = an array of N values that, upon entry to the subroutine, provides the initial conditions for the dependent variable. Upon return from the subroutine it is the solution for the dependent variable(s) at x = XEND.

YTT = a work array of N values. It is used only to store the solution for the last interval, which is then compared with the solution Y in making decisions about the next increment in the independent variable to use in satisfying the error criterion.

Observe that this subroutine calls RUKU4S three times; the first two times complete the solution over the increment Δx in two steps of length $\Delta x/2$ (this solution is stored in array Y), and the third time uses the increment Δx, i.e. uses the four sub-steps in the Runge-Kutta method (and this solution is stored in the work array YTT). The difference between these two solutions is used to determine the error ERR (or ERR_1); then, based on Eq. A.15, the Δx that should supply the desired accuracy is computed. If the accuracy is insufficient, then the solution is repeated, using the computed Δx (the statement GO TO 20 does this). Another test checks whether the solution has proceeded to XEND. If not, then the solution proceeds over the next increment with the newly computed Δx by going back to statement 1. The program is required to end the solution at XEND, and this is accomplished by adjusting the last Δx so it equals the difference between the current value of x and XEND.

Example Problem A.3

The bottom of the tank is defined by the ODE $dy/dx = x + y/2$, measured in a coordinate system that is rotated downward 45^{o} from the horizontal. The bottom is located between two vertical walls that are 2 m apart, and the tank is 5 m long. It contains water that is 5 m deep at the left wall. Find the vertical component of force on the bottom, and the location of its line of action.



To solve this problem, we must first solve the ODE to obtain the shape of the bottom and then numerically integrate from this bottom position to the water surface. The program below obtains the solution. We will need to establish the relation between the x-direction and the horizontal direction, denoted by x' in the sketch, to account for the rotated coordinate system. The differential area to integrate can be written $dA = hdx'$, in which h is the distance from the bottom of the tank to the water surface; from the sketch this distance is $h = H + x \sin \theta - y \cos \theta$, with $H = 5$ being the water depth at the origin. Also from the sketch $x' = x \cos \theta + y \sin \theta$. Since $\sin 45^o = \cos 45^o$, only $\sin 45^o$ will be used in the program. While there are alternate approaches, in this solution the ODE will be solved first to obtain the shape of the bottom, with the results stored in an array, YY; when needed, values from this array will be interpolated. We choose this approach because, although the numerical integration is in terms of x' and the corresponding value of x along the rotated axis will be needed, the only way to determine this x is to use $x'/\sin 45^o = x + y$. For any x', therefore, the table is searched for the entry where $x + y$ is just larger than $x'/\sin 45^o$, and then a linear interpolation is used to find x by writing $y = y_o + (\Delta y/\Delta x)(x - x_o)$ between the two entries in the table, with subscript o denoting the first entry. Thus $x'/\sin 45^o = y_o + (\Delta y/\Delta x)(x - x_o) + x$, and the solution is found to be $x = (x'/\sin 45^o - y_o + (\Delta y/\Delta x)x_o)/(\Delta y/\Delta x + 1)$. Once x is determined, y is interpolated as $y = y_o + (x - x_o)/[\Delta x(y_1 - y_o)]$. The ODE is solved first in the main program, and the function EQUAT performs the interpolations to obtain h so that Simpson's rule can properly evaluate the area. The solution produces a cross-sectional area of 9.44 m^2, leading to a force per unit length of 92.5 kN/m and a total vertical force F_V = 5x92.5 = 462.5 kN. The first moment of the area is determined by changing the EQUAT statement slightly, as the comment statement in the program shows, and the result of the integration gives $x_cA = 8.82$ m^3 so the line of action of this vertical force is at $x_c = 8.82/9.44 = 0.934$ m.

```
TANKODEK.FOR
      EXTERNAL EQUAT
      COMMON /TRAS/XX(30),YY(30),H,SIN45,DELX,II
      REAL Y(1),YTT(1)
      II=2
      WRITE(*,*)' GIVE XB,XE,H,Y0,GAMMA,ERR,MAX'
      READ(*,*) XB,XE,H,YO,GAMMA,ERR,MAX
      SIN45=SIN(0.7853982)
      SXY=(XE-XB)/SIN45
      TOL=0.000001
      DXS=0.01
      I=1
      DELX=(XE-XB)/50.
      XX(1)=XB
      YY(1)=YO
      Y(1)=YO
   10 X=XX(I)+DELX
      CALL RUKUST(1,DXS,XB,X,TOL,Y,YTT)
      I=I+1
      XX(I)=X
      YY(I)=Y(1)
      IF(X+Y(1).LT.SXY) GO TO 10
      CALL SIMPR(EQUAT,XB,XE,VALUE,ERR,MAX)
      WRITE(*,"(' AREA = ',F10.3,/,' FORCE = ',F10.3)")
     &VALUE,VALUE*GAMMA
      END
      SUBROUTINE SLOPE(X,Y,DY)
      REAL Y(1),DY(1)
      DY(1)=X+0.5*Y(1)
```

```
      RETURN
      END
      FUNCTION EQUAT(X)
      COMMON /TRAS/XX(30),YY(30),H,SIN45,DELX,II
      SX=X/SIN45
   10 IF(XX(II)+YY(II).GT.SX) GO TO 20
      II=II+1
      GO TO 10
   20 DYDX=(YY(II)-YY(II-1))/DELX
      XP=(SX-YY(II-1)+DYDX*XX(II-1))/(DYDX+1.)
      FAC=(XP-XX(II-1))/DELX
      YP=YY(II-1)+FAC*(YY(II)-YY(II-1))
      EQUAT=H+(XP-YP)*SIN45
C     EQUAT=(H+(XPYP)*SIN45)*X
      RETURN
      END
```

A.4.3. USE OF ODE SOLVER ODESOL

The subroutine ODESOL solves either a system of first-order ordinary differential equations (equations with first derivatives) or a higher-order ordinary differential equation. It utilizes an extrapolation with a modified midpoint that is called the Bulirsh-Stoer method. If a higher-order equation, of order N, is to be solved, it must first be reduced to a system (or coupled set) of N first-order differential equations. For example, if the second-order equation

$$\frac{d^2y}{dx^2} + f(x)\frac{dy}{dx} = g(x) \tag{A.16}$$

is to be solved, it is first rewritten as the following two first-order coupled equations:

$$\frac{dz}{dx} = g(x) - f(x)\cdot z(x) \tag{A.17a}$$

and

$$\frac{dy}{dx} = z(x) \tag{A.17b}$$

Subroutine ODESOL provides users considerable flexibility. A common use will call ODESOL repeatedly, with each new call over a new increment of the independent variable x, until the solution has been found over the desired range. Another use will call ODESOL once, with the end values of the desired range given for the independent variable. In this second use, intermediate values of the dependent variables (and the corresponding independent variable) can be stored and printed. In fact, these intermediate values can also be stored and examined when the first application, with several calls between the end values of the independent variable, is employed . The sizes of the arrays in ODESOL are established by integer values passed through arguments of the call; thus the amount of memory required by ODESOL is related to the size of the problem being solved. For a single first-order equation and a very limited storage of intermediate values, a very small amount of memory is required by ODESOL, e.g., that of its code and variables and the very small arrays that are passed as arguments. On the other hand, if a system of eight ordinary differential equations is being solved, the memory requirements for arrays will be larger. A smaller version that does not return intermediate values and does not use a blank COMMON statement is called ODESOLS.

The call in the driver program for ODESOL must contain a statement such as

```
CALL ODESOL(YBEG,DYDX,NV,X1,X2,ERR,H1,HMIN,NSTOR,XP,YP,WK1,SLOPE)
```
(A.18a)

or

```
CALL ODESOLS(YBEG,DYDX,NV,X1,X2,ERR,H1,HMIN,WK1,SLOPE)
```
(A.18b)

in which

YBEG = real array of dimension NV. Its elements are the dependent variables at X1 for which a solution is being sought. Before the call this array contains the starting values of the dependent variables, i.e. the values of the y's at X1. Upon return from the call, this array contains the dependent variables at X2.

DYDX = real array of dimension NV. Its elements are the derivatives of the dependent variables with respect to the independent variable x. The subroutine SLOPE defines these derivatives. The main program must dimension this array.

NV = integer variable, the number of first-order equations to be solved. In Eq. A.17 NV = 2.

X1 = the independent variable at the beginning of the solution interval.

X2 = the independent variable at the end of the solution interval. A solution will be obtained for x in the range between X1 and X2. Either X2 or X1 can be the smaller number.

ERR = real variable that defines the desired accuracy that ODESOL is to achieve. The step size will be changed as needed to achieve this accuracy.

H1 = the initial increment in x that ODESOL will use in obtaining the solution. It will be modified as needed to satisfy ERR. Normally ODESOL is called repeatedly to solve a problem over an extended range of the independent variable. When this is done, H1 will be used only in the first call to ODESOL. Thereafter the increment that was found to be appropriate in the previous call will be used. Therefore, it is best to provide H1 only in the first call to ODESOL.

HMIN = the minimum step size that will be allowed in seeking the solution. It may be set to zero (but this act may cause an infinite loop), and it is positive even if X2 is less than X1.

NSTOR = an integer that agrees with the dimensions of XP and the second dimension of YP. If KMAX in the common statement is 0, then NSTOR can be 1. NSTOR should never be less than 1.

XP = a one-dimensional real array of size NSTOR that contains the values of x upon return from ODESOL if KMAX is nonzero. These values will not be equally spaced but will range from X1 to X2.

YP = a two-dimensional real array of size NV (first subscript) and NSTOR (second subscript) that contains the values of the dependent variable upon return from ODESOL if NBETW is nonzero. The values in YP(I, J), with I between 1 and NV and J constant but between 1 and IBETW, will be the values of y corresponding to XP(J).

WK1 = a two-dimensional real array with NV as the first dimension and 13 as the second dimension. It is used for work space by ODESOL in obtaining the solution. Thus WK1 is dimensioned in the main program as WK1(NV, 13).

SLOPE must be declared as EXTERNAL in the main or driver program and is the name of the subroutine described below that defines the derivatives.

The main program only requires a COMMON statement when ODESOL is used; it must contain the five variables defined below. The statement should be similar to

```
COMMON NGOOD,NBAD,NBETW,IBETW,DXBETW
```
(A.19)

in which

NGOOD = an integer variable, the number of steps in the solution that equal or exceed the accuracy established by the error criterion ERR, i.e. good steps.

NBAD = an integer variable, the number of steps in the solution that did not meet the accuracy established by the error criterion ERR, i.e. bad steps.

NBETW = an integer variable, the maximum number of intermediate values of x and y in arrays XP and YP. If NBETW is zero, then no intermediate values will be returned. NBETW should not exceed the dimension of XP or YP, as defined by NSTOR, but the number of values in XP and YP will usually be less than NBETW. However, if the required step sizes are small, values will not be stored in XP and YP after NBETW values have been placed in these arrays.

IBETW = an integer variable, the number of intermediate values of x and y that are stored in XP and YP after returning from ODESOL . Hence the value of IBETW will never exceed NBETW.

DXBETW = a real variable, the smallest increment in the independent variable for which intermediate values will be stored in XP and YP. Should the increment that is needed to meet the error criterion ERR become less than DXBETW, then some values obtained in the solution process will not be stored.

The user of ODESOL must supply subroutine SLOPE to define the derivative(s) in the differential equation(s). The first statement should be

```
SUBROUTINE SLOPE(X, Y, DYDX)                    (A.20)
```

in which the name SLOPE is the EXTERNAL variable that is the last argument of the call to ODESOL. The arguments are

X = a real variable, the independent variable x. Its value will be passed from ODESOL to SLOPE to define the derivatives.

Y = a real array with the dimension NV. Values of this array pass from ODESOL to SLOPE to define the derivatives at X.

DYDX = a real array with dimension NV. Subroutine SLOPE must contain statements to define the elements of this array using X and the elements of Y to define each derivative of the individual dependent variables with respect to the independent variable x. The name must match the second argument of the call to ODESOL from the main program. Somewhere in subroutine SLOPE there must be a statement DYDX(J) = ... with J ranging from 1 through NV. SLOPE will be called repeatedly by ODESOL and must be written to provide the correct derivatives in the array DYDX which defines the system of ordinary differential equations that are being solved.

It would be a worthwhile exercise to use ODESOL to solve Example Problem A.2. To do so, first add SLOPE to the EXTERNAL declaration, and replace the REAL declaration at the beginning of the program with two statements:

```
COMMON NGOOD,NBAD,KMAX,ICOUNT,DXSAVE
REAL W(1,13),XP(1),YP(1,1),DY(1),Y(1)
```

Then replace DXS=0.01 with two statements H1=0.01 and HMIN=1.E-8, and replace the call to RUKUST with

```
CALL ODESOL(Y,DY,1,XB,X,TOL,H1,HMIN,1,XP,YP,W,SLOPE)
```

Example Problem A.4

A pipe connects two tanks, as shown below, that have different water surface elevations; at time $t = 0$ a valve in the pipe is instantaneously and completely opened. Analyze the ensuing unsteady flow for a pipe with a length L = 1000 ft and a diameter D = 10 in, with tank diameters $D_1 = D_2 = 4$ ft. Initially the water surface elevations in the tanks are $h_1 = 60$ ft and $h_2 = 30$ ft. Assume all inertial effects are negligible in the tanks.

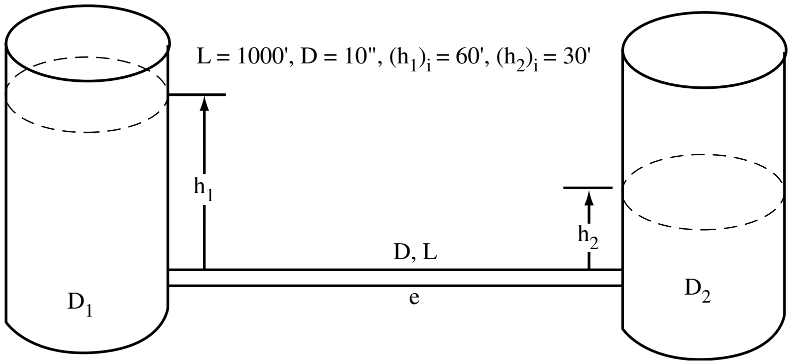
Obtain a solution for several situations:

(a) the fluid is idealized as inviscid, creating no entrance or exit losses;

(b) the fluid is idealized as inviscid, but assume that the velocity head into the second tank is lost;

(c) the fluid is real, and the pipe has a Darcy friction factor $f = 0.02$;

(d) the fluid is real, and the Darcy friction factor is to be determined for a pipe with an equivalent sand grain roughness $e = 0.005$ in.



The Fortran program TANKPI (a C version is on the CD) has been written to solve all four cases. It calls the solver RUKUST to obtain solutions to the ODE for the problem. The general ordinary differential equation for this problem is

$$\frac{dV}{dt} = \frac{g}{L}\left\{h_1 - h_2 - \left(K_e + 1 + f\frac{L}{D}\right)\frac{V|V|}{2g}\right\}$$

in which K_e is the entrance loss coefficient and V is the velocity in the pipe. The velocity in each tank is found by multiplying the pipe velocity by the ratio of the pipe and tank cross-sectional areas; for example, for the first tank $dh_1/dt = -(D/D_1)^2V$. Thus the changes in tank water surface elevation over any time increment Δt, based on the trapezoidal rule for numerical integration, are given by

$$\Delta h_1 = -\left(D/D_1\right)^2\left(V_p + V_{pi}\right)\Delta t/2$$

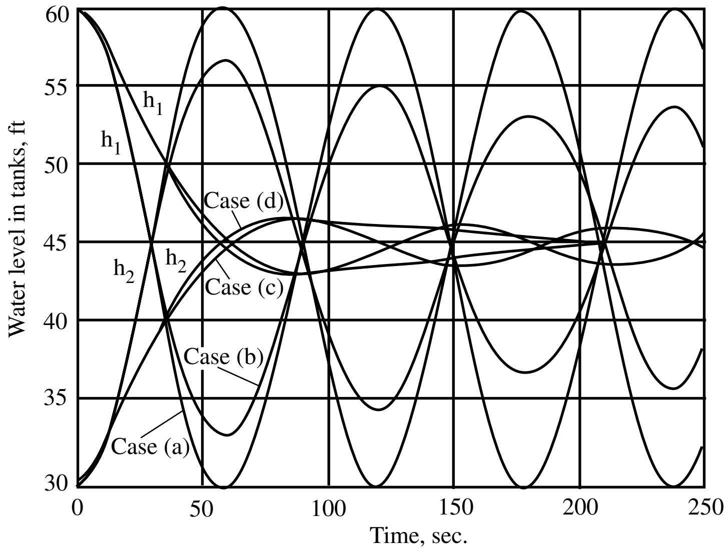
and

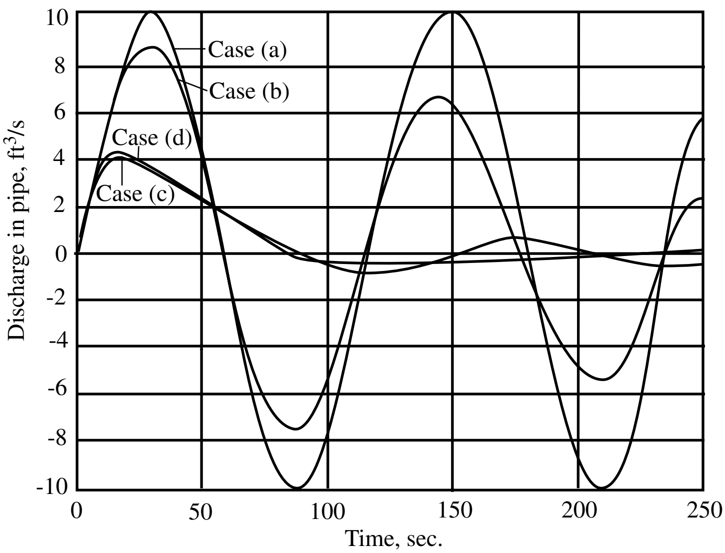
$$\Delta h_2 = +\left(D/D_2\right)^2\left(V_p + V_{pi}\right)\Delta t/2$$

This numerical integration is needed in the main program, where Δt is the time increment specified in the input, and within subroutine SLOPE since it will be called for times that will be determined by the ODE solver RUKUST.

For case (a) all terms after h_2 in the ODE are ignored, so that $dV/dt = g(h_1 - h_2)/L$ is to be solved. Case (b) is obtained by setting K_e and f to zero. For case (c) the program uses EK1 = $K_e + 1 + fL/D$ as the multiplier of the velocity head. In case (d) the Colebrook-White equation is solved for f, unless the Reynolds number is below 2100, in which case $f = 64/Re$. If $Re < 100$, then $f = 0.64$.

The results for the four cases are plotted on the two graphs which follow. In case (a) the water surface elevations in the tanks oscillate repeatedly between elevations 60 ft and 30 ft. For all of the other cases the magnitude of the oscillations is damped, or reduced, with time, but the rates of change differ from case to case.

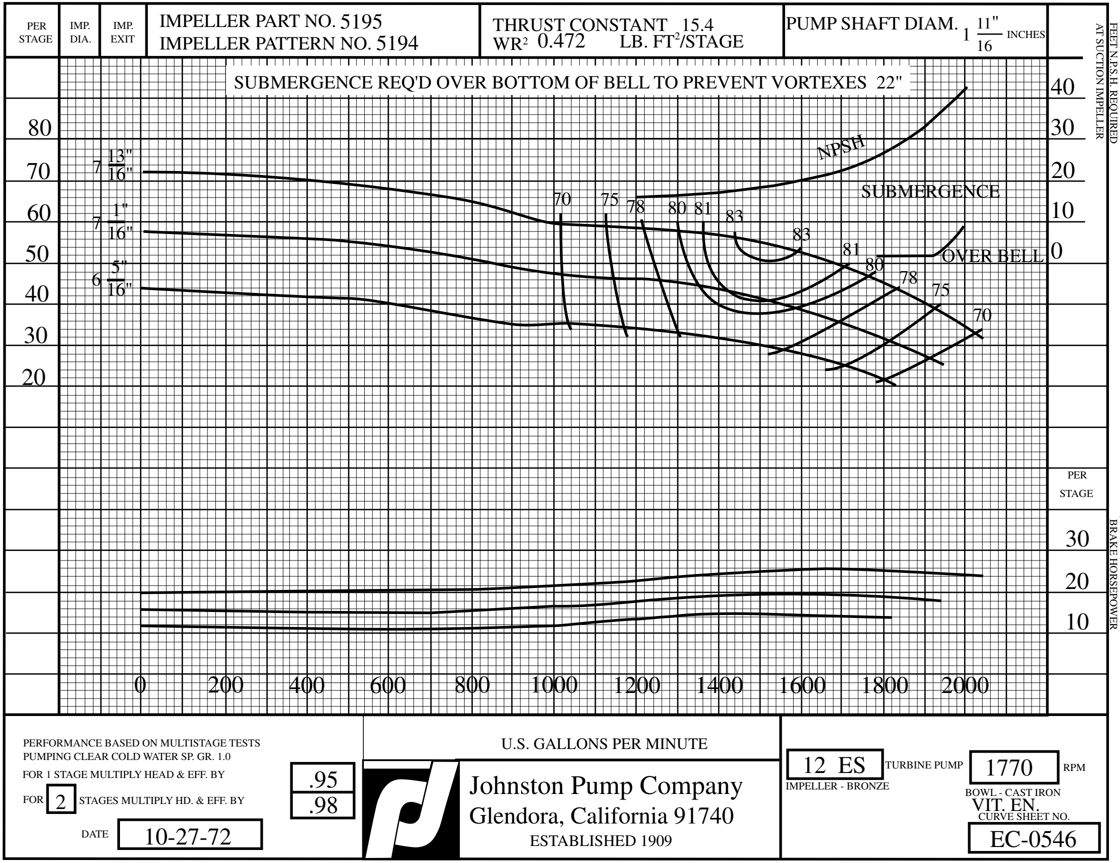




The variables that are read by the program have the following meanings: IOUT = number of the logic unit for written output, L = pipe length (ft or m), E = equivalent sand grain roughness (ft or m), D = pipe diameter (ft or m), D1 = diameter of tank one (ft or m), D2 = diameter of tank two (ft or m), VI = initial velocity in pipe (ft/s or m/s), H1 = head in tank one (ft or m), H2 = head in tank two (ft or m), G = acceleration of gravity, KE = entrance loss coefficient, NT = number of time steps for the solution, DELT = time increment (s), VISC = fluid kinematic viscosity (ft^2/s or m^2/s).

```
TANKPI.FOR
      REAL V(1),VTT(1)
      COMMON D,FL,GL,AR1,AR2,H1,H2,COE,VISC,VI,SF,G2,EK1,EK,E,TIM1
C IF E=-10 THEN THE DOWNSTREAM VELOCITY HEAD IS LOST; IF E = - ABS(F),
C THEN F = CONSTANT; OTHERWISE KE IS THE ENTRANCE LOSS AND E IS THE
C EQUIVALENT SAND GRAIN ROUGHNESS
      WRITE(*,*) 'GIVE IOUT,L,E,D,D1,D2,VI,H1,H2,G,EK,NT,DELT,VISC'
      DTS=0.05
      V(1)=VI
      G2=2.*G
      GL=G/FL
      AR1=(D/D1)**2/2.
      AR2=(D/D2)**2/2.
      AREA=0.78539816*D**2
      IF(ABS(EK).GT.1.E-7) EK=(1.+EK)/G2
      IF(E.LT.0.) EK=EK+ABS(E)*FL/(D*G2)
      EK1=EK
      IF(E.LT.-9.) EK1=1./G2
      TIM1=0.
      WRITE(IOUT,100) TIM1,VI,AREA*VI,H1,H2
  100 FORMAT(F5.1,4F10.3)
      DO 10 I=1,NT
      TIME=DELT*FLOAT(I)
      CALL RUKUST(1,DTS,TIM1,TIME,1.E-6,V,VTT)
      H1=H1-AR1*V(1)+VI)*DELT
      H2=H2+AR2*(V(1)+VI)*DELT
      WRITE(IOUT,100) TIME,V(1),AREA*V(1),H1,H2
      VI=V(1)
   10 TIM1=TIME
      END
      SUBROUTINE SLOPE(T,V,DVT)
      REAL V(1),DVT(1)
      COMMON D,FL,GL,AR1,AR2,H1,H2,COE,VISC,VI,SF,G2,EK1,EK,E,TIM1
      IF(E.GT.1.E-7) THEN
      RE=V(1)*D/VISC
      IF(RE.LT.100.) THEN
      F=0.64
      ELSE IF(RE.LT.2100.) THEN
      F=64./RE
      ELSE
    1 SF1=SF
      SF=1.14-2.*ALOG10(E/D+9.35*SF/RE)
      IF(ABS(SF-SF1).GT.1.E-6) GO TO 1
      F=1./SF/SF
      ENDIF
      EK1=EK+F*FL/D/G2
      ENDIF
      DVT(1)=GL*(H1-H2-(V(1)+VI)*(AR2+AR1)*(T-TIM1)-EK1*V(1)*ABS(V(1)))
      RETURN
      END
```

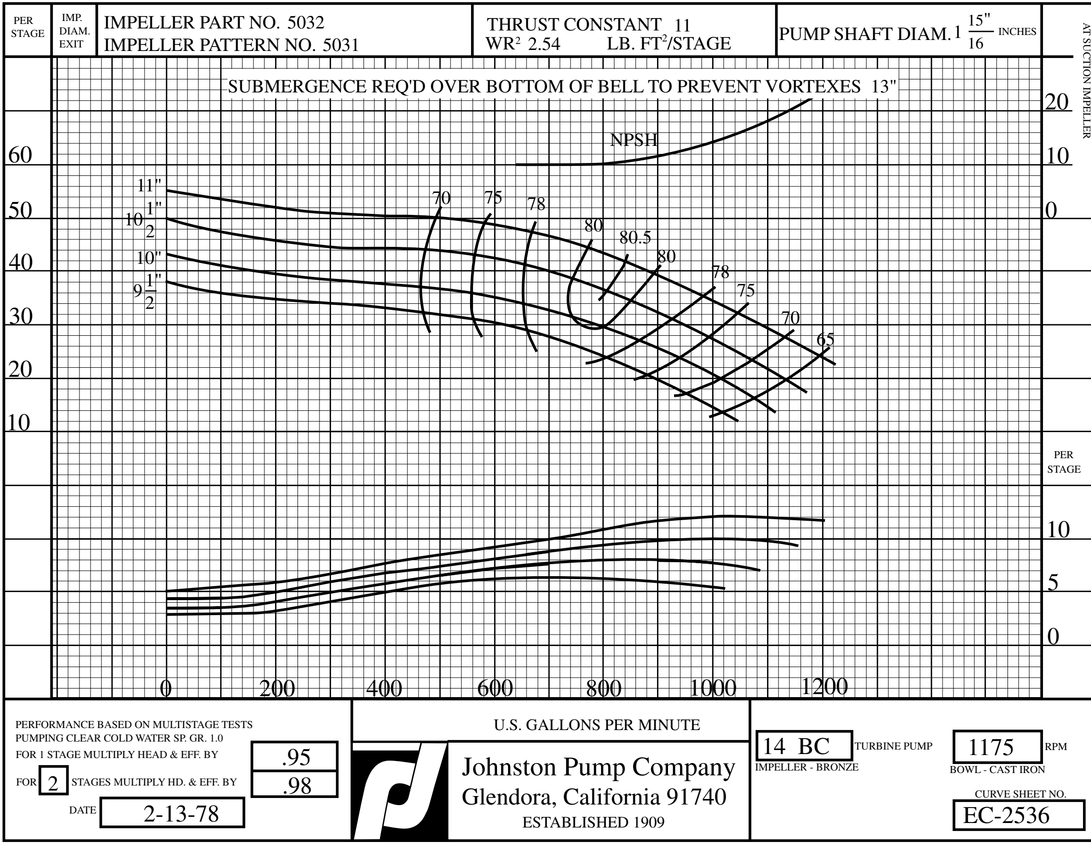
Modifying this program to use ODESOL and/or DVERK (part of IMSL) in place of RUKUST is an instructive exercise.



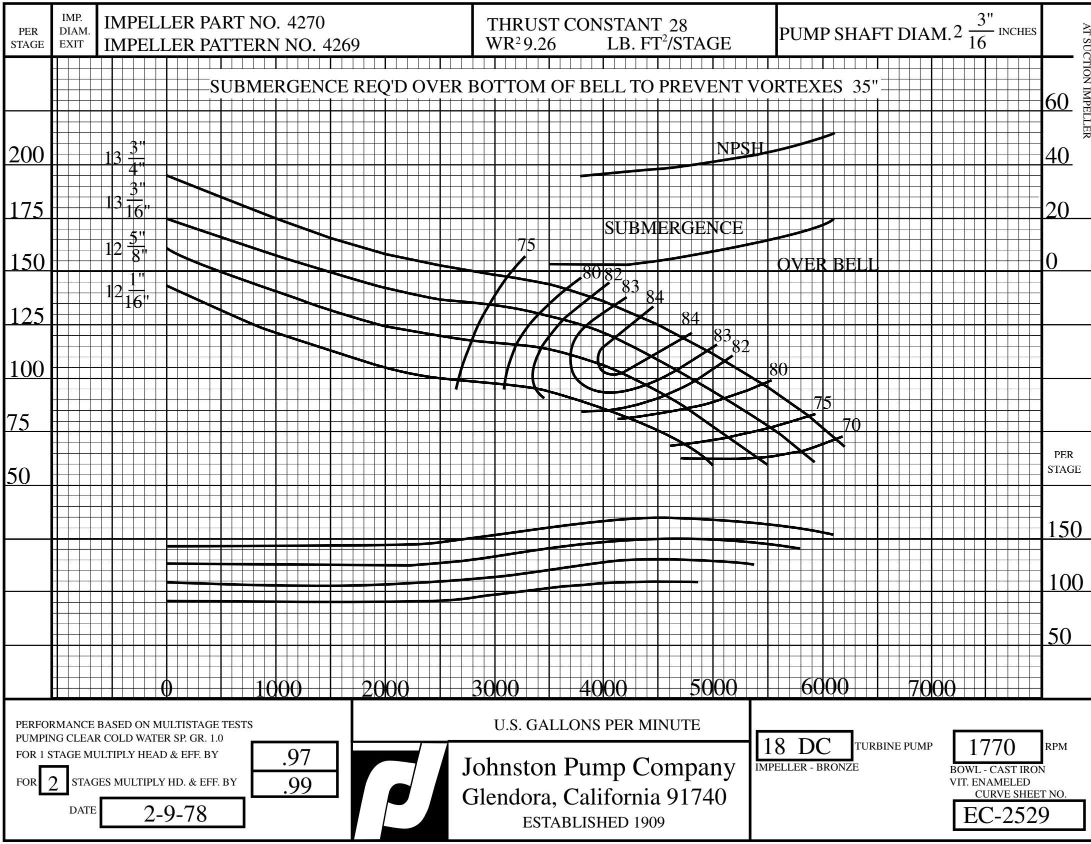
(Courtesy of Johnston Pump Co.)

APPENDIX B

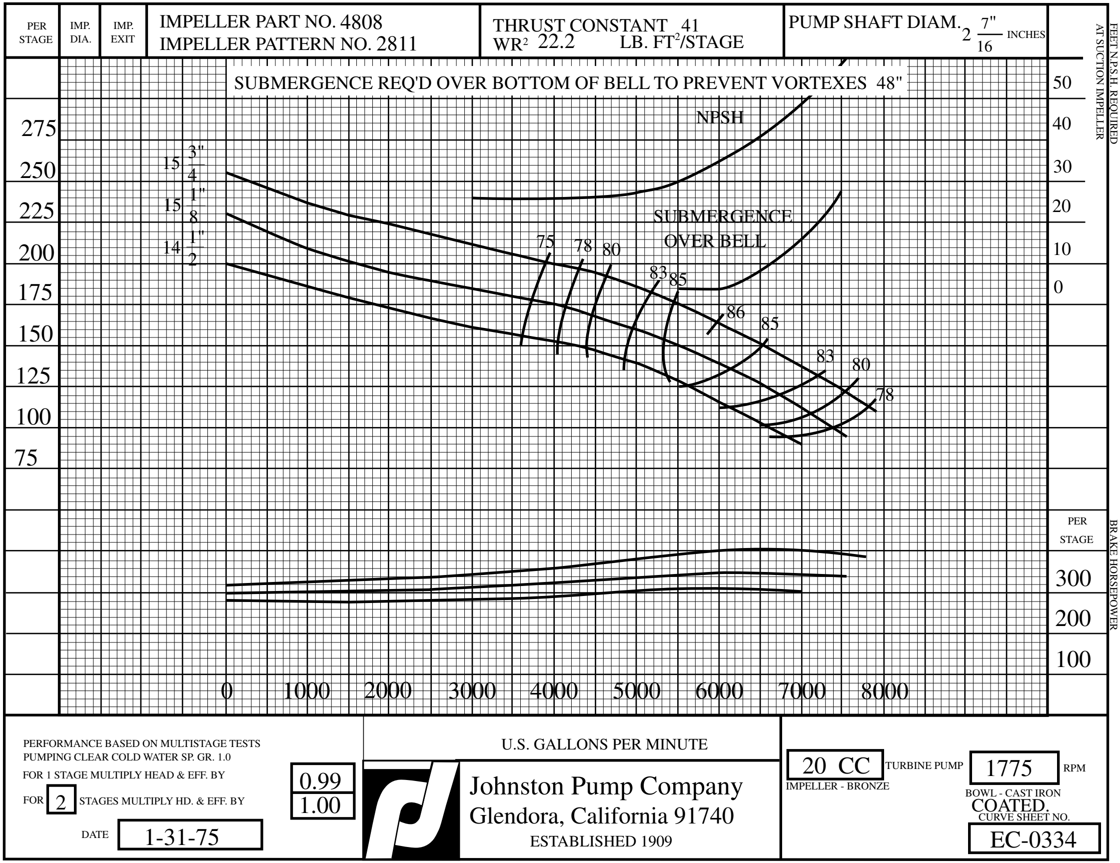
PUMP CHARACTERISTIC CURVES



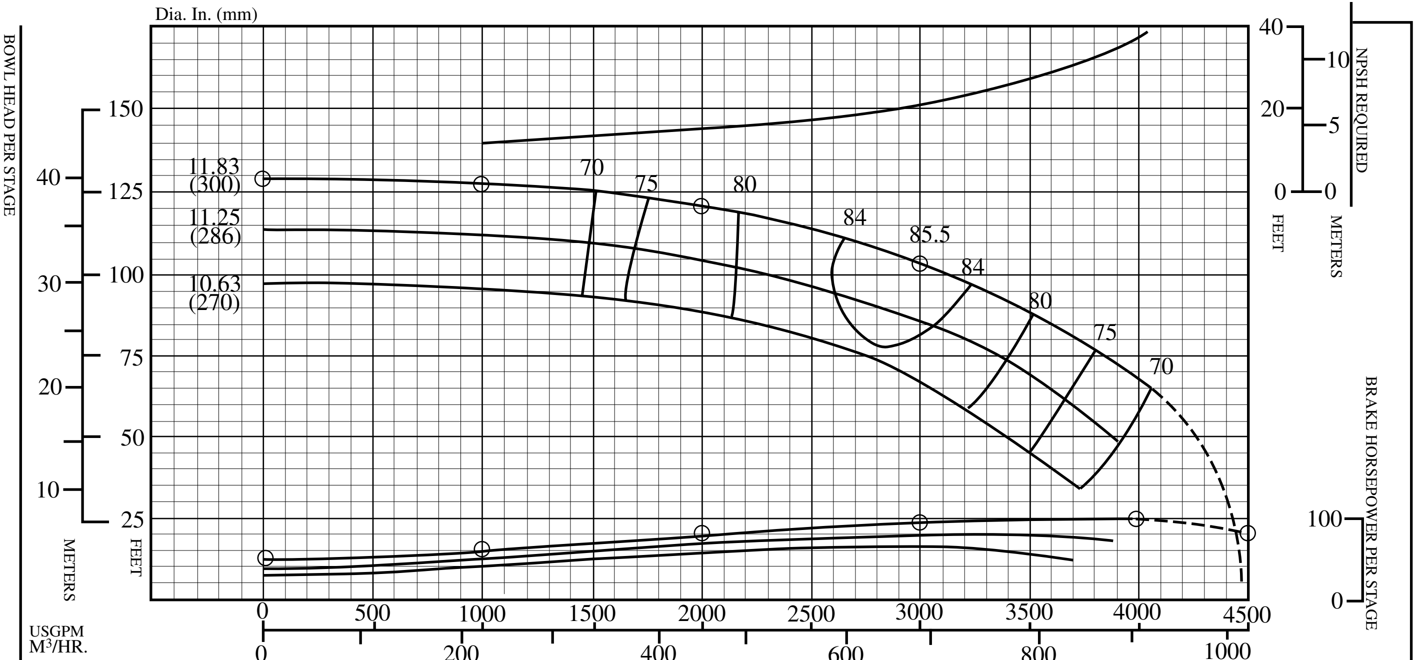
(Courtesy of Johnston Pump Co.)



(Courtesy of Johnston Pump Co.)



(Courtesy of Johnston Pump Co.)



PERFORMANCE BASED ON PUMPING CLEAR WATER BELOW 85 F. WITH LISTED MATLS. COLUMN LOSSES NOT INCLUDED

BOWL - CAST IRON - LINED

IMPELLER - BRONZE

CHANGE STAGES	IN EFF. POINTS
1	-3
2	-1.5
3	-1

Ingersoll-Dresser Pumps

THRUST FACTORS AT BEP.		
IMPELLER	LBS/FT	KG/M
STANDARD	12.0	17.9
BALANCE	---	---

EYE AREA - 34.4 SQ. IN.

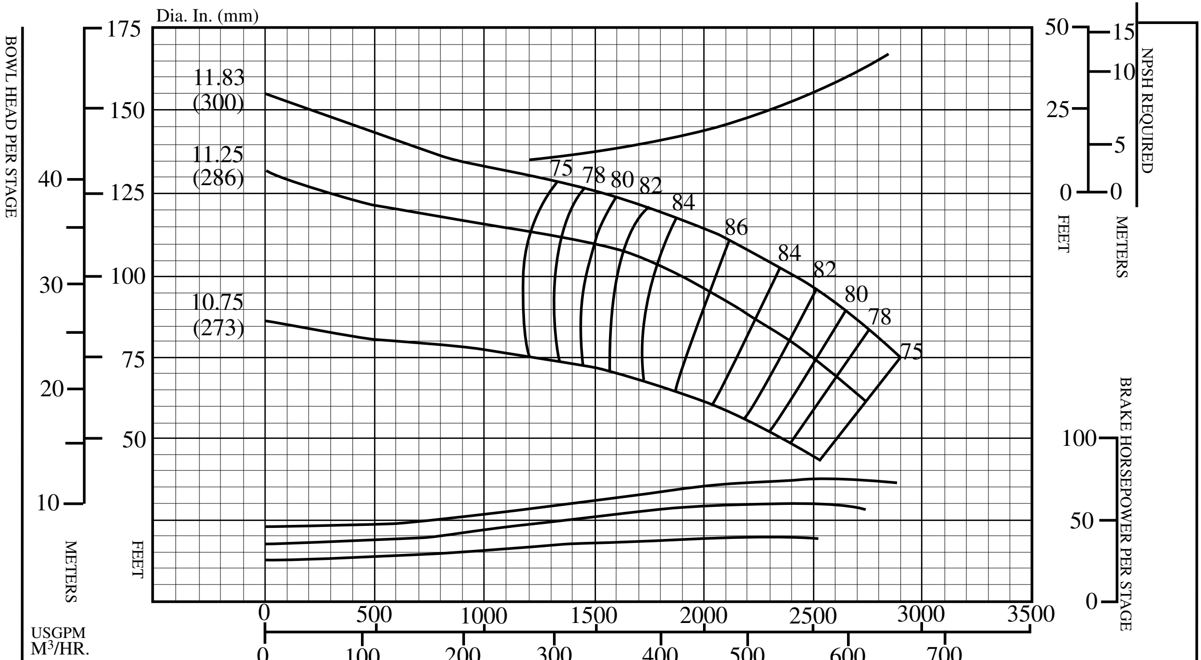
RPM 1775

ENCLOSED IMPELLER

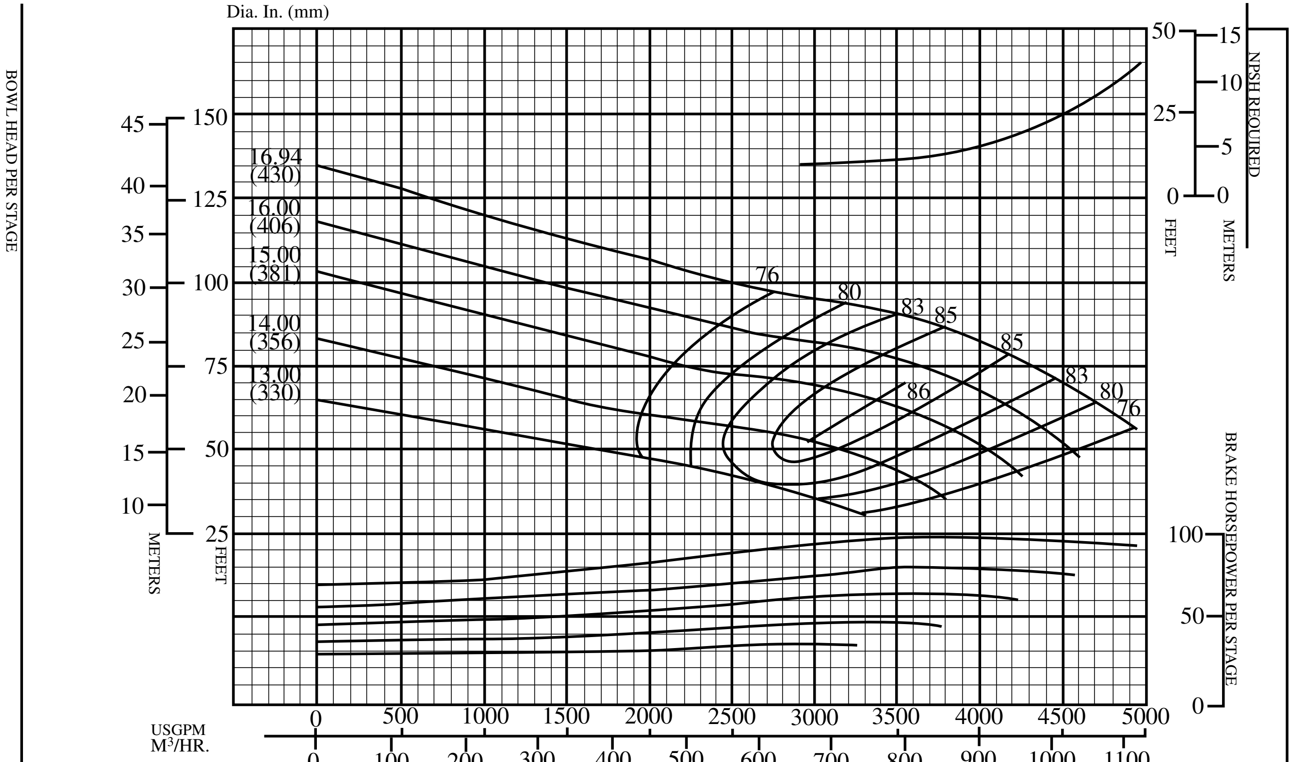
$N_S = 3020$

PUMP 15H277

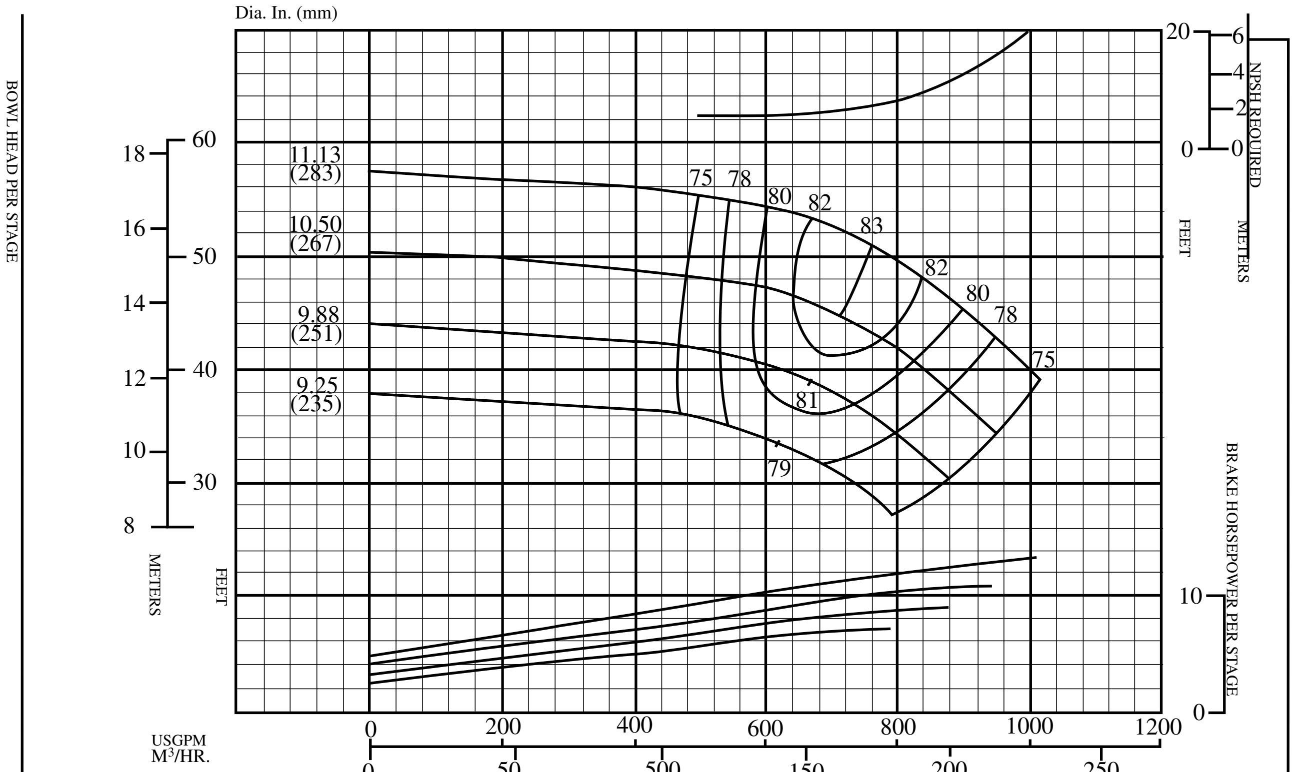
CURVE NO. EC - 2392



PERFORMANCE BASED ON PUMPING CLEAR WATER BELOW 85 F. WITH LISTED MATLS. COLUMN LOSSES NOT INCLUDED	CHANGE STAGES	IN EFF. POINTS	**Ingersoll-Dresser Pumps**	THRUST FACTORS AT BEP. IMPELLER	LBS/FT	KG/M	RPM 1775	PUMP 15M185
BOWL - CAST IRON - LINED	1	-1.5		STANDARD	8.57	12.8	ENCLOSED IMPELLER	CURVE NO. EC - 2377
	2	-1		BALANCE	---	---		
IMPELLER - BRONZE	3	0		EYE AREA - 26.9 SQ. IN.			$N_S = 2380$	



PERFORMANCE BASED ON PUMPING CLEAR WATER BELOW 85 F. WITH LISTED MATLS. COLUMN LOSSES NOT INCLUDED	CHANGE STAGES	IN EFF. POINTS	Ingersoll-Dresser Pumps	THRUST FACTORS AT BEP. IMPELLER / LBS/FT / KG/M	RPM 1180 / PUMP 20KKH
BOWL - CAST IRON - LINED	1	2		STANDARD / 19.7 / 29.3	ENCLOSED IMPELLER / CURVE NO. EC - 1362
	2	1		BALANCED / 10.0 / 14.9	
IMPELLER - BRONZE	3	0		EYE AREA - 60.5 SQ. IN.	$N_S = 2740$



PERFORMANCE BASED ON PUMPING CLEAR WATER BELOW 85 F. WITH LISTED MATLS. COLUMN LOSSES NOT INCLUDED

BOWL - CAST IRON - LINED

IMPELLER - BRONZE

CHANGE STAGES	IN EFF. POINTS
1	-3
2	-2
3	-1
4	0

Ingersoll-Dresser Pumps

THRUST FACTORS AT BEP.		
IMPELLER	LBS/FT	KG/M
STANDARD	5.4	8.0
BALANCED	2.6	3.9

EYE AREA - 18.6 SQ. IN.

RPM 1175

ENCLOSED IMPELLER

$N_S = 1700$

PUMP 14JKH

CURVE NO. EC - 1453

APPENDIX D

ANSWERS TO SELECTED PROBLEMS

CHAPTER 2

2.1 (a) Re = 834, laminar; (b) $Re = 3.94 \times 10^6$, e/D = 0.00173, wholly rough, turbulent.
2.2 (a) f = 0.0174, transitional; (b) $e = 1.05 \times 10^{-4}$m; (c) 18.6 kW.
2.3 p = 744 kPa.
2.4 Q = 0.607 m^3/s, P = 397 kW.
2.5 Q = 0.559 m^3/s.
2.6 D = 143.9 mm.
2.7 (a) K = 22.907, n = 1.852; (b) K = 2868.7, n = 1.852, (c) K = 14.410, n = 1.852.
2.9 Darcy-Weisbach h_f = 4.986 ft; Hazen-Williams h_f = 4.803 ft; difference = 0.092 ft; Manning h_f = 5.101 ft; difference = - 0.205 ft.
2.13 Q = 0.00698 m^3/s.
2.14 h_f = 3.47 m.
2.15 113.5 m vs. 34.6 m (about 3 times as large).
2.16 f = 0.0195, h_f = 26.6 m.
2.17 P = 109.7 kW.
2.18 P = 5608 kW.
2.19 h_f = 28.95 m.
2.20 Use 10% of z for h_f, D = 2.58 m.
2.21 f = 0.0279, D = 0.513 m.
2.22 h_p = 109.2 ft, HP = 99.1, N_s = 1775 (centrifugal).
2.23 f = 0.0127, D = 326 mm.
2.24 f = 0.0148, D = 0.181 m.
2.26 h_L = 0.162 m saved.
2.27 h_L = 3.59 m, 2.94 m, 1.96 m.
2.28 h_L = 4.077 m, 0.155 m, 8.155 m.
2.29 Q = 0.0082 m^3/s.
2.30 Q = 0.55 m^3/s (net return for pumping = 40.45/16.18 = 2.5).

CHAPTER 4

4.2 Q_1 = 3.1 ft^3/s, f = 0.0181.
4.3 (a) K = 0.8318, n = 1.960.
4.4 (d) h_{f5} = 20.37 m, H_3 = 300.4 m; (e) Q_6 = 0.112 m^3/s, H_2 = 306 m.
4.7 Q_1 = 1.75 ft^3/s, h_{f1} = 16.82 ft.
4.8 Q_1 = 0.849 ft^3/s, Q_2 = 2.151 ft^3/s.
4.9 Q_1 = 1.97 ft^3/s, Q_2 = 2.47 ft^3/s, H_1 = 96.9 ft, H_2 = 116.6 ft.
4.10 Q_1 = 0.496 ft^3/s, Q_2 = 2.504 ft^3/s, Q_3 = 3.504 ft^3/s, h_p = 154.4 ft.
4.11 HGL = 69.93 ft, PRV dissipates 7.37 ft.
4.12 Q_1 = 0.143, Q_2 = 0.622, Q_3 = 0.265, Q_4 = 0.735, Q_5 = 1.857, all in ft^3/s.
4.15 Q_1 = 0.202, Q_2 = 0.109, Q_3 = 0.029, Q_4 = 0.033, Q_5 = 0.012, all in ft^3/s.
4.19 Q_3 = 0.0446, Q_4 = 0.0196, Q_5 = 0.0554, Q_6 = 0.0254, all in m^3/s.

4.23 (**a**) $Q_1 = 0.606$, $Q_2 = 0.560$, $Q_3 = 0.106$, $Q_4 = 0.0462$, $Q_5 = 0.0602$, $Q_6 = 0.394$, $Q_7 = 0.190$, all in ft^3/s; (**b**) $Q_1 = 0.0061$, $Q_2 = 0.0323$, $Q_3 = 0.0228$, $Q_4 = 0.1572$, $Q_5 = 0.1177$, $Q_6 = 0.1412$, $Q_7 = 0.5239$, all in m^3/s.

4.25 $Q_1 = 4.146$, $Q_2 = 1.783$, $Q_3 = 0.783$, $Q_4 = 1.258$, $Q_5 = 0.606$, $Q_6 = 0.718$, $Q_7 = 0.658$, $Q_8 = 0.058$, $Q_9 = 0.554$, all in ft^3/s.

CHAPTER 5

5.1 (**1**) $f = 0.0156$, $D = 6.055$ in, $D = 6.090$ in; (**2**) 0.0265, 6.677 in, 7.034 in; (**3**) 0.0179, 9.925 in, 9.811 in.

5.2 (**1**) $f = 0.0125$, $D = 0.344$ m, $D = 0.346$ m; (**2**) 0.0232, 0.428 m, 0.445 m; (**3**) 0.0132, 0.709 m, 0.669 m.

5.5 $D_1 = 5.65$ in, ... $D_{10} = 14.48$ in.

5.10 Partial results:

Pipe	Diameter	Discharge	Velocity	Head loss	*Re*
1	0.0846	0.000965	1.718	0.200	1345
5	0.0446	0.000149	0.674	0.200	196
.	.	.	.	.	.
.	.	.	.	.	.
45	0.0446	0.000156	0.476	0.200	196

5.11 $D = 7.52$ in, $f = 0.0192$.

5.12 $D = 7.25$ in.

5.13 QJ = 0.327 ft^3/s, $f_1 = 0.0144$, $f_2 = 0.0145$.

5.14 $h_p = 177.24$ ft, $P = 114.47$ kW, cost/day = \$274.72.

5.17 Darcy-Weisbach: $Q_1 = 1.438$ ft^3/s, $Q_2 = 0.938$ ft^3/s;

Hazen-Williams: $Q_1 = 1.527$ ft^3/s, $Q_2 = 1.027$ ft^3/s.

5.18 $D_2 = 4.73$ in, $f_2 = 0.0207$.

5.19 $h_{f1} = 12.11$ ft, $D_2 = 4.71$ in.

5.22 Pump 1: Q = 15.829 ft^3/s; Pump 2: Q = 11.123 ft^3/s.

5.23 $Q_1 = 22.786$ ($f = 0.0143$); $Q_1 + Q_2 = 22.786 + 11.123 = 33.909$ ft^3/s into the reservoir.

5.27 Pump 2 must supply 13.0 ft^3/s, producing a negative flow in pipe 4; a solution is not possible because $H_2 > H_3$.

5.28 WS elevation = 605 - 39.66 = 565.34 ft; $H_{14} = 572.4$ ft.

5.29 Cases 1, 2 and 4 fail.

5.30 $h_p = 51$ ft.

5.32 A pump with $Q_{new} = 3.1$ ft^3/s and $h_p = 50.4$ ft should be selected.

5.35 For 2 ft^3/s demands the deficit pressures are 17.8 lb/in^2 at node 4, 34.9 lb/in^2 at node 6, and 17.8 lb/in^2 at node 9; increase the diameters of pipe 4 to 18 in, pipe 7 to 6 in, pipes 9 and 10 to 10 in, and pipes 11 and 12 to 8 in.

5.36 (2) $Q_1 = 7.104$ ft^3/s, $Q_3 = 1.053$ ft^3/s, and $Q_6 = 2.406$ ft^3/s; also $Q_1 = 7.104$ ft^3/s, $f_1 = 0.0128$ and $h_p = 63.72$ ft.

5.37 $QJ_2 = 0.976$ ft^3/s, $h_t = 403.4$ ft.

5.38 $N_{ra} = 1.124$, $N_{rb} = 1.132$.

5.45 Energy = 81.5×10^6 ft-lb = 110,600 kWh.

CHAPTER 6

6.9 Partial results: 45.1 kWh, \$767/day, energy cost present worth = \$2,700,000.

6.12 1 in smaller, difference = \$12.98/day; 1 in larger, difference = \$14.38/day.

6.13 PVC pipe increases discharge to reservoir by 192 gal/min and reduces total power from 266 kW to 261 kW.
6.14 $712.80/day, reduced by $53.80/day; flow to reservoir is reduced by 15.5 acre feet.
6.15 Energy cost/year = $2027; present worth = $228,200.

CHAPTER 7

7.1 Δt = 85.2 min; Δt = 76.9 min.
7.2 t_f = 10.5 min.
7.3 t = 117 sec.
7.4 $A(h) = (-Ca\sqrt{2g}/K)h^{1/2}$, with $K = dh/dt$ = negative constant.
7.5 η = 3.19 ft and t = 146 sec when the upper tank is closed; when η = 1.0 ft, t = 711 sec.
7.6 V_0 = 9.51 ft/s; t_{50} = 15.6 sec; t_{99} = 75 sec.
7.7 t_{99} = 75 sec.
7.8 t_{99} = 9.8 sec.
7.9 t_{99} = 9.6 sec.
7.10 t_{99} = 38.1 sec.
7.11 t_{99} = 54.5 sec.
7.12 Minimum pressure head = 113.9 ft at 10 sec on downstream side of valve; maximum pressure head = 200 ft at 0 sec at the valve; final valve loss coefficient = 95.8.
7.13 Minimum pressure head = 32.2 ft at 9.2 sec at the valve.
7.14 Pressure head at valve needs to reach 209 ft but cannot exceed 200 ft; proposed scheme will not work.
7.17 Maximum pressure head = 32.2 ft at t = 7.75 sec.
7.18 V_0 = 7.23 ft/s, maximum pressure head = 74.2 ft at t = 26.4 sec.
7.19 p_{max} = 68 lb/in^2 at 28 sec.

CHAPTER 8

8.1 (**a**) Case (a) a = 4190 ft/s, Case (b) a = 4210 ft/s, Case (c) a = 4170 ft/s; (**b**) p_{max} = 1316 lb/in^2 with stress of 18,600 lb/in^2 in steel pipe; stress too high.
8.2 (**a**) Choose Case (c) restraint with a = 860 ft/s; (**b**) 1.12%; (**c**) 0.068 in.
8.3 (**a**) Case (a) a = 2830 ft/s, Case (b) a = 2870 ft/s, Case (c) a = 2790 ft/s; (**b**) though the pipe is connected with ring gaskets, soil friction may not permit the pipe to slip at the joints, so select Case (b) as a conservative approach.
8.4 Liquid compression, 37%; pipe expansion, 63%.
8.5 (a) a = 3450 ft/s; (b) a = 3970 ft/s; (c) a = 3320 ft/s; (d) a = 3580 ft/s; (e) a = 1070 ft/s.
8.6 Percent change = 0.89%.
8.7 (a) Volume inflow = 369 ft^3; (b) liquid compression, 156 ft^3; longitudinal stretching, 22 ft^3; radial stretching, 191 ft^3.
8.8 a = 4210 ft/s.
8.9 Case (a) a = 3870 ft/s, Case (b) a = 3900 ft/s, Case (c) a = 3850 ft/s.
8.10 Pipe volume change = 0.82%; liquid density change = 0.18%.
8.11 (a) a = 1050 ft/s; (b) p_{max} = 241 lb/in^2; (c) σ_1 = 0 lb/in^2.
8.12 a = 2710 ft/s.
8.13 (**a**) Case (a) a = 960 ft/s, Case (b) a = 960 ft/s, Case (c) a = 860 ft/s; (**b**) percent area change = 0.80%.
8.14 a = 930 ft/s.
8.15 (**a**) Case (a) a = 4160 ft/s, 0.65% error; Case (b) a = 4180 ft/s, 0.69% error; Case (c) a = 4140 ft/s, 0.68% error; (**b**) Case (a) - 0.62%; Case (b) - 0.69%;

Case (c) - 0.70%; (**c**) no, the thin-wall formulas give more conservative results for head increase.

8.16 (**a**) Case (a) a = 2760 ft/s, - 2.5%; Case (b) a = 2790 ft/s, - 2.7%; Case (c) a = 2720 ft/s, - 2.6%; (**c**) in all cases the thin-wall formulas are more conservative.

8.17 a = 930 ft/s.

8.18 a = 4560 ft/s.

8.19 a = 4300 ft/s.

8.20 a = 4600 ft/s.

8.21 a = 4530 ft/s.

8.22 a = 3260 ft/s.

8.23 With stress-free concrete, a = 1780 ft/s; otherwise a = 3720 ft/s.

8.24 a = 3850 ft/s.

8.25 a = 3480 ft/s.

CHAPTER 9

Answers in this chapter may vary, owing to slightly differing input parameters.

9.1 t_c = 0 sec, $(p/\gamma)_{max}$ = 1010 ft at x = 0.1, $(p/\gamma)_{min}$ = - 38 ft at x = 1.0 (impossible);
t_c = 4 sec, $(p/\gamma)_{max}$ = 925 ft at x = 1.0, $(p/\gamma)_{min}$ = - 38 ft at x = 1.0 (impossible);
t_c = 8 sec, $(p/\gamma)_{max}$ = 668 ft at x = 1.0, $(p/\gamma)_{min}$ = 408 ft at x = 1.0;
t_c = 12 sec, $(p/\gamma)_{max}$ = 589 ft at x = 1.0, $(p/\gamma)_{min}$ = 283 ft at x = 1.0.

9.2 Minimum t_c = 7.7 sec.

9.3 (a) Yes, $(p/\gamma)_{min}$ = 2 ft; (b) $(p/\gamma)_{max}$ = 703 ft, $(p/\gamma)_{min}$ = 462 ft.

9.6 (a) $h_p = -2.86\times10^{-5}Q^2 + 0.0085Q + 55$; (c) a = 3980 ft/s;
(d) t_c = 3 sec:
At valve p_{max} = 377 lb/in^2 at 2.92 sec, p_{min} = 3.5 lb/in^2 at 6.36 sec;
At check valve p_{max} = 250 lb/in^2 at 4.24 sec, p_{min} = 140 lb/in^2 at 0 sec;
t_c = 6 sec:
At valve p_{max} = 214 lb/in^2 at 4.24 sec, p_{min} = 3.9 lb/in^2 at 0 sec;
At check valve p_{max} = 188 lb/in^2 at 6.89 sec, p_{min} = 140 lb/in^2 at 0 sec;
t_c = 9 sec:
At valve p_{max} = 206 lb/in^2 at 9.01 sec, p_{min} = 3.9 lb/in^2 at 0 sec;
At check valve p_{max} = 188 lb/in^2 at 9.81 sec, p_{min} = 140 lb/in^2 at 0 sec;
t_c = 12 sec:
At valve p_{max} = 186 lb/in^2 at 11.93 sec, p_{min} = 3.9 lb/in^2 at 0 sec;
At check valve p_{max} = 188 lb/in^2 at 13.26 sec, p_{min} = 140 lb/in^2 at 0 sec.

9.7 (a) $h_p = -1.048\times10^{-6}Q^2 - 0.00938Q + 254$; (c) a = 2790 ft/s;
(d) At valve p_{max} = 233 lb/in^2 at 14.4 sec, p_{min} = 13.0 lb/in^2 at 0 sec;
At check valve p_{max} = 152 lb/in^2 at 19.7 sec, p_{min} = 80 lb/in^2 at 0 sec.

9.8 (a) Q = 3600 gal/min and V_0 = 10.21 ft/s;
(b) $h_p = -1.59\times10^{-6}Q^2 - 0.00730Q + 175$; (c) a = 3720 ft/s;
(g) At valve p_{max} = 277 lb/in^2 at 28.1 sec, p_{min} = 17 lb/in^2 at 0 sec;
At pump p_{max} = 124 lb/in^2 at 34.1 sec, p_{min} = 77 lb/in^2 at 0 sec.

9.9 (a) $hp = -2.07\times10^{-6}Q^2 - 0.00310Q + 180$;
At valve p_{max} = 185 lb/in^2 at 2.00 sec, p_{min} = 100 lb/in^2 at 0 sec;
At check valve p_{max} = 155 lb/in^2 at 2.57 sec, p_{min} = 101 lb/in^2 at 0 sec.

9.10 (a) $hp = -1.14\times10^{-4}Q^2 + 0.0343Q + 220$;

(c) $t_c = 3$ sec:

At valve p_{max} = 286 lb/in^2 at 1.71 sec, p_{min} = 2.6 lb/in^2 at 0 sec;

At check valve p_{max} = 211 lb/in^2 at 3.75 sec, p_{min} = 70 lb/in^2 at 0 sec;

$t_c = 5$ sec:

At valve p_{max} = 194 lb/in^2 at 5.11 sec, p_{min} = 2.6 lb/in^2 at 0 sec;

At check valve p_{max} = 193 lb/in^2 at 5.97 sec, p_{min} = 70 lb/in^2 at 0 sec;

$t_c = 7$ sec:

At valve $\mathrm{p_{max}}$ = 162 lb/in^2 at 5.11 sec, p_{min} = 2.6 lb/in^2 at 0 sec;

At check valve p_{max} = 134 lb/in^2 at 7.84 sec, p_{min} = 70 lb/in^2 at 0 sec;

9.11 (a) $hp = -\ 1.43\text{x}10^{-4}Q^2 + 0.029Q + 275$;

(c) At valve p_{max} = 144 lb/in^2 at 2.26 sec, p_{min} = 13.9 lb/in^2 at 0 sec;

At check valve p_{max} = 111 lb/in^2 at 4.87 sec, p_{min} = 81 lb/in^2 at 0 sec.

9.12 At valve p_{max} = 129 lb/in^2 at 4.00 sec, p_{min} = 13.9 lb/in^2 at 0 sec;

At check valve p_{max} = 109 lb/in^2 at 4.80 sec, p_{min} = 81 lb/in^2 at 0 sec.

9.13 (a) $hp = -\ 1.176\text{x}10^{-5}Q^2 + 0.00647Q + 72$; (c) (c) 3670 ft/s;

(d) $t_c = 3$ sec:

At valve p_{max} = 357 lb/in^2 at 3.00 sec, p_{min} = 75 lb/in^2 at 0 sec;

At check valve p_{max} = 303 lb/in^2 at 4.36 sec, p_{min} = 109 lb/in^2 at 0 sec;

$t_c = 6$ sec:

At valve p_{max} = 223 lb/in^2 at 3.82 sec, p_{min} = 75 lb/in^2 at 0 sec;

At check valve p_{max} = 153 lb/in^2 at 5.18 sec, p_{min} = 109 lb/in^2 at 0 sec.

9.14 (a) $(p/\gamma)_{max}$ = 800 ft (b) $(p/\gamma)_{min} \approx -$ 28 ft (d) $(p/\gamma)_{max}$ = 800 ft.

CHAPTER 10

Answers in this chapter may vary, owing to slightly differing input parameters.

10.1 p_{max} = 828 lb/in^2 at 30.1 sec at the valve; p_{min} = column separation at 34.6 sec at a point 1670 ft downstream from the upper reservoir.

10.2 (a) p_{max} = 374 lb/in^2; (b) it occurs 600 ft downstream of low point at 20.7 sec; (c) column separation does occur; (d) it occurs at the valve at 24.3 sec.

10.3 t_c = 20 sec: p_{max} = 390 lb/in^2 at 21 sec at low point; p_{min} = column separation at 26 sec at valve;

t_c = 40 sec: p_{max} = 263 lb/in^2 at 41 sec at low point; p_{min} = column separation at 48 sec at valve;

t_c = 60 sec, p_{max} = 221 lb/in^2 at 55 sec at low point; p_{min} = column separation at 69 sec at valve.

10.4 Minimum valve closure time is 13 sec; phase 1 closes to 8% open in 1 sec, and phase 2 completes closure in 12 sec more; p_{max} = 193 lb/in^2, p_{min} = - 11 lb/in^2.

10.5 Minimum valve closure time is 14 sec; phase 1 closes to 1.5% open in 1 sec, and phase 2 completes closure in 13 sec more; p_{max} = 198 lb/in^2, p_{min} = - 12 lb/in^2.

10.6 Minimum valve closure time is 5 sec; phase 1 closes to 20% open in 1 sec, and phase 2 completes closure in 4 sec more; p_{max} = 191 lb/in^2, p_{min} = 3 lb/in^2.

10.7 Valve A: p_{max} = 90 lb/in^2 at low point; p_{min} = 22 lb/in^2 at upper reservoir;

Valve B: p_{max} = 124 lb/in^2 at low point; p_{min} = column separation at 39 sec at valve; valve A is the better choice.

10.8 (a) p_{max} = 374 lb/in^2 at 16 sec at the low point;

p_{min} = column separation at 18 sec near midpoint of 2000 ft line;

(b) p_{max} = 312 lb/in^2 at 16 sec at the low point;
p_{min} = column separation at 18 sec near midpoint of 2000 ft line;
(c) p_{max} = 373 lb/in^2 at 21 sec at the low point;
p_{min} = column separation at 23 sec near midpoint of 2000 ft line;
(d) p_{max} = 276 lb/in^2 at 21 sec at the low point;
p_{min} = column separation at 23 sec near midpoint of 2000 ft line
(e) p_{max} = 312 lb/in^2 at 21 sec at the low point;
p_{min} = column separation at 23 sec near midpoint of 2000 ft line.

10.9 (a) p_{max} = 197 lb/in^2 at 5 sec at the low point;
p_{min} = - 11 lb/in^2 at 19 sec at the valve;
(b) p_{max} = 231 lb/in^2 at 5 sec at the low point;
p_{min} = 5 lb/in^2 at 19 sec at the valve;
(c) p_{max} = 274 lb/in^2 at 5 sec at the low point;
p_{min} = - 8 lb/in^2 at 8 sec at the upstream end of the 2000 ft pipe;
(d) p_{max} = 202 lb/in^2 at 3.5 sec at the low point;
p_{min} = 1 lb/in^{2}at 24 sec at the valve;
(e) p_{max} = 283 lb/in^2 at 3.5 sec at the low point;
p_{min} = - 8 lb/in^2 at 6 sec at the upstream end of the 2000 ft pipe.
With a working pressure of 200 lb/in^2, the only clear-cut option is schedule (a), but schedule (d) is close enough to consider.

CHAPTER 11

Answers in this chapter may vary, owing to slightly differing input parameters.

11.1 Column separation occurs at the midpoint of the first pipe segment at 4 sec. At that time the pump is still turning at 525 rev/min, producing 129 gal/min at a head increase of 77.5 ft.

11.2 Column separation occurs at the midpoint of the first pipe segment at 2.7 sec. At that time the pump is turning at 600 rev/min, producing 263 gal/min at a head increase of 100 ft.

11.3 Column separation occurs at the downstream end of the 2600 ft pipe at 2.19 sec. At that time the pumps were turning at 352 rev/min, producing 994 gal/min at a head increase of 6 ft.

11.4 Column separation occurs at the upstream end of the last pipe at 0.9 sec. At that time the pumps were turning at 1194 rev/min, producing 1300 gal/min at a head increase of 329 ft.

11.5 Column separation occurs 2700 ft downstream in the 4000 ft pipe at 1.6 sec. At that time the pumps were turning at 760 rev/min, producing 564 gal/min at a head increase of 117 ft.

11.6 p_{max} = 162 lb/in^2 at 12.6 sec at the pump discharge; p_{min} = 8.5 lb/in^2 at 2.2 sec near the downstream reservoir.

11.7 p_{max} = 55 lb/in^2 occurs under steady flow conditions at the pump discharge; p_{min} = 10 lb/in^2 at 7 sec 3700 ft downstream of the pumps.

11.8 Vapor pressure is reached throughout most of the pipeline, but the largest cavity appears at the “knee” at elevation 1350 ft. Cavity volume reaches 34 ft^3 at 17 sec but has collapsed by 29 sec. The maximum pressure of 415 lb/in^2 occurs at the pump discharge at 44 sec.

11.9 Vapor pressure is reached throughout most of the pipeline, except at the “knee” where an air-vacuum valve causes a 151 ft^3 air cavity to form. The air cavity begins to form at 3 sec, reaches its maximum size at 59 sec, and finally is exhausted at 135 sec. The maximum pressure of 435 lb/in^2 occurs at the pump discharge at 11 sec.

11.10 Vapor pressure is reached throughout most of the pipeline, but the largest cavities are at the "knees" at elevations 4440 ft and 4470 ft. Maximum cavity volumes reach 111 ft^3 and 34 ft^3 at 34 sec and 22 sec, respectively. By 45 sec, major cavity formation has ceased. The maximum pressure of 364 lb/in^2 occurs at the pump discharge at 66 sec.

11.11 Vapor pressure is reached throughout the 2000 ft pipe but is not extensive owing to the air-vacuum valve at the "knee". At that point a 58 ft^3 air cavity forms, reaching maximum size at 5 sec. The air cavity continues to form and exhaust periodically until its size is down to 10 ft^3 at 60 sec. The maximum pressure of 492 lb/in^2 occurs at the pump discharge at 10 sec.

11.12 Vapor pressure is reached throughout the downstream half of the pipeline; however, the presence of the air-vacuum valve at the "knee" prevents the formation of large vapor cavities. The air cavity reaches a maximum volume of 66 ft^3 at 6 sec, reforming several more times until dying out completely by 50 sec. The maximum pressure of 478 lb/in^2 is at the pump discharge at 15 sec.

CHAPTER 12

Answers in this chapter may vary, owing to slightly differing input parameters.

12.2 (b) $Q_3 = 0.74$ ft^3/s, $H_2 = 72.11$ ft.

12.4 (a) One real loop equation, $Q_2 = 2.13$ ft^3/s, $H_2 = 98.81$ ft; (b) 5 continuity equations + 6 ODE's = 11 equations; (d) at $t = 5$ sec $Q_2 = 2.98$ ft^3/s, $H_2 = 97.94$ ft.

12.5 (a) $Q_5 = 0.17$ ft^3/s, $H_4 = 194.98$ ft; (b) 6 ODE's and a total of 11 equations; (c) at $t = 6$ sec $Q_5 = 0.49$ ft^3/s, $H_4 = 154.48$ ft.

12.6 (a) $Q_2 = 3.29$ ft^3/s, $H_2 = 452.74$ ft; (b) 6 ODE's and a total of 10 equations; (c) at $t = 5$ sec $Q_2 = 3.21$ ft^3/s, $H_2 = 448.49$ ft.

12.7 (a) Four equations, plus 3 Colebrook-White equations; (b) one continuity equation and 3 ODE's; $H_1(0) = 64.3$ ft, $H_1(4) = 156.3$ ft, $H_1(15) = 89.0$ ft.

12.12 At $t = 0$ $Q_1 = Q_3 = 0.248$ ft^3/s, $Q_2 = 0$; $H_1 = 34.55$ ft; at $t = 3.5$ sec $Q_1 = 0.160$ ft^3/s, $Q_2 = 0.127$ ft^3/s, $Q_3 = 0.033$ ft^3/s, $H_1 = 51.54$ ft, $z = 62.33$ ft, $H_2 = 61.32$ ft.

12.13 (a) $Q_1 = 2.62$, $Q_2 = 1.22$, $Q_3 = 3.66$, $Q_4 = 7.50$, all in ft^3/s; $H_1 = 68.81$ ft, $H_2 = 38.30$ ft; (b) at $t = 5$ sec $Q_1 = 2.26$, $Q_2 = 1.04$, $Q_3 = 1.90$, $Q_4 = 3.49$, all in ft^3/s, $H_1 = 85.11$ ft; (c) at $t = 5$ sec $V_1 = 0.243$ ft/s, $V_2 = -1.047$ ft/s, $V_3 = 0.222$ ft/s, $V_4 = 0$, $H_1 = 201.4$ ft, $H_2 = 698.1$ ft; at $t = 30$ sec $V_1 = 1.763$ ft/s, $V_2 = 0.589$ ft/s, $V_3 = -1.340$ ft/s, $V_4 = 0$, $H_1 = H_2 = 85.1$ ft.

12.15 (a) $Q_1 = 2.78$ ft^3/s, $Q_5 = 1.38$ ft^3/s, $H_3 = 479.1$ ft; (e) at $t = 30$ sec $Q_1 = 1.34$, $Q_2 = 0.73$, $Q_3 = -0.88$, $Q_4 = -0.97$, $Q_5 = 3.66$, all in ft^3/s; $H_1 = 493.7$ ft, $H_2 = 488.1$ ft, $H_3 = 498.7$ ft; at $t = 100$ sec $Q_1 = 1.42$, $Q_2 = 0.75$, $Q_3 = -0.83$, $Q_4 = -0.95$, $Q_5 = 3.58$, all in ft^3/s; $H_1 = 497.3$ ft, $H_2 = 494.2$ ft, $H_3 = 502.2$ ft.

12.16 (a) $H_1 = 3.85$ ft; (b) $p_{air} = 240$ lb/ft^2, $\rho = 0.0144$ $slugs/ft^3$; (d) at $t = 3$ sec $Q_1 = 1.16$ ft^3/s, $Q_2 = 0.61$ ft^3/s, $Q_3 = 0.54$ ft^3/s, $H_1 = 86.5$ ft, $H_2 = 86.4$ ft, $H_3 = 38.7$ ft, Volume = 50.83 ft^3, $x = 0.042$ ft; at $t = 37.5$ sec $Q_1 = Q_3 = 0.045$ ft^3/s, $Q_2 = 0$, $H_1 = 58.2$ ft, $H_2 = 58.2$ ft, $H_3 = 60.7$ ft, Volume = 64.09 ft^3, $x = 0.704$ ft.

12.18 At t = 10 sec $Q_1 = 3.44$, $Q_2 = 0.68$, $Q_3 = 0.55$, $Q_4 = 1.83$, $Q_5 = 1.44$, $Q_6 = 3.48$, $Q_7 = 3.26$, $Q_8 = 0.22$, all in ft^3/s; $z = 124.1$, $H_1 = 111.7$, $H_2 = 111.2$, $H_3 = 111.3$, $H_4 = 126.9$, $H_5 = 139.1$, all in ft.

12.19 At $t = 12.5$ sec $Q_1 = 2.52$, $Q_2 = -0.71$, $Q_3 = 1.02$, $Q_4 = 0.71$, $Q_5 = 0.98$, $Q_6 = 3.69$, $Q_7 = 3.48$, $Q_8 = 0.21$, all in ft^3/s; $z = 124.1$, $H_1 = 135.1$, $H_2 = 143.5$, $H_3 = 131.5$, $H_4 = 132.0$, $H_5 = 137.4$, all in ft.

12.25 (a) $Q_1 = 823$, $Q_8 = 170$, $Q_{14} = 110$, $Q_{18} = 99$, all in gal/min; $p_{12} = 81.7$ lb/in^2; (b) column separation occurs in pipe 4 one third of the distance from the upstream end at 14 sec.

12.26 Column separation occurs instantaneously at node 1.

12.27 Max. pressure head = 298 ft in pipe 7 at upstream side of closed valve at 2.0 sec; Min. pressure head = 47 ft at the same location at 10.7 sec.

12.28 (a) $Q_7 = 3910$, $Q_{24} = 1535$, $Q_{77} = 80$, $Q_{555} = 1040$, all in gal/min; pressure at node 500 = 76.3 lb/in^2; (b) Max. pressure head = steady state value of 190 ft at node 99, Min. pressure head = steady state value of 20 ft at the reservoir at the upstream end of pipe 7.

12.29 Max. pressure = 388 lb/in^2 at the upstream side of the closed valve at the instant of closure; Min. pressure causes column separation at the downstream side of the valve at the same time.

12.30 (a) $Q_7 = 3620$, $Q_{24} = 1390$, $Q_{77} = 390$, $Q_{555} = 970$, all in gal/min; pressure at node 500 = 77.8 lb/in^2; (b) Max. pressure head = 194 ft near midpoint of pipe 88 at 11 sec; Min. pressure head = the steady state value of 20 ft at the reservoir at the upstream end of pipe 7.

12.31 Column separation occurs at the downstream end of pipe 88 at 3.3 sec. The max. pressure head before this time is 493 ft at the same location at 1.1 sec.

12.32 (a) $Q_1 = 940$, $Q_4 = 1033$, $Q_8 = 890$, $Q_{14} = 407$, all in gal/min; pressure at node 5 = 43.2 lb/in^2; (b) column separation occurs at the upstream end of pipe 1 at 2.0 sec. The max. pressure head before this is the steady state value of 110 ft at the pump discharge.

12.33 Max. pressure head = 197 ft at the downstream end of pipe 14 at 0.37 sec; Min. pressure head = - 19 ft near the downstream end of pipe 6 at 2.7 sec.

CHAPTER 13

Answers in this chapter may vary, owing to slightly differing input parameters.

13.1 Approximately 310 ft^3, including a safety factor.

13.2 (a) About 935 ft^3, including a safety factor; (b) an air chamber of approximately 450 ft^3 and a one-way surge tank 8 ft in diameter and 25 ft tall.

13.3 (a) Approximately 630 ft^3, including a safety factor; (b) there is no practical means of accomplishing this design objective; (c) an air chamber of approximately 210 ft^3 and a one-way surge tank 6 ft in diameter and 15 ft tall.

13.4 Approximately 950 ft^3, including a safety factor.

13.5 The air chamber size is approximately 1036 ft^3; a one-way surge tank 12 ft in diameter and 35 ft tall is needed at the first summit, with another 10 ft in diameter and 20 ft tall at the second summit.

13.6 Approximately 52 ft^3, including safety factor.

13.7 $Wr^2 = 460$ lb-ft^2 to prevent column separation; $Wr^2 = 795$ lb-ft^2 to prevent negative pressures.